9 The Oblique Plane

SYLLABUS OUTLINE

Areas to be studied:

• Definition of planes, simply inclined and oblique. • Determination of oblique and *tangent* planes. • True shape and inclinations of planes to principal planes of reference. • Intersection of oblique planes, lines and *dihedral angle.* • Sectioning of right solids by oblique planes. • *Treatment of planes as laminar surfaces given rectangular coordinates.* • *Properties and projections of skew lines.* • *Spatial relationships between lines and planes.*

Learning outcomes

Students should be able to:

Higher and Ordinary levels

- Distinguish between simply inclined and obliquely inclined plane surfaces.
- Determine the angle of inclination between given planes and the principal planes of reference.
- Determine the true length and inclination of given lines.
- Establish the true shape of an obliquely inclined plane.
- Determine the line of intersection between two planes.
- Determine the projections and true shape of sections of solids resulting from simply inclined and oblique cutting planes.

Higher level only

- *Construct obliquely inclined planes given the angles of inclination to the principal planes of reference and to include a given line or point.*
- *Establish the dihedral angle between two intersecting planes.*
- *Display knowledge of the relationships between planes and lines.*
- *Understand the concept of a laminar surface defined by spatial coordinates.*
- *Solve a variety of problems involving the intersection, inclination and positioning of laminar plane surfaces.*
- *Define the concept of skew lines and their use in solving practical problems.*
- *Establish various spatial relationships between skew lines and other lines and planes, including distance, inclination and direction.*

A plane is usually represented by its lines of intersection with the principle planes of reference. These lines are called the **traces** of the plane. The line of intersection between the plane and the vertical plane is called the **vertical trace** and the line of intersection between the plane and horizontal plane is called the **horizontal trace**.

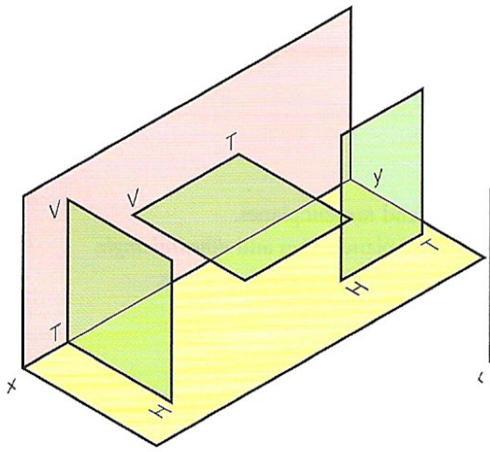

Fig. 9.1

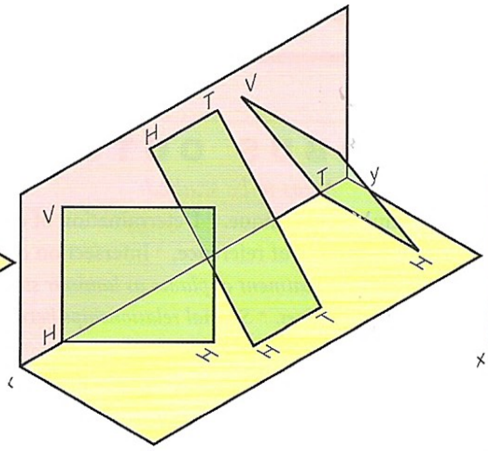

Fig. 9.2

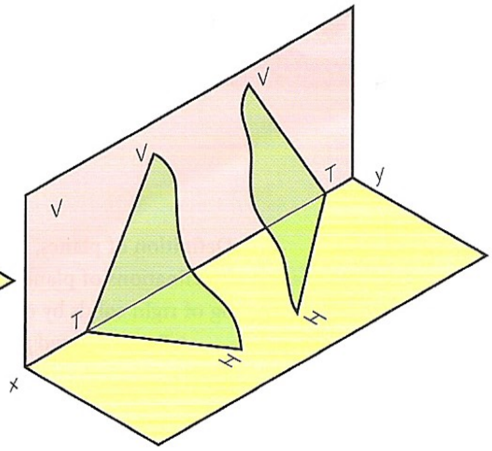

Fig. 9.3

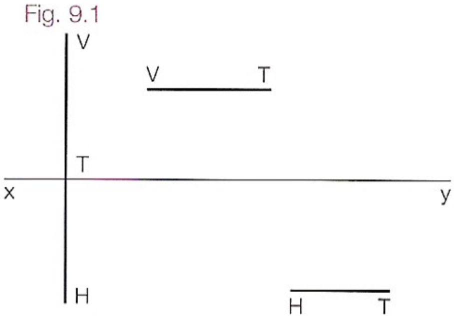

Fig. 9.1a

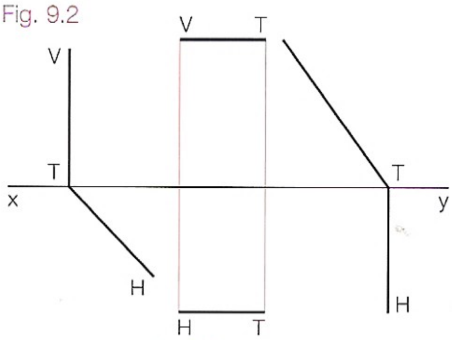

Fig. 9.2a

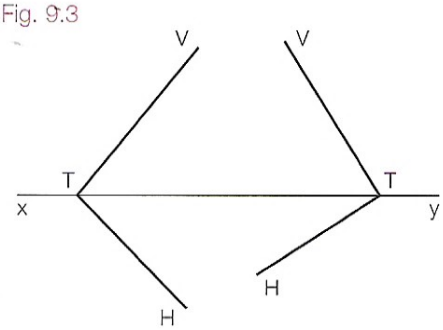

Fig. 9.3 a

Given the plan of a point P on an oblique plane VTH. Find the elevation of this point. Fig. 9.4

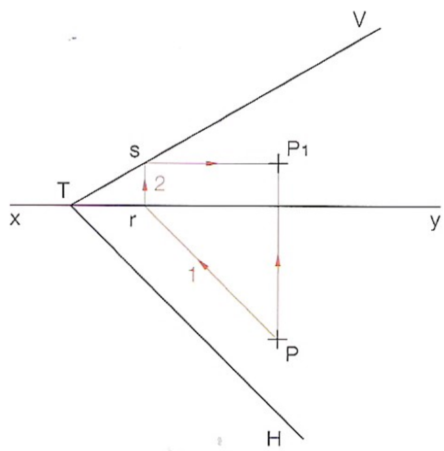

Fig. 9.4

(1) Draw a line parallel to the horizontal trace from point P to hit the xy line at r.

