Three Methods of Constructing a Hyperbola

Method 1: Rectangle Method

Fig. 8.80

(1) The two rectangles must be of equal size and share an axis as shown.
(2) V_1 and V_2, the vertices of the hyperbolas, are located.
(3) The edge CB is divided into a number of equal parts, five in this example.
(4) These points are joined to V_2.
(5) The edge AB is now divided into the same number of parts as edge CB.
(6) These points are joined to V_1.
(7) Where the two sets of construction lines cross, plot points on the curve as shown.
(8) The rest of the curve is found in a similar way.

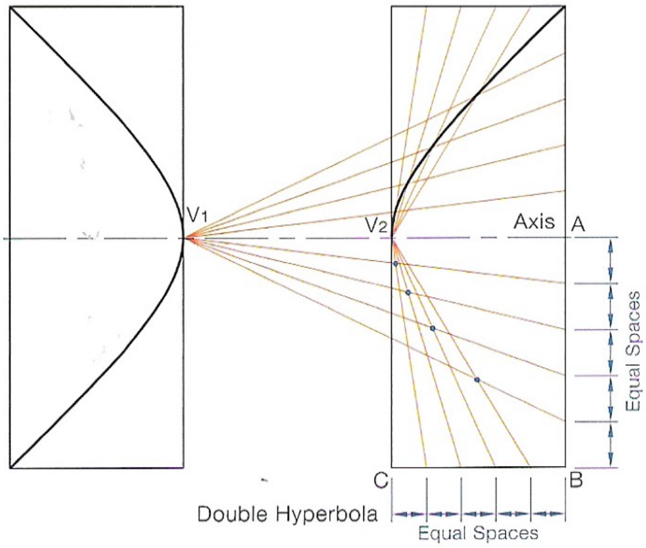

Fig. 8.80

Method 2: Compass Method

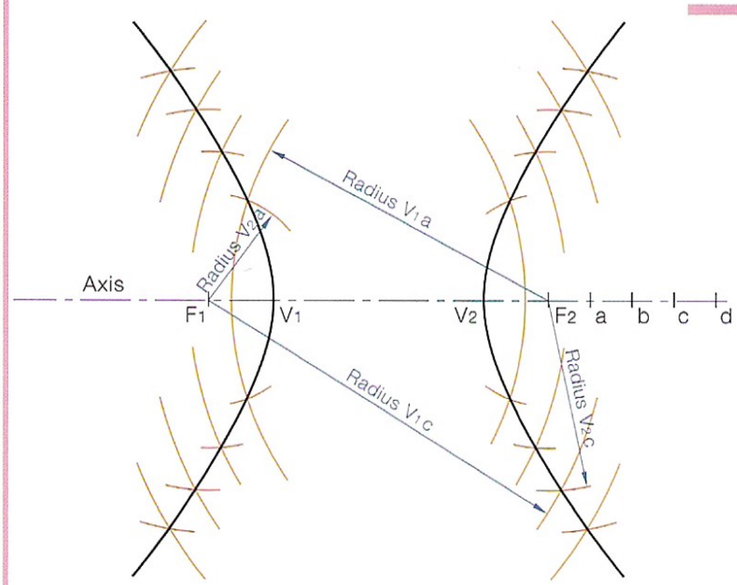

Fig. 8.81

Fig. 8.81

(1) Draw an axis and mark the two vertices V_1 and V_2.
(2) Locate the focal points F_1 and F_2. The distance from F_1 to V_1 must be the same as from F_2 to V_2.
(3) Choose points a, b, c, d etc. on the axis beyond F_2.
(4) With radius V_1a and centre F_1 draw an arc.
(5) With F_2 as centre draw another arc.
(6) With radius V_2a and centre F_2 draw arcs above and below the axis to cut the previous arc giving two points on one branch of the curve.
(7) Using the same radius repeat this process using F_1 as centre giving two points on the second branch of the curve.
(8) Repeat this process using points b, c, d etc. plotting the path of the curve.

