A tangent drawn to an ellipse will cross the major auxiliary circle in two places, s and t. Perpendiculars drawn to the tangent from these two points will always pass through the focal points F_1 and F_2.
Fig. 8.115

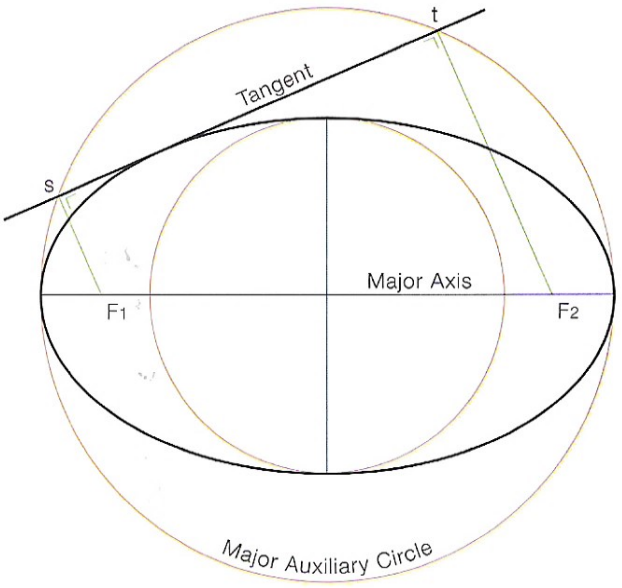

Fig. 8.114

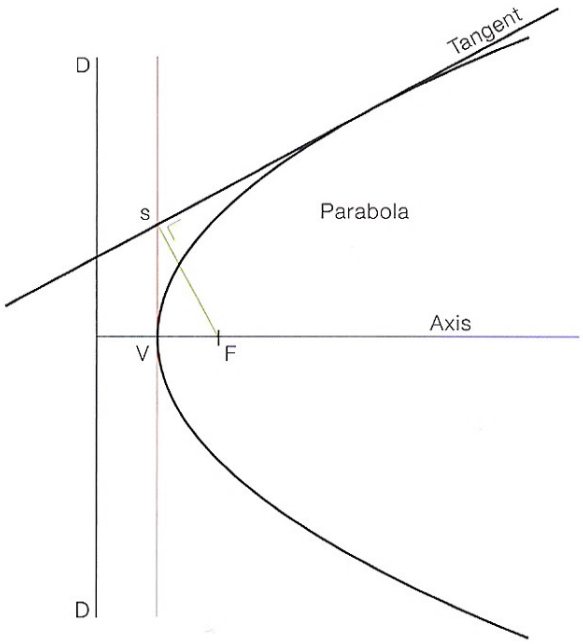

Fig. 8.115

A tangent drawn to the parabola will cross the tangent at the vertex at one point, point s. If a perpendicular is drawn to the tangent at s it will pass through the focus. For a parabola, the tangent at the vertex should be considered as a circle of radius infinity, a small section of which will look like a straight line.

HIGHER LEVEL

Centre of Curvature and Evolute

The centre of curvature is a specific point inside the curve relating to a specific point P on the curve. If the point of the compass is placed on the centre of curvature C and a radius taken of CP, the curvature of the resulting arc would match the curvature of the conic at point P. Each point on the conic will have its own centre of curvature. If all the centres of curvature are plotted and joined, the resulting path is the evolute of that conic. An evolute is the locus of the centres of curvature.

To draw the centre of curvature for a point P on a parabola. Fig. 8.116

(1) Construct a tangent at point P.
(2) Draw the normal to the tangent.
(3) Join P to the focus and extend.
(4) Where the normal crosses the axis at point A construct a perpendicular to intersect the PF line extended. These lines intersect at B.
(5) At B draw a perpendicular to PB to intersect the normal at C. Point C is the centre of curvature for point P.