(2) Erect a perpendicular at r to hit the vertical trace at s.

(3) From s, project horizontally (parallel to the xy line).

(4) Project vertically from P in plan to intersect this horizontal thus locating P_1 in elevation, Fig. 9.5.

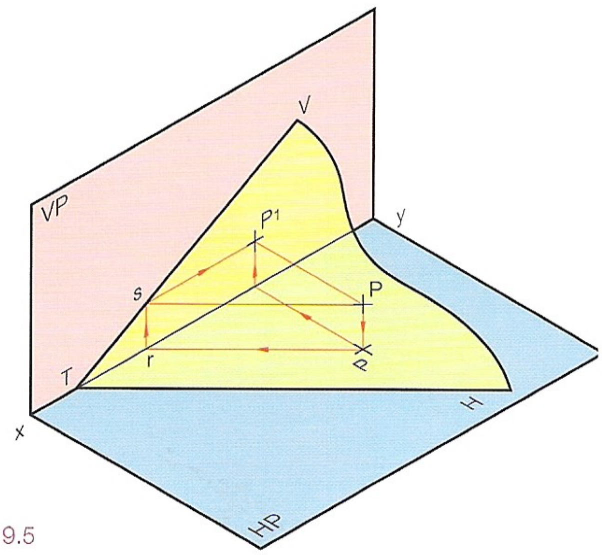

Fig. 9.5

Given the elevation of a line AB on an oblique plane VTH. Find its plan. Fig. 9.6

(1) From A, project parallel to the VT line to meet the xy line at r.
(2) Project vertically down from r to s on the HT line.
(3) Project s parallel to the xy line.
(4) Drop a perpendicular from A in elevation to find A_1.
(5) Point B is done in a similar manner, Fig. 9.7.

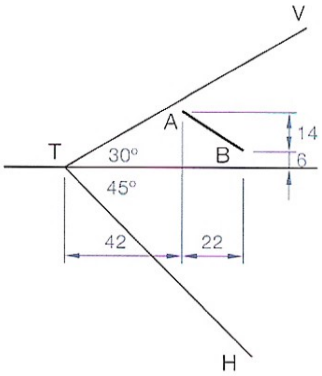

Fig. 9.6

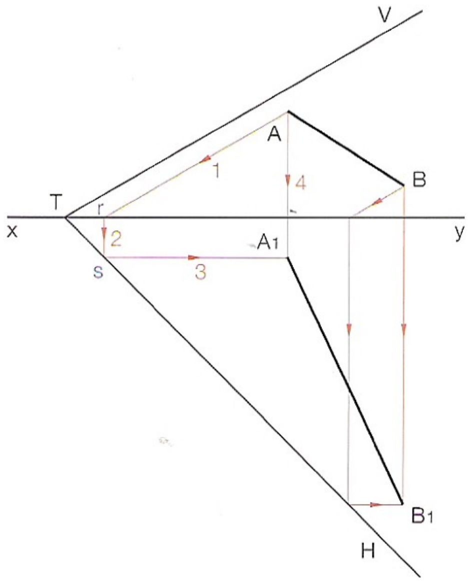

Fig. 9.7

Lines drawn parallel to the vertical trace in elevation will be parallel to the xy line in plan.

Lines drawn parallel to the horizontal trace in plan will be parallel to the xy line in elevation.

Given the plan of a lamina ABC on an oblique plane VTH. In plan AB = 32 mm, BC = 55 mm and AC = 44 mm. Find the elevation. Fig. 9.8

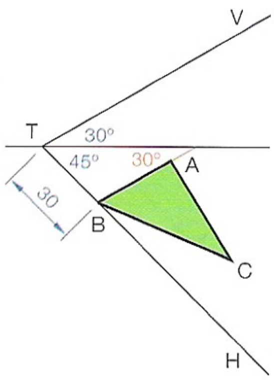

Fig. 9.8

(1) Each point is brought up as in previous examples and found in elevation.
(2) Index each point and join finding $A_1B_1C_1$ in elevation, Fig. 9.9.

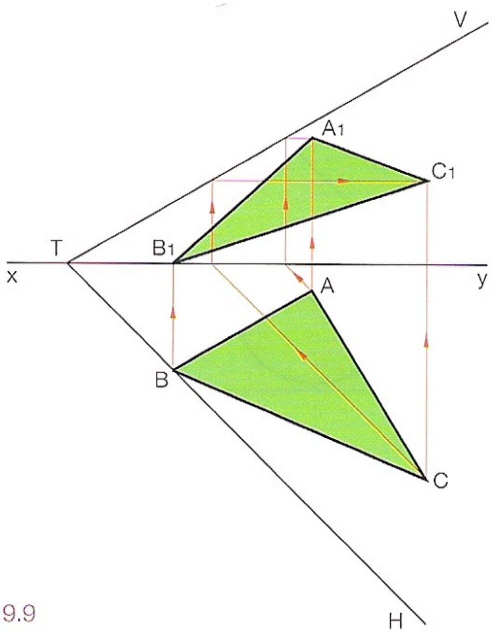

Fig. 9.9

Point B is on the HT. Since the horizontal trace is the line of intersection between the oblique plane and the horizontal trace, every point on the HT will also be on the horizontal trace.