Method 3: Eccentricity Method

Fig. 8.82

The eccentricity for a hyperbola is always **greater than one**, e.g. 7/4, 12/5, 1.6 etc.

The eccentricity line for a hyperbola will always make an angle of greater than 45° with the axis.

Eccentricity is a ratio between two distances

$$= \frac{\text{Distance from focus to a point on the curve}}{\text{Distance from the same point to directrix}}$$

$$= \frac{\text{F to P}}{\text{P to DD}}$$

The ratio is a constant for a particular curve, no matter where on the curve the point P is chosen.

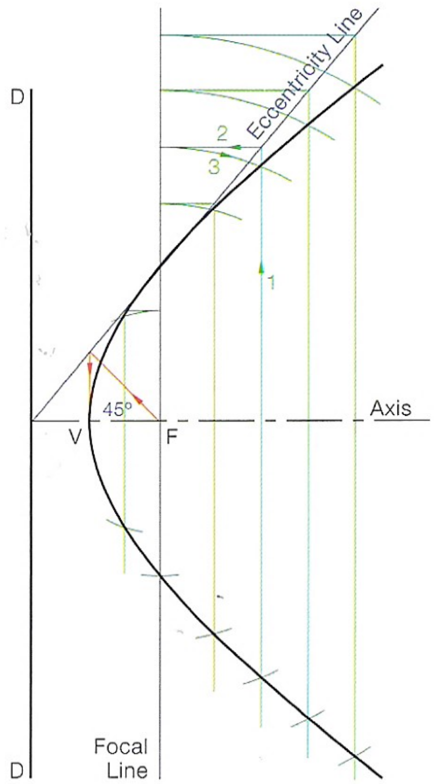

Fig. 8.82

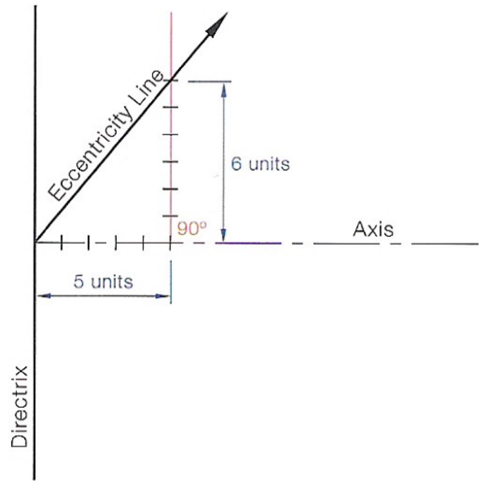

Fig. 8.83

CONSTRUCTION

Given directrix focus and eccentricity of 1.2.

The advantage of the eccentricity method is that once the eccentricity line is set up correctly the process of construction is the same for all three conics.

(1) Draw the directrix, axis and focus.
(2) Eccentricity of 1.2 can be written as a fraction 6/5.
(3) Measure out from the directrix five units, e.g. 5 × (5 mm units) = 25 mm, and mark a point on the axis.
(4) Draw a perpendicular to the axis at this point and measure up from the axis six units, e.g. 6 × (5 mm units) = 30 mm.
(5) The eccentricity line can now be drawn.

Note: The bottom portion of the eccentricity fraction is measured out along the axis and the top is measured up on a perpendicular.

(6) A line drawn up from the focus, at 45° to the axis, to hit the eccentricity line and dropped to the axis finds the vertex.
(7) Construction for the curve is the same as for the ellipse and parabola.