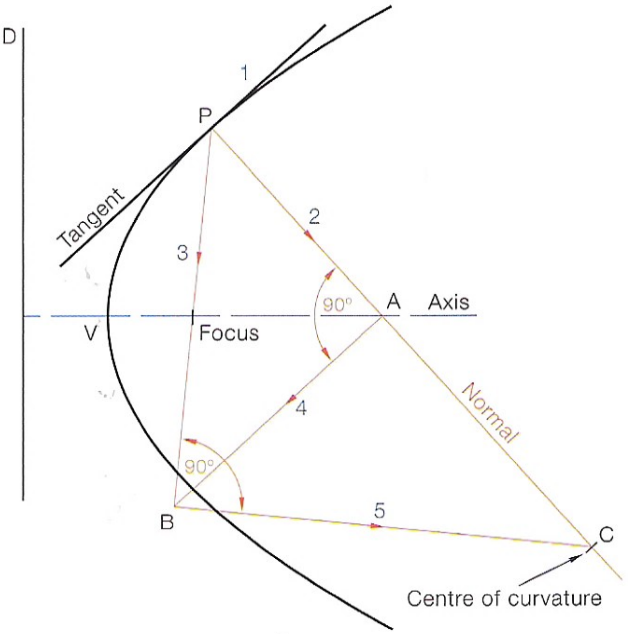

Fig. 8.116

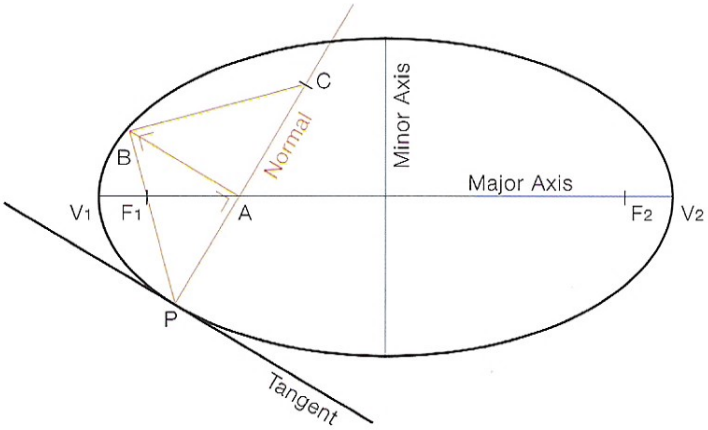

Fig. 8.117

Note: The construction for the centre of curvature for a point P on a hyperbola is the same as above.

To find the centre of curvature at the vertex of a parabola. Fig. 8.118

The method discussed previously will not work at the vertex. A separate construction must be used.

For a parabola, the distance from V to F will equal the distance from F to C, the centre of curvature.

To draw the centre of curvature for a point P on an ellipse. Fig. 8.117

(1) The construction for an ellipse is the same as that for a parabola. Construct a tangent at P.
(2) Draw the normal to the tangent.
(3) Join P to F_1 and extend.
(4) Where the normal crosses the major axis at A construct a perpendicular to intersect PF_1 at B. At B construct a perpendicular to intersect the normal at C. C is the centre of curvature for P.

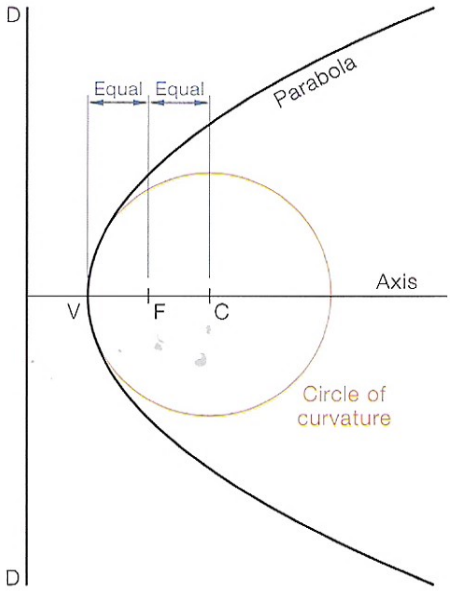

Fig. 8.118

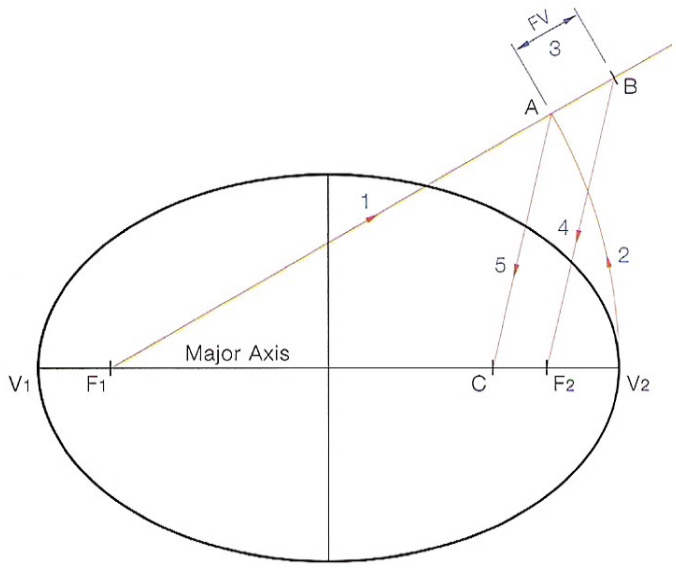

Fig. 8.119

To find the centre of curvature at the vertex of a hyperbola. Fig. 8.120

(1) Draw any sloping line from F_1.
(2) With F_1 as centre and radius F_1V_2, draw an arc to hit this line at point A.
(3) Step the distance FV past point A to find B.
(4) Join A back to F_2.
(5) From B draw a line parallel to AF_2 finding C the centre of curvature at the vertex.