Given the plan of a figure resting on the oblique plane VTH. Find its elevation. Fig. 9.10

The construction is the same as in previous examples.

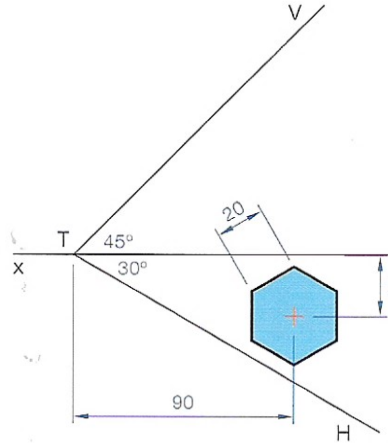

Fig. 9.10

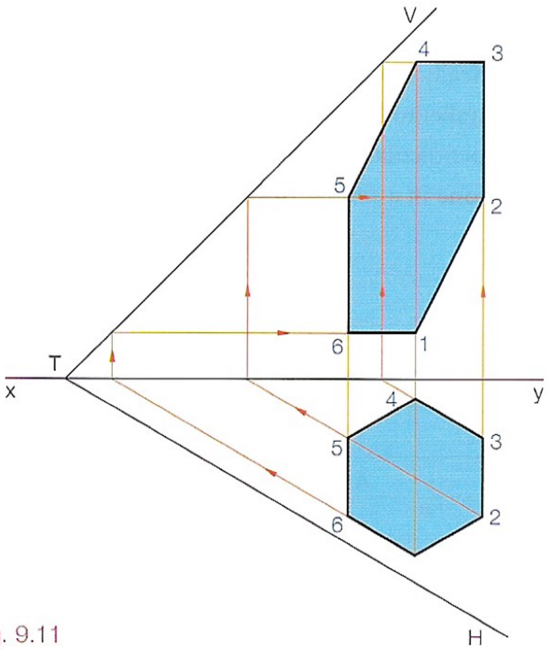

Fig. 9.11

Given the elevation of a figure resting on the oblique plane VTH. Find its plan. Fig. 9.12

Since this figure is not made up of straight lines we must choose points on the curve. In Fig. 9.13 we have chosen eight points. Once the points are found in plan they are joined freehand with a fair curve, Fig. 9.13.

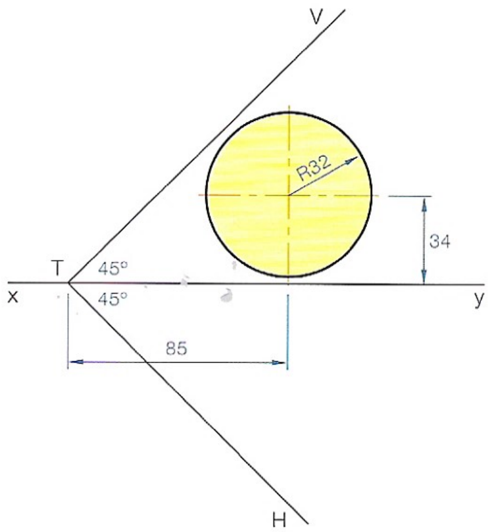

Fig. 9.12

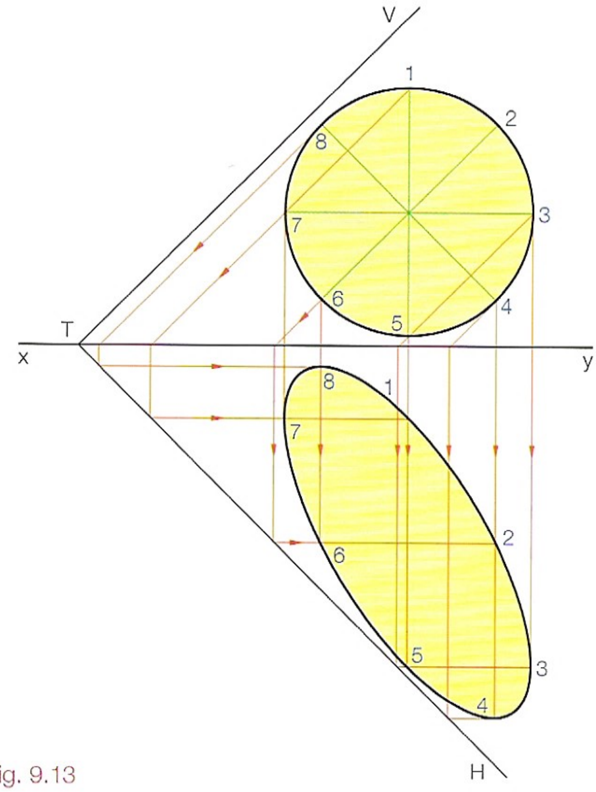

Fig. 9.13

True Inclination of an Oblique Plane to the VP and the HP

To find the true inclination of an oblique plane to the HP. Fig. 9.14

To find the true angle, insert a half cone underneath the plane and tangential to it. The base angle of this cone will be the true angle of the plane to the horizontal plane.