Tangent to a Hyperbola from a Point on the Curve

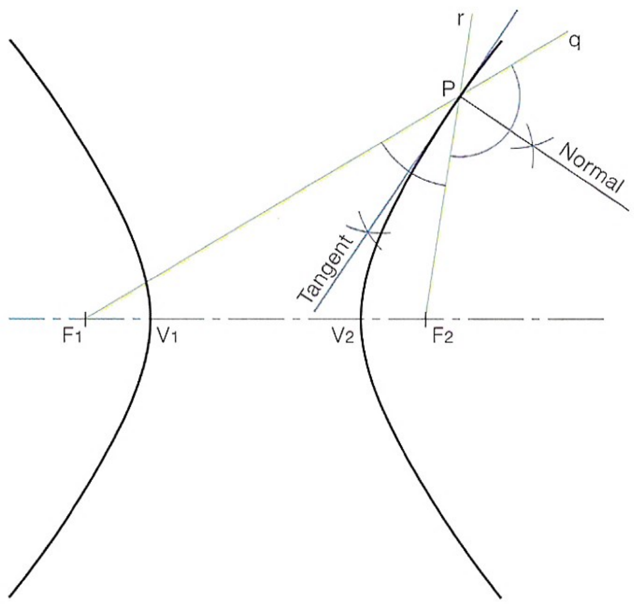

Fig. 8.84

Method 1

Fig. 8.84

(1) Join F_1 to P and extend to q.

(2) Join F_2 to P and extend to r.

(3) The line that bisects angle F_1PF_2 or angle qPr will be the tangent.

(4) The normal is perpendicular to the tangent and can be found by bisecting the angle F_2Pq.

Note: The similarity to the method used for the parabola, Fig. 8.10, and the ellipse, Fig. 8.44.

Method 2

Fig. 8.85

This method works for all three conics and has already been shown in Fig. 8.8 for the parabola and Fig. 8.43 for the ellipse.

(1) Join P back to the focus.

(2) At the focus draw a perpendicular to PF to hit the directrix at q.

(3) Point q is a point on the tangent. Join P to q, forming the tangent.

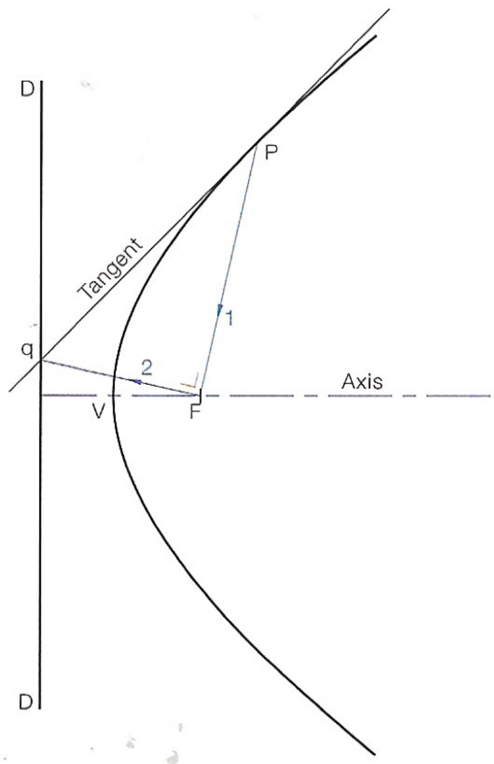

Fig. 8.85

Tangent to a Hyperbola from a Point P outside the Curve

Method 1

Fig. 8.86

(1) Join P to F_2 and place a circle on this line having PF_2 as its diameter.

(2) Draw the auxiliary circle for the double hyperbola, i.e. the circle which has V_1V_2 as its diameter.

(3) The circles cross at q and r which will be points on the tangents.

(4) Draw the tangent p to q extended.

Note: The similarity to construction for a parabola, Fig. 8.14, and construction for an ellipse, Fig. 8.46.

