(2) GIVEN ONE POINT ON THE LINE OF INTERSECTION

Given the coordinates of two planes ABC and ADE. Determine the line of intersection and the dihedral angle between the planes. Fig. 9.79.

Point A is a shared point and therefore must be on the line of intersection.

1) Draw a horizontal cutting plane in elevation to cut both planes. In Fig. 9.79 this cutting plane gives points 1 and 2 on plane ABC and points D and 3 on plane ADE.

2) Find these points in plan giving lines 1, 2 and D, 3. Where these two lines cross is a point on the line of intersection, i.e. a shared point. Thus find the line of intersection Ai in plan and elevation. The line of intersection must stop when it hits the edge of a plane. The dihedral angle follows as before.

(3) GIVEN NO POINTS ON THE LINE OF INTERSECTION

Given the coordinates of two planes ABC and DEF to find the line of intersection and the dihedral angle between the planes. Fig. 9.80

\[
\begin{align*}
A &= 140 5 80 & B &= 95 90 25 & C &= 30 25 55 \\
D &= 80 10 15 & E &= 130 55 95 & F &= 20 70 35
\end{align*}
\]

This method is the same as in the previous example except that two separate horizontal cutting planes are used.

These horizontal cutting planes can be drawn at any level as long as they cut both planes.
**Note 1:** Line E,3 in plan will be parallel to 4,5. Also line 1,2 will be parallel to C,6 in plan.

**Note 2:** If the lines do not intersect they are extended until they do intersect.

---

**To draw the shortest horizontal line to a plane from a given point P outside it, Fig. 9.81.**

\[
\begin{align*}
A &= 75 & 75 & 10 & B &= 20 & 50 & 90 \\
C &= 40 & 5 & 115 & D &= 95 & 20 & 45 \\
P &= 110 & 45 & 80
\end{align*}
\]

1. Draw an auxiliary showing the plane as an edge view. Project P onto this view.
2. In the auxiliary draw the horizontal line from P to hit the plane at i. Project i to plan.
3. Since the horizontal line from P to i in plan will be seen as a true length, then the line iP in plan must be the shortest distance from P to the line projected from the auxiliary elevation. Draw from P perpendicular to the projection lines to the auxiliary to find i.
4. iP will be horizontal in elevation.

---

**To draw the perpendicular to a plane ABC from a point P outside it, Fig. 9.82**

\[
\begin{align*}
A &= 120 & 88 & 2 & B &= 50 & 50 & 94 \\
C &= 160 & 20 & 84 & P &= 148 & 76 & 96
\end{align*}
\]

1. Draw an auxiliary elevation showing the plane as an edge view and project point P onto this view.
2. In the auxiliary, draw the perpendicular from P to the plane finding point q.
3. It should be noted that a perpendicular to a plane will appear perpendicular to the traces of that plane. We can therefore draw the required line PQ in plan as it will appear perpendicular to the level line on the plane. The level line will be parallel to the HT line.
4. Line PQ can be found in elevation as shown in Fig. 9.82.
(1) This problem is solved under the principle that any line generator on the surface of a right cone will have the same inclination to the horizontal plane as the cone base angle. Furthermore all generators on a right cone will have the same length.

Draw an auxiliary showing ABCD as an edge view. Project point E onto this view.

(2) Draw a cone in this view having E as apex, base angle of 45° and side length of 90 mm.

(3) Where the base of the cone is cut by the plane will give the required line when projected back.

To find the angle that a line PC makes with a given plane ABC. Fig. 9.84

A = 45 14 66  B = 56 96 10  
C = 128 40 33  P = 105 72 56

(1) Draw an auxiliary elevation showing an edge view of the plane.

(2) Project a true shape of this plane by viewing the edge view at 90°.

(3) Find the line PC on both of these views. If PC is used as a generator of a cone with P as vertex then the base angle of that cone will equal the angle the line PC makes with the plane.

(4) With PC as radius and P as centre draw the plan of this cone in the second auxiliary plan. The edges of this circle produced back to the first auxiliary will give the required angle.
Given a plane ABC and a point E outside it. To draw a line from E that is 40 mm long, is parallel to the plane ABC and the vertical plane. Fig. 9.85

A = 110 15 105  B = 166 85  20  C = 50 70 85  E = 65 100 30

(1) With E as centre draw a disc in elevation of 40 mm radius. The disc will be parallel to the vertical plane. All radii will be 40 mm long and parallel to the VP.

(2) Project an auxiliary elevation of the plane showing it as an edge view. Project the disc onto this auxiliary.

(3) In the auxiliary the line EP can be easily found which is parallel to ABC.

(4) Project EP back to plan and elevation. It satisfies all parameters.