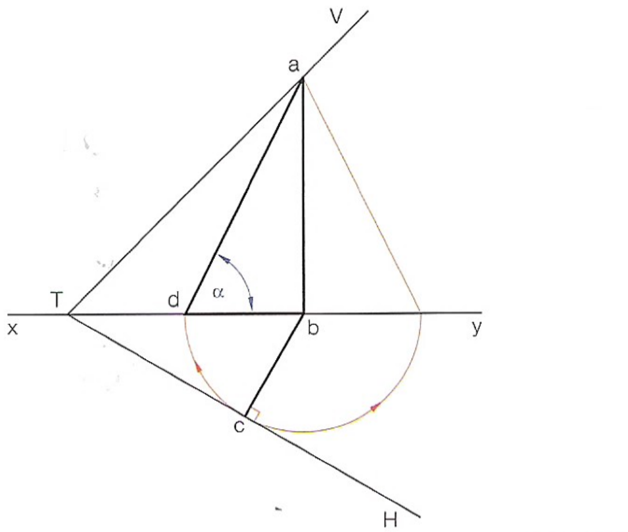

Fig. 9.14

(1) Draw a vertical line ab in elevation.

(2) From b on the xy line, draw a perpendicular to the HT finding c.

(3) Line bc equals the radius of the cone. Draw the half cone in plan.

(4) Draw the elevation of the cone. The base angle of the cone is the true angle of the plane to the horizontal plane, Fig. 9.15.

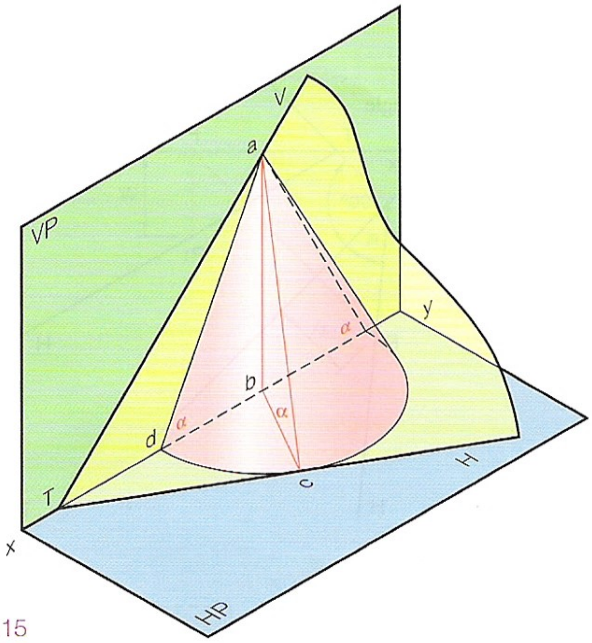

Fig. 9.15

Alternative Method

The true angle an oblique plane makes with the horizontal plane can be found by using an auxiliary elevation. Fig. 9.16

Note: The point P on the oblique plane is usually chosen at position A as it reduces the amount of projection involved.

(1) Choose any point P on the plane in plan and project it to elevation.

(2) Take an auxiliary elevation viewing along the horizontal trace.

(3) x_1y_1 is drawn perpendicular to the line of sight.

(4) The horizontal trace projects as a point view.

(5) Point P is projected to the auxiliary elevation and its height h is taken from the elevation.

(6) The oblique plane is seen as an edge view showing its true angle to the horizontal plane.

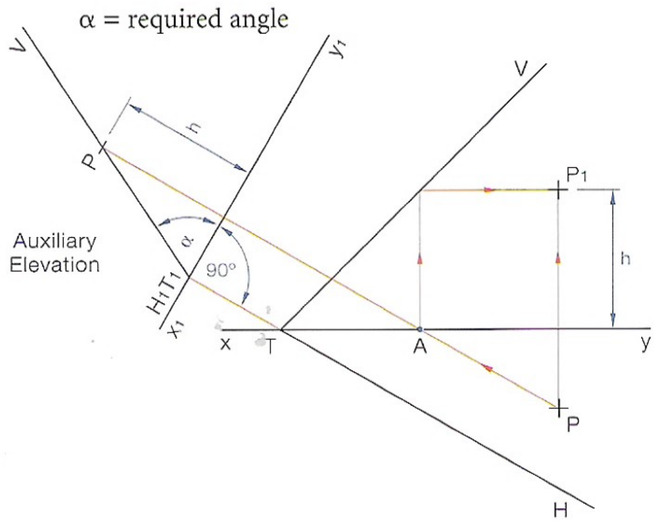

Fig. 9.16

To find the true inclination of an oblique plane to the vertical plane. Fig. 9.17

The construction is similar to Figures 9.14 and 9.15.

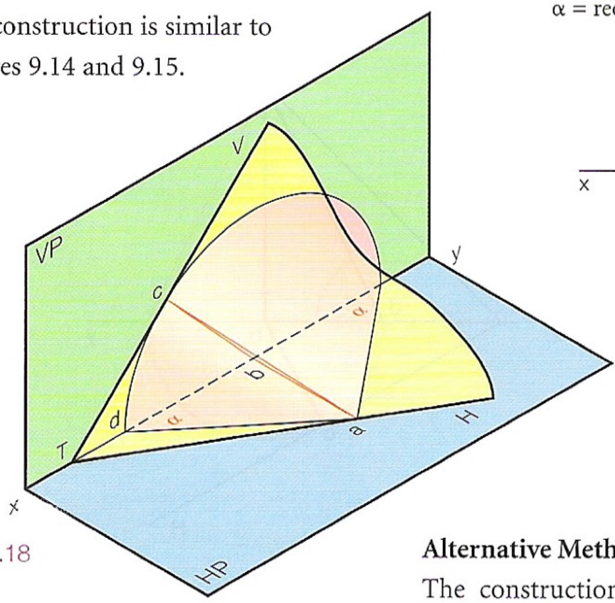

Fig. 9.18

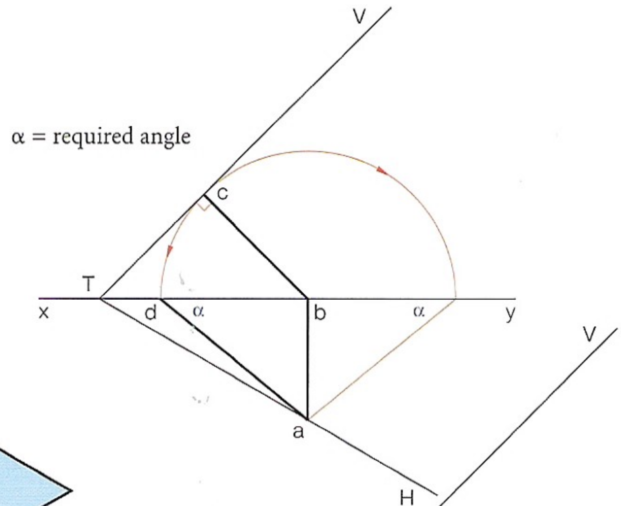

Fig. 9.17

Alternative Method

The construction is similar to that for Fig. 9.16. Point P is chosen on the xy line. View along the VT line.