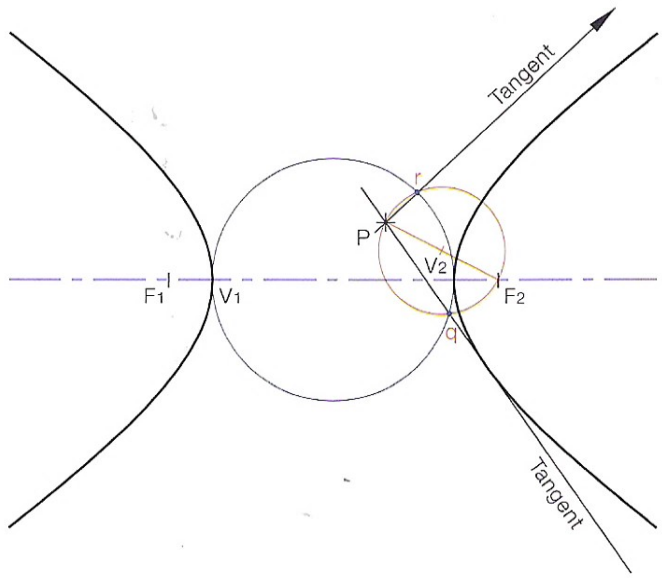

Fig. 8.86

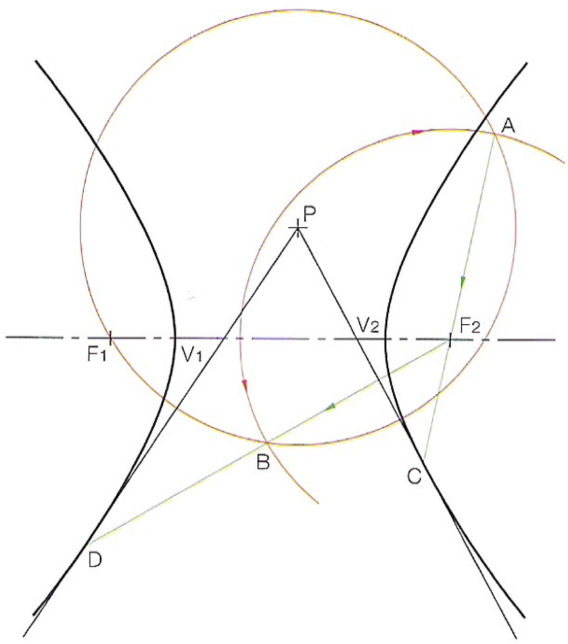

Fig. 8.87

Method 2

Fig. 8.87

(1) Draw a circle with P as centre and PF_1 as radius.

(2) With radius V_1V_2 (transverse axis) and F_2 as centre, scribe an arc to cut this circle at A and B.

(3) Join A to F_2 and extend to C, join B to F_2 and extend to D.

(4) Draw the tangents PC and PD.

Note: Similar to the construction for an ellipse, Fig. 8.47.

Activities

Q1. Given the plan and elevation of a cone which has been sectioned as shown. Draw the views and determine the true shape of the section. Hyperbola, Fig. 8.88.

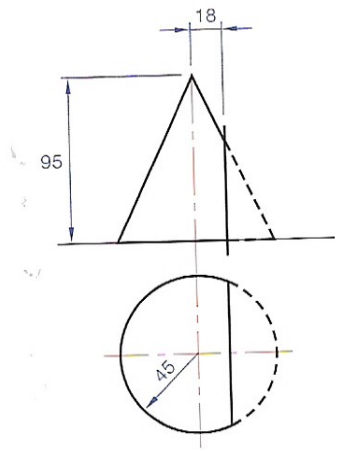

Fig. 8.88

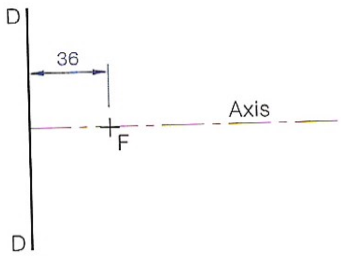

Fig. 8.89

Q2. Given the directrix, axis, focus and eccentricity of 6/5. Draw a portion of the hyperbola, Fig. 8.89.

Q3. Given the axis, focus, point P on the curve and eccentricity of 1.25. Draw a portion of the hyperbola Fig. 8.90.