To find the centre of curvature at the vertex of an ellipse. Fig. 8.119

(1) Draw any sloping line from F_1.
(2) With F_1 as centre and radius F_1V_2, draw an arc to hit this line at point A.
(3) Add the distance between the focus and vertex, FV, beyond point A. This locates point B.
(4) Join B to F_2.
(5) Draw a line parallel to BF_2 starting at point A. This locates point C on the axis, the centre of curvature at the vertex.

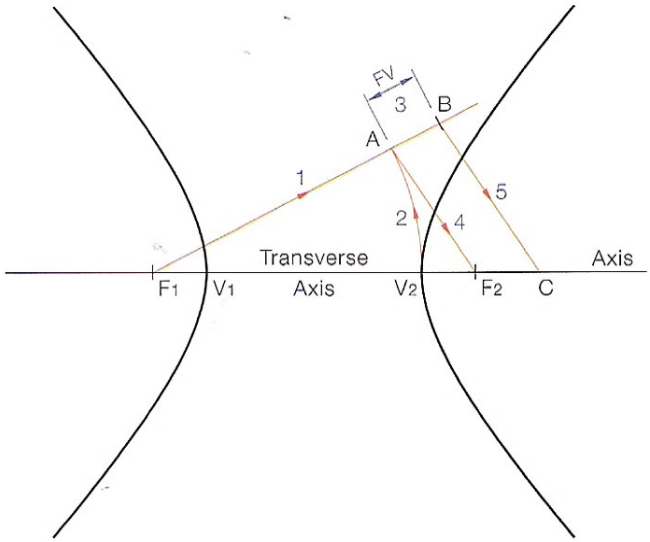

Fig. 8.120

To draw the evolute to a parabola, an ellipse or a hyperbola. Figures 8.121, 8.122 and 8.123

Locate a number of centres of curvature to plot the locus of the evolute.

The evolute will always be symmetrical about the axis.

Note: To locate the centre of curvature at the vertex of a hyperbola when the second focus is unavailable you can use the construction shown. The ratio CF:FV will be equal to the eccentricity of the curve.

Eccentricity = Focus to point:Point to directrix, or FV:VDD.