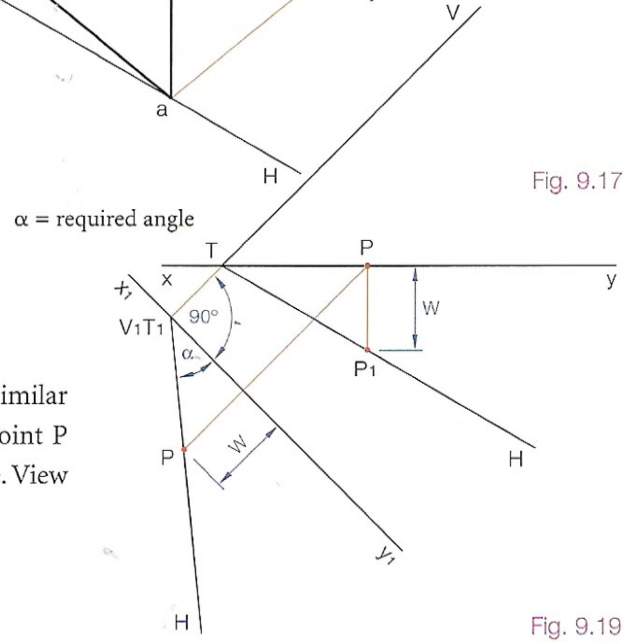

Fig. 9.19

Solids Cut by Oblique Planes

Given the plan and elevation of a square-based prism and the traces of a plane VTH. Show the projections of the prism when it is cut by the oblique plane VTH. Fig. 9.20

The prism when cut will still look the same in the plan. The cut surface in plan can be projected to elevation as has been explained earlier, Fig. 9.21.

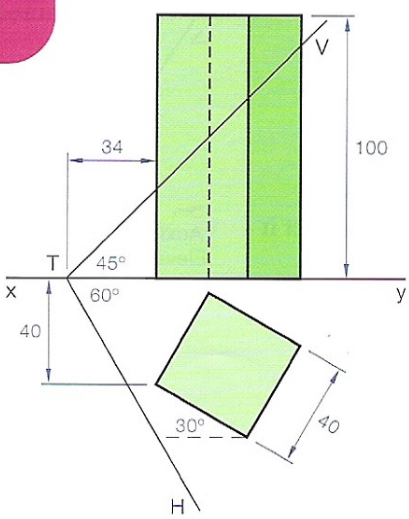

Fig. 9.20

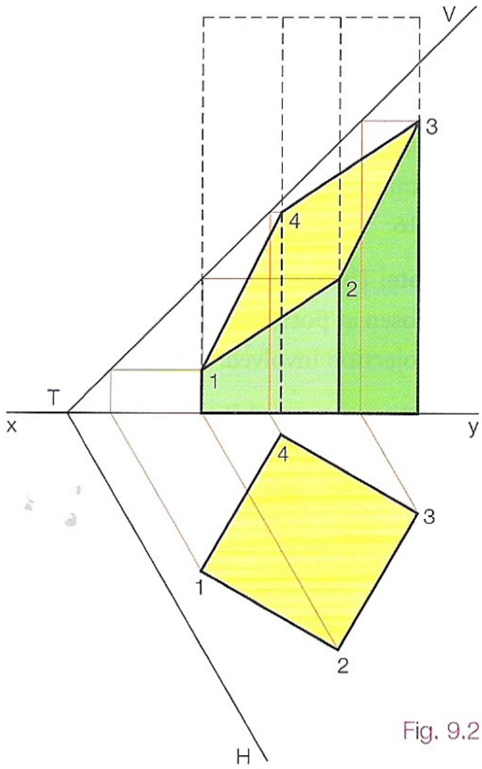

Fig. 9.21

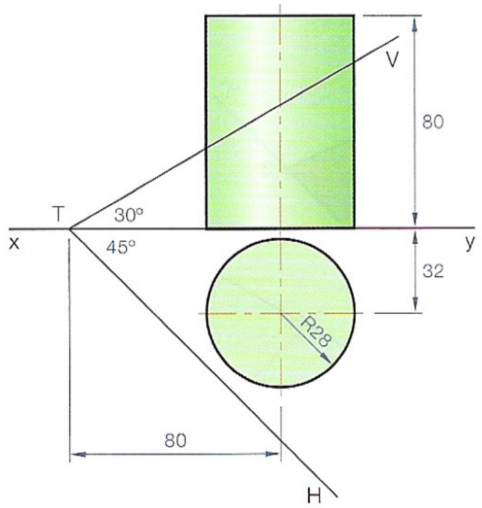

Fig. 9.22

Given the plan and elevation of a cylinder and the traces of a plane VTH. Show the projections of the cylinder when it is cut by the oblique plane VTH. Fig. 9.22

As in the last example, the plan of the solid is unchanged after it has been cut by the plane. Divide the plan up into a number of divisions. Index the points and project each point to elevation as usual, Fig. 9.23.