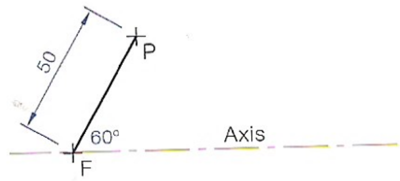

Fig. 8.90

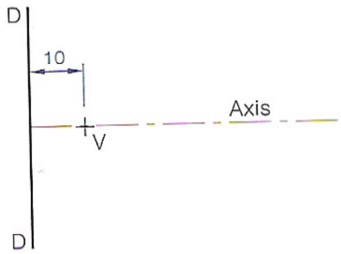

Fig. 8.91

Q4. Given the axis, vertex and directrix. Construct a hyperbola having an eccentricity of 7/5. Construct a tangent to this hyperbola from a point P which is 40 mm from the directrix, Fig. 8.91.

Q5. Construct the triangle P_1P_2F where P_1 and P_2 are points on the curve of a hyperbola and F is its focus. The eccentricity is 10/7, Fig. 8.92.

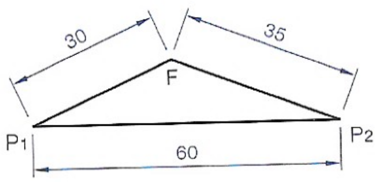

Fig. 8.92

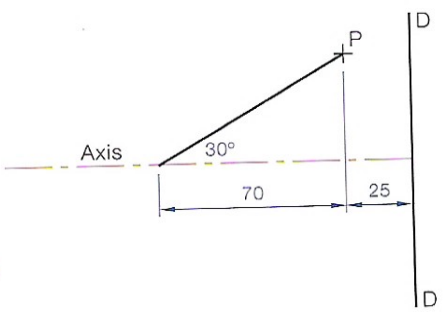

Fig. 8.93

Q6. Given the normal, axis, point of contact and directrix. Find the focus and construct a portion of the curve, Fig. 8.93.

Q7. Given the focus and vertex of a hyperbola having an eccentricity of 1.1. Draw a portion of the curve, Fig. 8.94.

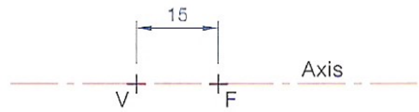

Fig. 8.94

Q8. Given the axis, directrix, tangent and point of contact P. Draw a portion of the hyperbola, Fig. 8.95.

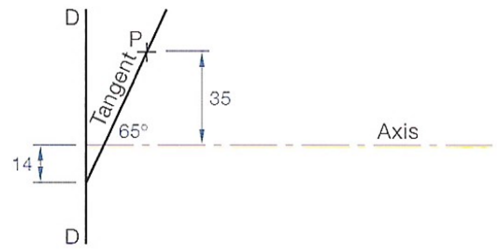

Fig. 8.95

Q9. Given the cone in Fig. 8.96 which is sectioned as shown. Construct the focal sphere and hence find the focus, vertex and directrix of the hyperbola. Draw the hyperbola.

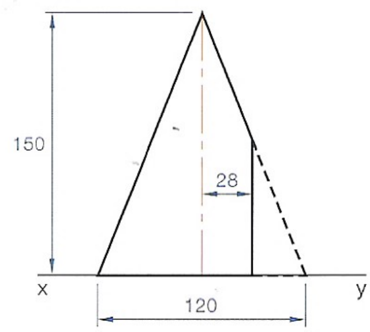

Fig. 8.96

Q10. Construct a double hyperbola in the rectangles shown, Fig. 8.97.

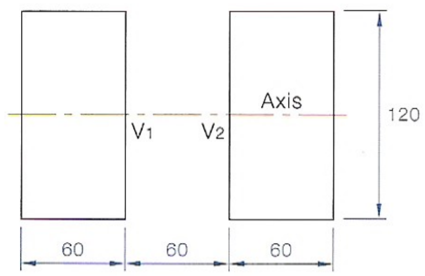

Fig. 8.97

Q11. Given the directrix, vertex, axis and focus. Draw the hyperbola. Construct a tangent to the curve from the given point P, Fig. 8.98.