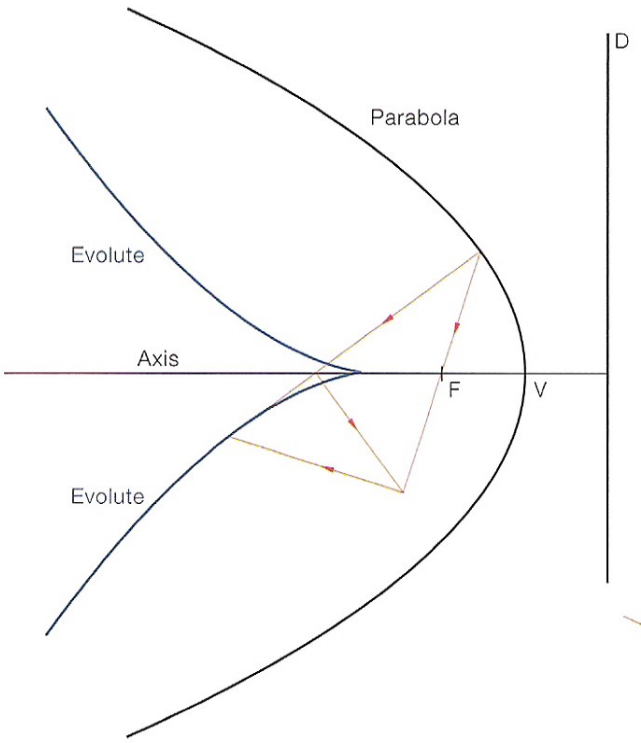

Fig. 8.121

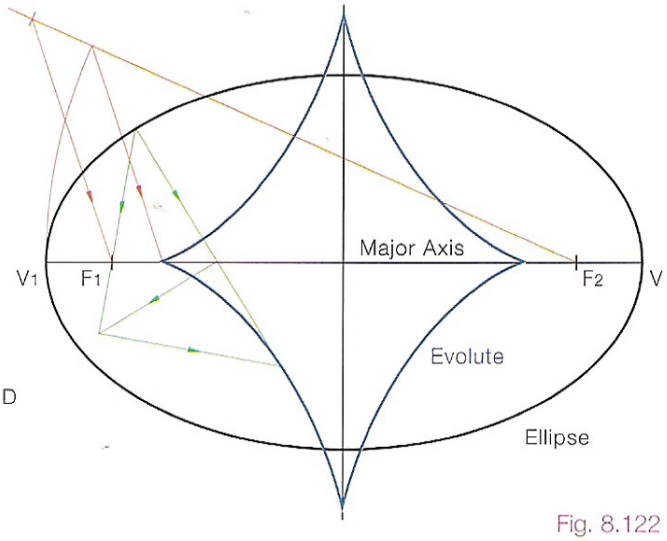

Fig. 8.122

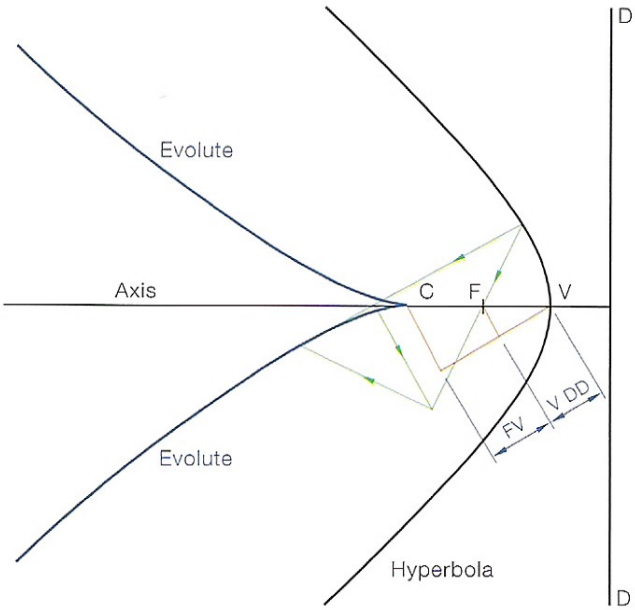

Fig. 8.123

Activities

Q1. Given the vertex, axis and directrix construct the parabola.
Construct a tangent from point P.
Find the centre of curvature for the point of contact, Fig. 8.124.

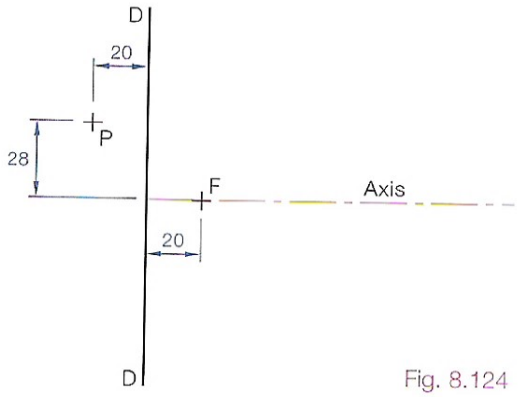

Fig. 8.124

Q2. Draw a parabola having its focus and vertex 18 mm apart. Construct the evolute to this curve.

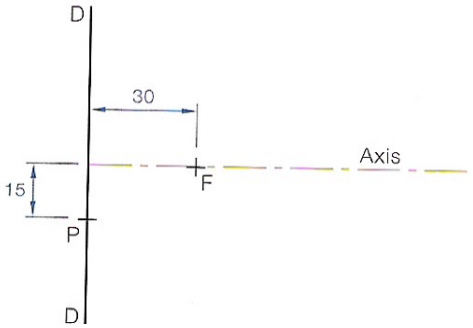

Fig. 8.125

Q3. Given the directrix and focus of an ellipse having an eccentricity of 0.75. Construct a portion of the curve. Draw a tangent to the ellipse from the point P on the directrix. Find the centre of curvature for the point of contact, Fig. 8.125.

Q4. Construct an ellipse having a major axis of 160 mm and a minor axis of 80 mm. Draw the evolute to this curve.

Q5. Given the vertex and focus of a hyperbola having an eccentricity of 6/5. Draw the curve. Draw a tangent to the curve from a point P which is 40 mm from the focus. Find the centre of curvature for point P, Fig. 8.126.

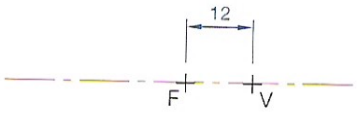

Fig. 8.126

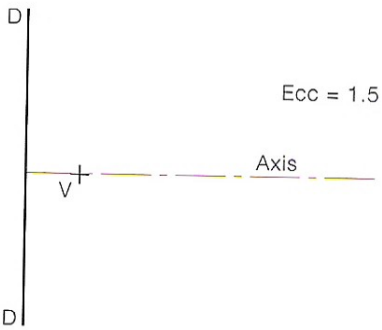

Fig. 8.127

Q6. Given the axis, vertex, directrix and eccentricity of 1.5. Construct a portion of the curve and draw an evolute to it, Fig. 8.127.