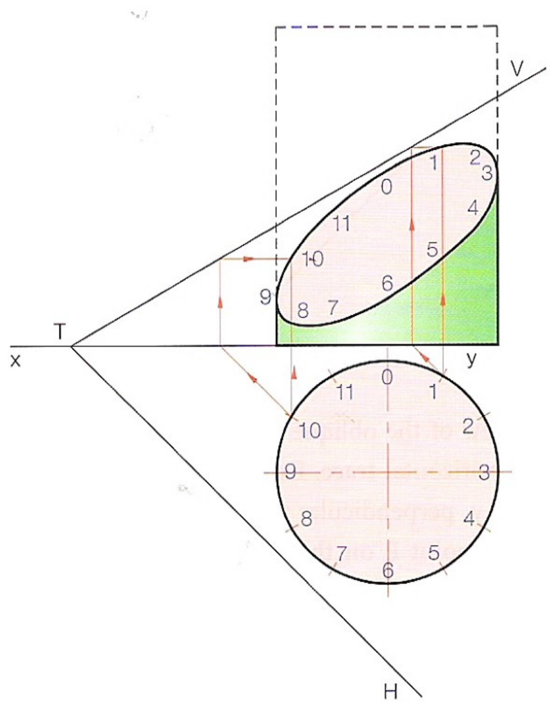

Fig. 9.23

Given the plan and elevation of a square-based pyramid and the traces of a plane VTH. Draw the projections of the solid after it has been cut by the oblique plane. Fig. 9.24

The plan does not show the cut surface in this case. This type of problem is solved by getting an edge view of the oblique plane and hence determining the cut points on the solid.

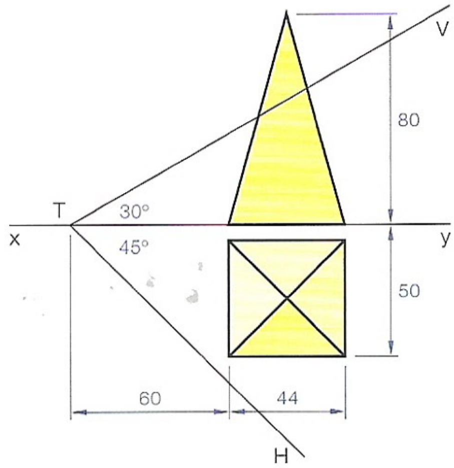

Fig. 9.24

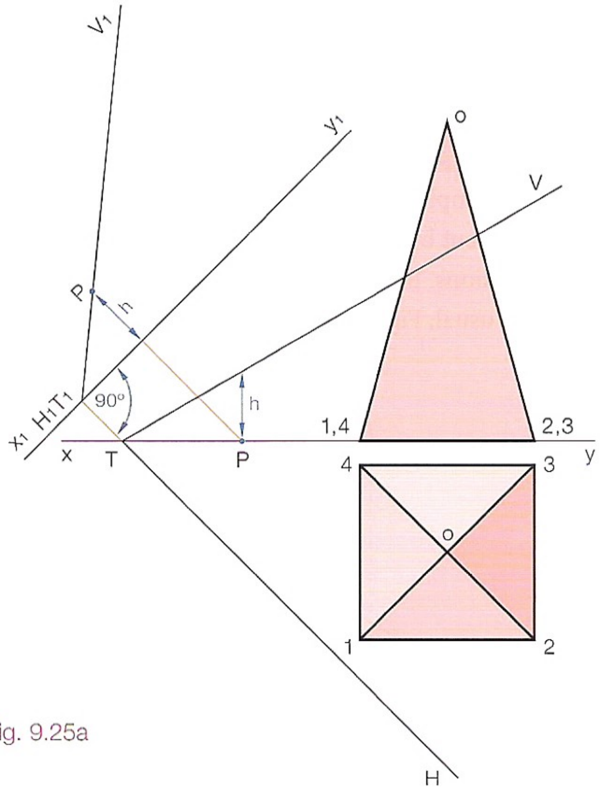

Fig. 9.25a

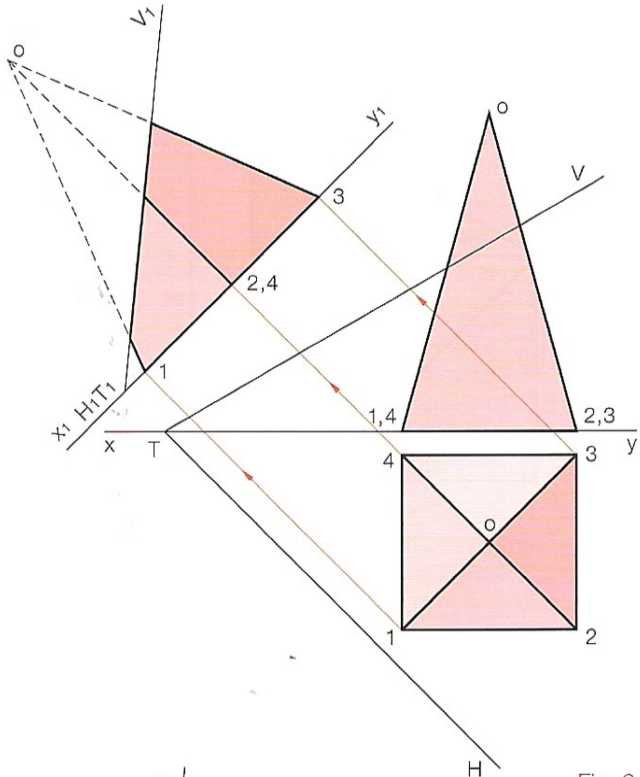

Fig. 9.25b

(1) An edge view of the oblique plane is found by viewing along the horizontal trace. Extend the horizontal trace. Draw the x_1y_1 perpendicular to HT, Fig. 9.25a.