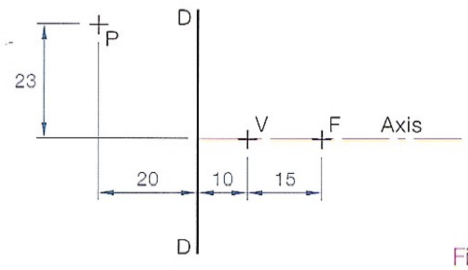

Fig. 8.98

Q12. Given the foci and vertices of a double hyperbola. Construct a portion of each curve. Construct a tangent to the curve from a point 50 mm from F_1, Fig. 8.99.

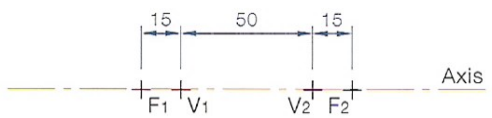

Fig. 8.99

Q13. Given the transverse axis of 80 mm and the distance between F and V of 20 mm. Construct a double hyperbola, directrices, asymptotes, auxiliary circle and conjugate diameter.

Q14. Given a tangent, point of contact and focus of a double hyperbola. Draw the double curve, Fig. 8.100.

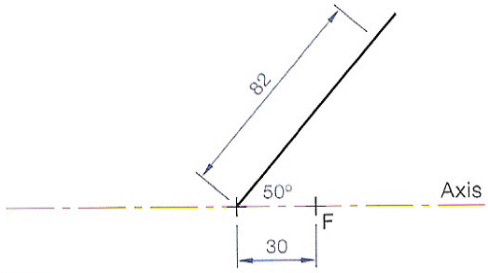

Fig. 8.100

Q15. Given the line P_1FP_2 as shown in Fig. 8.101. P_1 and P_2 are points on the curve of a hyperbola, F is the focus and the eccentricity is 7/5. Draw the curve.

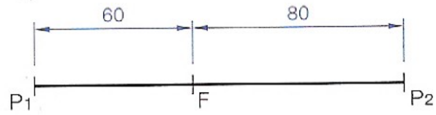

Fig. 8.101

Q16. Given the line FPA as shown in Fig. 8.102. If F is the focus, P is a point on the curve and A is a point on the directrix. Construct the hyperbola if the eccentricity is 6/5.

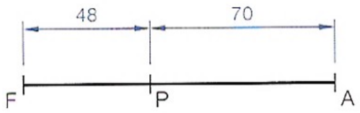

Fig. 8.102

Q17. Given the triangle FP_1P_2. F is one of the focal points on a double hyperbola, P_1 is a point on one branch of the curve and P_2 is a point on the other branch. The transverse axis is 60 mm long. Find the other focal point and draw both curves, Fig. 8.103.

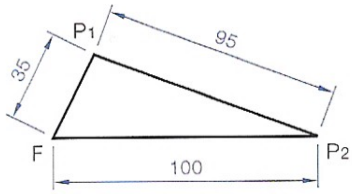

Fig. 8.103

Q18. Given a tangent, the point of contact P and the focal point of a double hyperbola. The transverse axis is 50 mm. Draw a portion of the double hyperbola, Fig. 8.104.

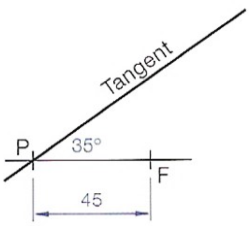

Fig. 8.104

Q19. Given the two focal points of a double hyperbola and a point P on one branch of the curve. Draw the double curve, Fig. 8.105.

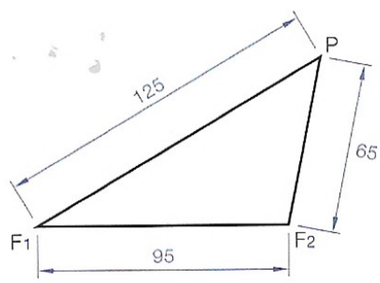

Fig. 8.105

HIGHER LEVEL

Q20. Given the line P_1FP_2. P_1 and P_2 are points on one branch of a hyperbola and F is its focus. If the transverse axis is 30 mm long construct the double curve, Fig. 8.106.