Given the coordinates of a plane ABC. Draw the projections of a line on the plane ABC, that passes through A and makes an angle of 60° with the edge BC. Fig. 9.86

A = 125 85 30  B = 170 10 100  C = 80 75 65

(1) Draw the plan and elevation of the plane.

(2) Get an edge view of the plane in the usual way.

(3) By viewing perpendicular to the edge view we can project a true shape of the lamina.

(4) On the true shape draw the required line. Project point P back through the views to find the line in plan and elevation as shown in Fig. 9.86.
Skew Lines

Definition: Two lines are called skew lines if they are neither parallel nor intersecting.

In many practical areas of engineering, the shortest level distance between skew lines or the shortest perpendicular distances between skew lines, is often required. For example, in pipework, mining, structural frames etc., it is often necessary to connect two skew pipes, with another new pipe; two mining shafts, with another new tunnel; or two skew members of a frame with another new member. In cases like these it is of great advantage to know the shortest horizontal distance, or the shortest perpendicular distance, between the two elements. At a later stage we will apply the principles learned here about skew lines, to solve problems on mining and on the hyperbolic paraboloid.
If we produce a plane that contains one of the lines and has an edge that is parallel to the other line, then an edge view of that plane will show both lines as parallel.

1. Draw the plane to contain AB and be parallel to CD. Draw a level line from A in elevation. From B draw a line parallel to the other skew line CD. These two lines intersect at O. This completes the plane in elevation.

2. Drop O to plan.

3. From B in plan draw a line parallel to CD in plan. This line intersects the line dropped from O in elevation to give point O in plan.

4. Join O back to A thus completing the plan of the plane.

5. An auxiliary elevation viewing along AO will show both lines as parallel.

6. Project a second auxiliary plan by projecting horizontally, i.e. parallel to the $x,y$. Both lines appear to cross. Where they appear to cross is the location of the shortest horizontal line.

7. Project the line back through the views as shown.
(1) The initial part of this problem is solved in the same way as the previous example up to the stage of projecting an auxiliary showing the two lines appearing parallel.

(2) A view is taken of this auxiliary which is perpendicular to the two parallel lines.

(3) The second auxiliary shows the two lines appearing to cross. Where they appear to cross is the location of the required line. The line is projected back to all views.

**Alternative Method: Line Method**

Given the same problem,

(1) Project an auxiliary from plan that will show one of the lines as a true length. Fig. 9.90 shows $x_1y_1$ drawn parallel to CD thus showing line CD as a true length in the auxiliary elevation.

(2) Project a second auxiliary viewing along the true length line. This new auxiliary shows CD as a point view.

(3) The shortest line between two skew lines will always appear as a true length in a view that shows one of the lines as a true length. When projected back to the first auxiliary the shortest line must therefore be parallel to the $x_2y_2$ line.

(4) Project back to all views.
Given the coordinates of the centre lines of two 15 mm diameter pipes. Determine the clearance between them using the line method. Fig. 9.91

\[
\begin{align*}
A &= 125.55.80 & B &= 175.10.10 \\
C &= 100.25.10 & D &= 165.75.50
\end{align*}
\]

The construction is as in the previous example.
The pipes need only be drawn in the secondary auxiliary view.

1. Draw the plan and elevation of the centre lines.
2. Find the true length of one of these, e.g. CD, by auxiliary projection.
3. Project a second auxiliary viewing along the true length CD.
4. A point view of centre line CD is found. Draw in the pipe details which will show clearly the clearance between the pipes.

Fig. 9.92

1. Using the plane method project the auxiliary which shows the two lines appearing parallel.
2. View parallel to \(x_Oy_O\) (horizontally) for the second auxiliary. The lines appear to cross which is the location of the required brace.
3. Project back through the views.

Given the coordinates of two struts. Show the projections of the shortest horizontal brace strut between them. Fig. 9.92

\[
\begin{align*}
A &= 30.8.16 & B &= 80.70.44 \\
C &= 38.50.7 & D &= 90.8.10
\end{align*}
\]
The construction is nearly identical to that described for shortest perpendicular or shortest horizontal distance between two skew lines. The first auxiliary must be viewed at 30° to the x₁y₁. The resulting auxiliary on x₂y₂ shows the lines crossing thus showing where the required line is located.

**Activities**

**DIHEDRAL ANGLE**

Q1. Given two planes VTH and V₁T₁H₁, Find the line of intersection between the planes. Determine the dihedral angle between the planes using the triangle method, Fig. 9.94.

Q2. Given two planes VTH and V₁T₁H₁, Find the line of intersection between the planes. Determine the dihedral angle between the planes using the point view method, Fig. 9.95.