(2) Choose any point P on the xy line and project to the elevation giving height h.

(3) Project P to the auxiliary elevation and use the height h. Draw the edge view of the plane.

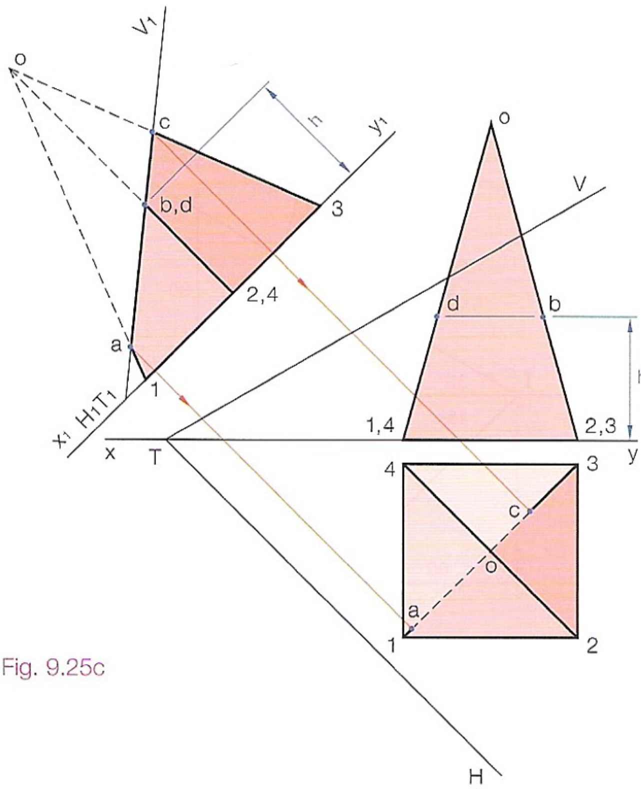

Fig. 9.25c

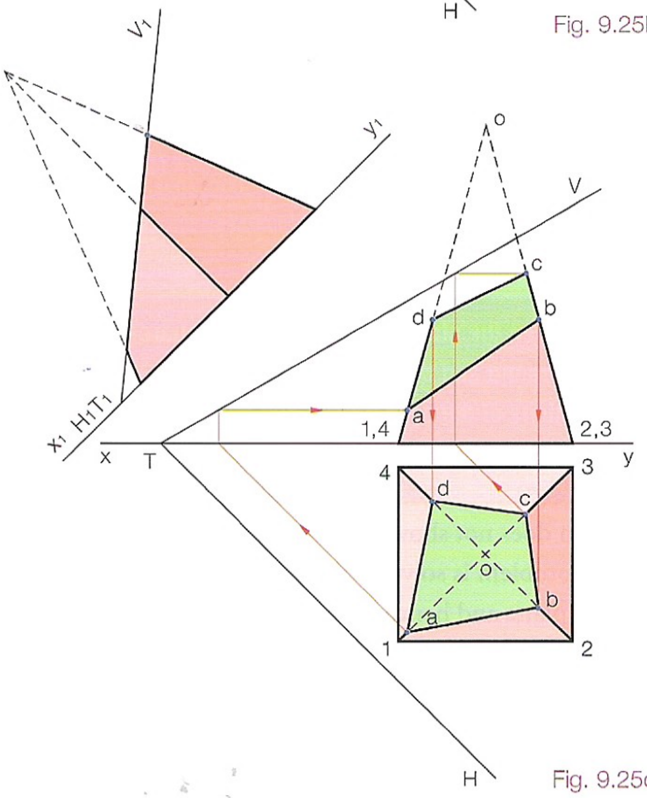

Fig. 9.25d

(4) Project the auxiliary elevation of the pyramid, Fig. 9.25b.

(5) The auxiliary elevation shows the cut points a, b, c and d. Points a and c are projected to plan, the heights of b and d are used to find them in elevation, Fig. 9.25c.

(6) Complete the plan and elevation by projection, Fig. 9.25d.

Shown in Fig. 26 is a regular hexagonal-based pyramid which is to be cut by the plane VTH. Draw the plan and elevation of the cut solid. Fig. 9.26

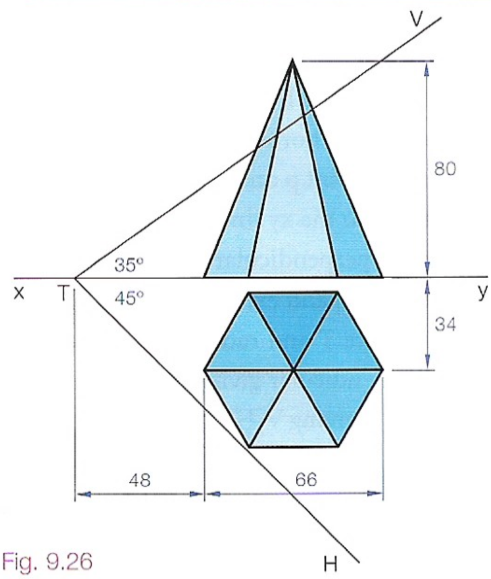

Fig. 9.26

(1) Draw the given plan and elevation and the traces of the plane VTH.
(2) Index the corners.
(3) Project the auxiliary elevation to find the points on the cut surface.
(4) Project to plan and up to the elevation, Fig. 9.27.

Some points, when projected down from the auxiliary elevation to the plan, do not show a clear intersection, e.g. the point on line 5,0. In cases like this the height of the point can be taken from the auxiliary and used in the elevation to find the point which may then be projected to plan.