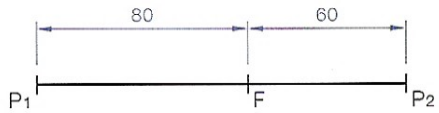

Fig. 8.106

Q21. Given the two focal points and a tangent as shown. Construct the double curve, Fig. 8.107.

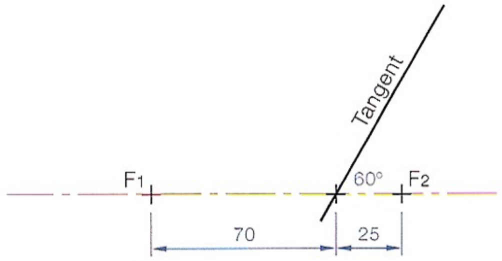

Fig. 8.107

Q22. Given two asymptotes and a point P on the curve of a hyperbola. Draw the curve, Fig. 8.108.

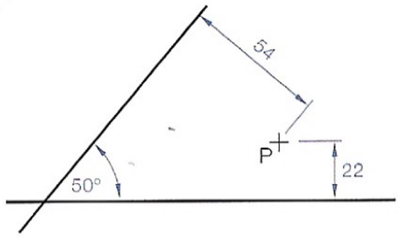

Fig. 8.108

Constructions Common to all Conics

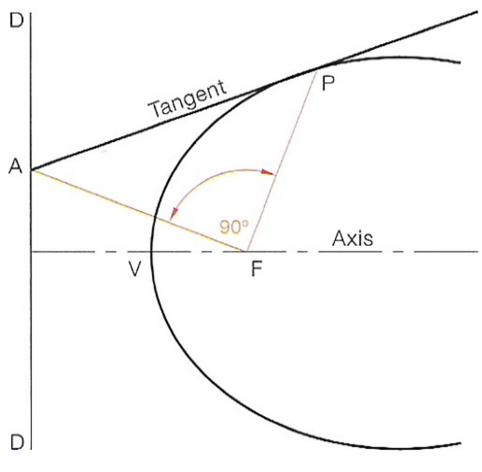

Fig. 8.109

Fig. 8.109
For all conic sections this construction can be used to draw the tangent at a given point P on the curve.
(1) Join P to the focus.
(2) At the focus draw a line FA at 90° to FP. Extend this line to intersect the directrix at A.
(3) Point A is a point on the tangent. Draw the tangent.

Fig. 8.110
Tangents drawn at both ends of a focal chord will always meet on the directrix.
This construction is a follow-on from Fig. 8.109.
P_1FA forms a 90° angle as does P_2FA. These two angles added together make 180°, a straight line, a focal chord.

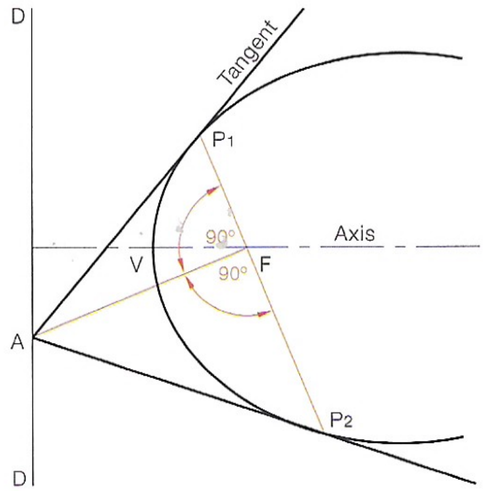

Fig. 8.110

Fig. 8.111
For any two focal chords, when the ends are joined and extended, the lines meet on the directrix.
AC extended and BD extended meet at G on the directrix.
AD extended and CB extended meet at E on the directrix.
Furthermore, the angle GFE will always be 90°.

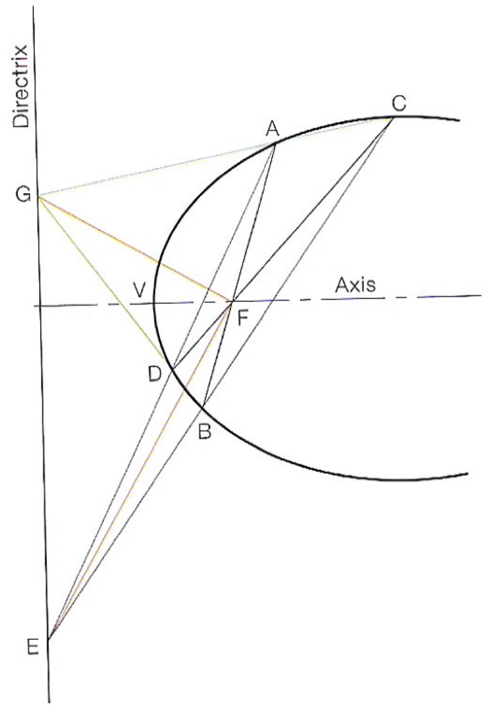

Fig. 8.111

Fig. 8.112
For any point A outside the curve. Tangents drawn from point A to the curve will give points of contact P_1 and P_2. Angles P_1FA and P_2FA will always be equal.

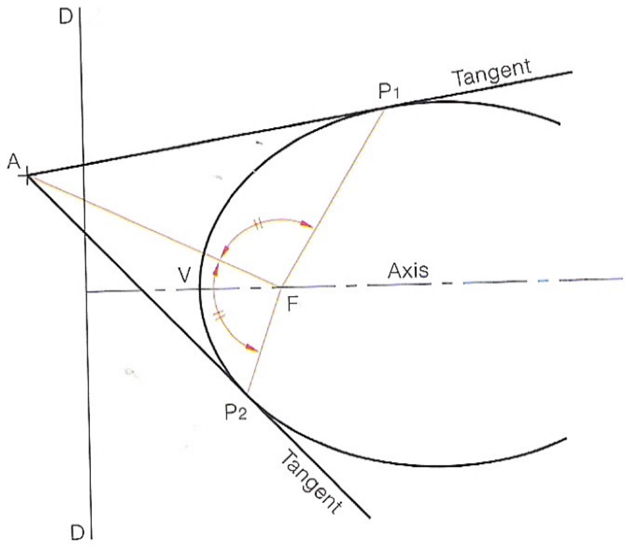

Fig. 8.112

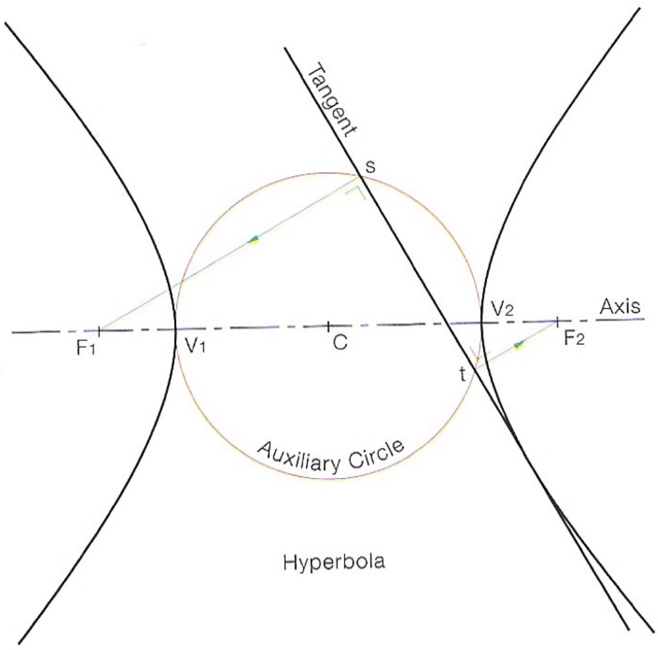

Fig. 8.113
A tangent is drawn to a double hyperbola and will cut through the auxiliary circle in two places, s and t. Perpendicular lines are drawn to the tangent to form these two points. These perpendiculars will always pass through F_1 and F_2, the focal points of the curves.
Fig. 8.114

Fig. 8.113