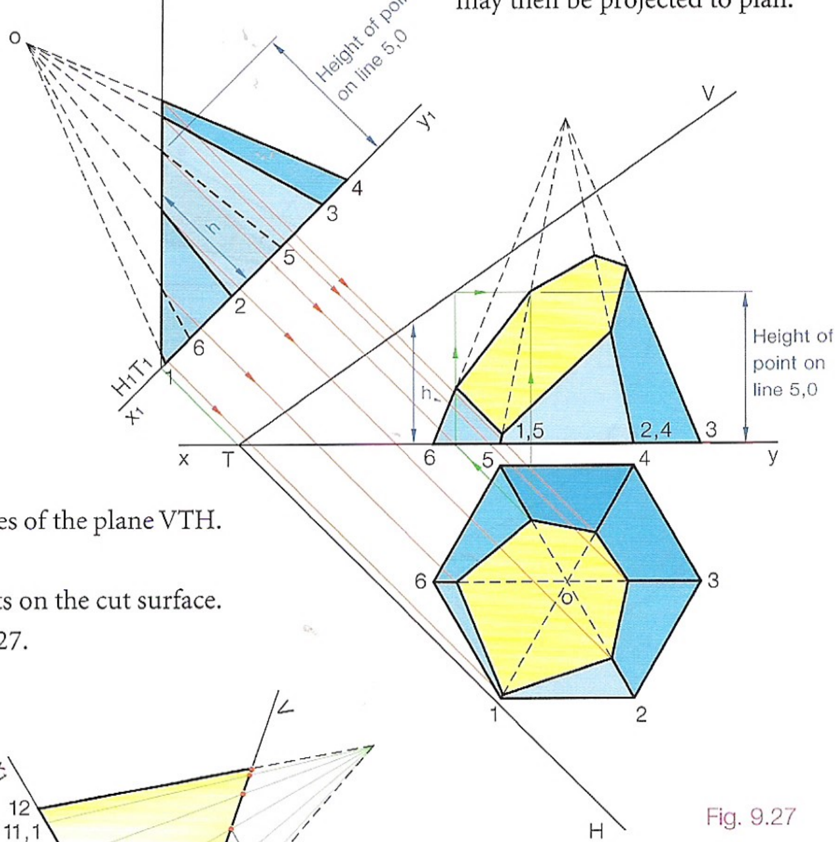

Fig. 9.27

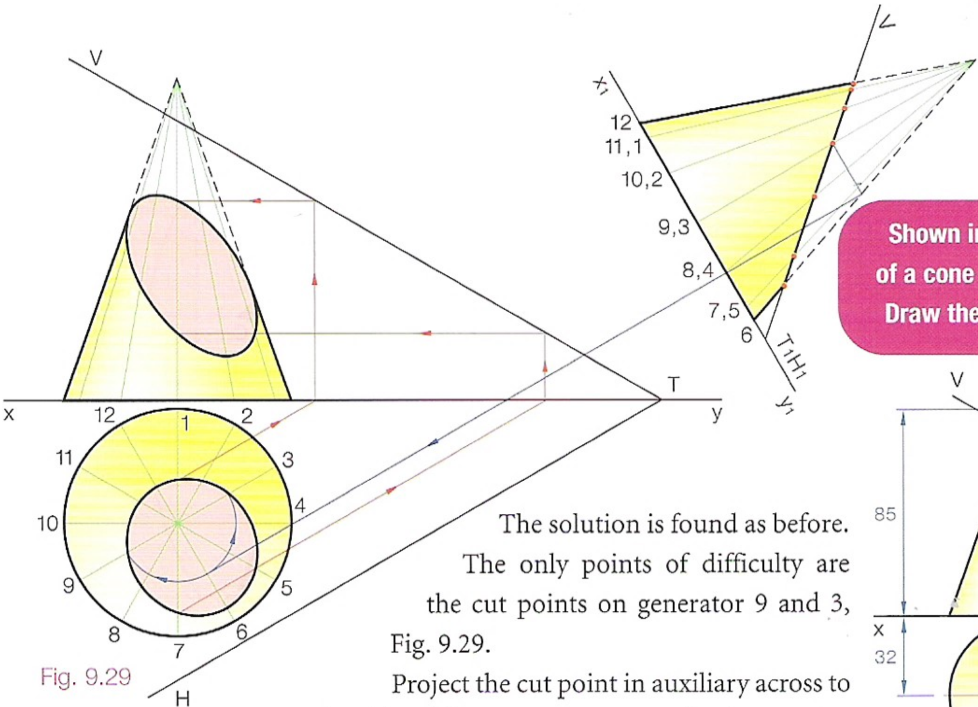

Fig. 9.29

The solution is found as before. The only points of difficulty are the cut points on generator 9 and 3, Fig. 9.29.

Project the cut point in auxiliary across to the side of the cone, generator 6. Project to plan on generator 6 and rotate onto generator 9 and 3. The cut points on generators 1 and 7 in plan must be projected around via the xy line and VT as shown.

Shown in Fig. 9.28 is the plan and elevation of a cone which is to be cut by the plane VTH. Draw the plan and elevation of the cut solid.

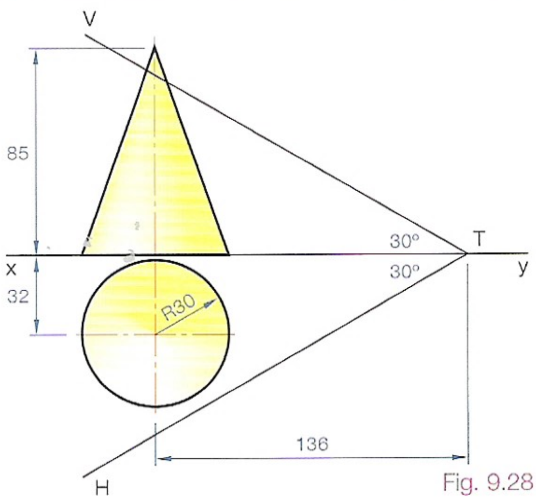

Fig. 9.28