Tangent to an Ellipse from a Point P outside the Curve

To draw a tangent to an ellipse from a point P outside the directrix. Fig 8.45

Method 1

Fig. 8.45

With P as centre and PF_1 as radius, swing an arc to hit the directrix at R and q. Join from R and q back to F_2 giving the points of contact. Draw the tangents.

Note the similarity with construction for parabola, Fig. 8.13.

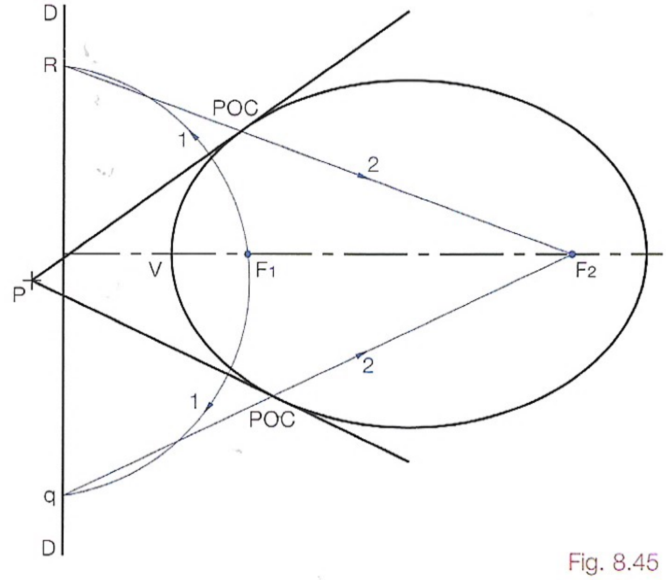

Fig. 8.45

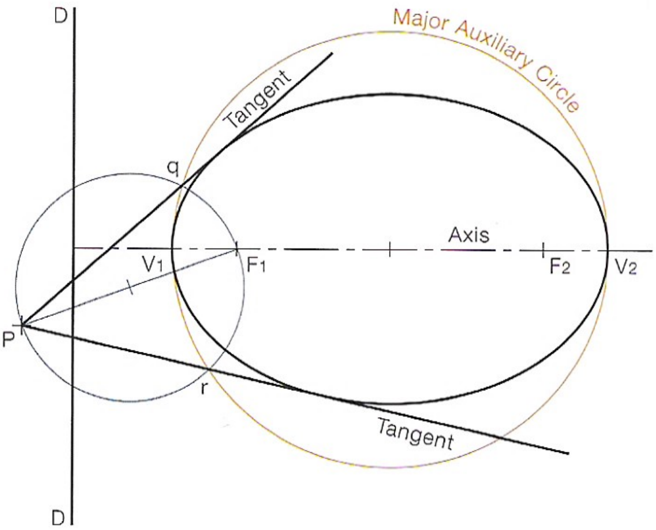

Fig. 8.46

Method 2

Fig. 8.46

Join P to F_1 and place a circle on this line having PF_1 as its diameter. Draw the major auxiliary circle for the ellipse, i.e. the circle which has V_1V_2 as a diameter. These two circles intersect at points r and q, which will be points on the tangents. Draw the tangents.

Note the similarity to construction for a parabola, Fig. 8.14.

Method 3

Fig. 8.47

With point P as centre and PF_1 as radius, draw an arc. Now take the length of the major axis as radius V_1V_2. Using F_2 as the centre point swing an arc to cut your first arc in two places, q and r. Points q and r are joined back to F_2. Where these lines cross the ellipse give the points of contact for the tangents. Draw the tangents.

Note the similarity between the construction for an ellipse and for a hyperbola.

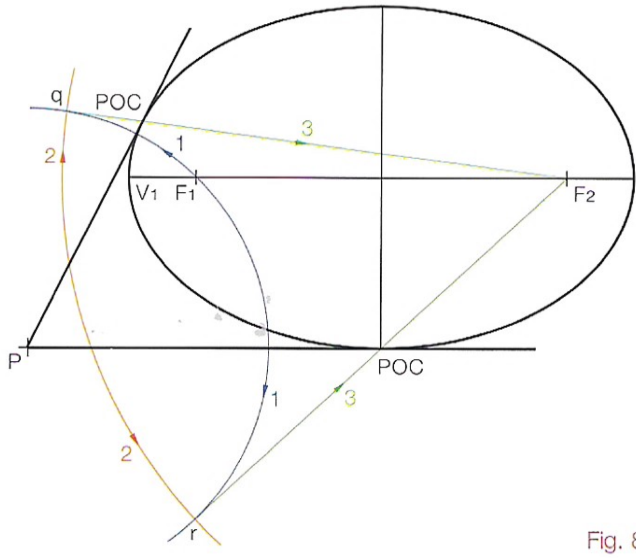

Fig. 8.47

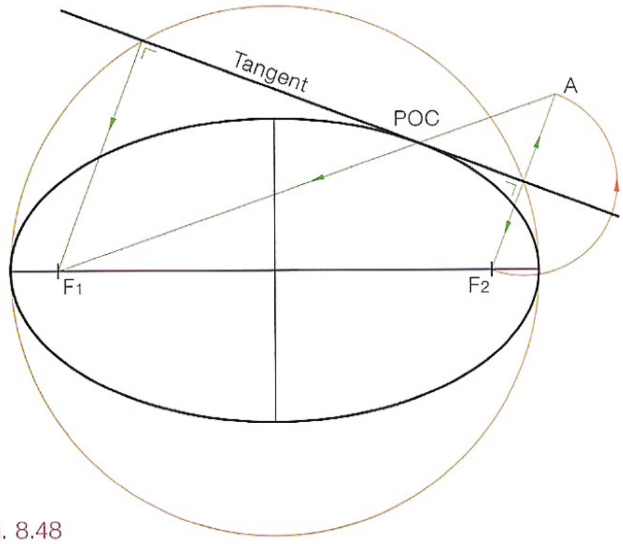

Fig. 8.48

Given a tangent to an ellipse to find the point of contact (POC). Fig 8.48

Fig. 8.48
(1) Draw the major auxiliary circle.
(2) Where the tangent and auxiliary circle intersect draw perpendiculars to the tangent. These perpendiculars will pass through the focal points F_1 and F_2.
(3) On either of these perpendiculars you double its length as shown, finding point A.
(4) Join A back to the focus to find the POC.

Given an ellipse to find its axes.

Fig. 8.49
(1) Draw any two parallel chords.
(2) Bisect these chords giving A and B.
(3) Join A and B.
(4) Bisect the diameter giving C the ellipse centre.
(5) With C as centre draw a circle to cut the ellipse in three places.
(6) Lines joining these points will be parallel to the axes.
(7) Draw the axes through point C.

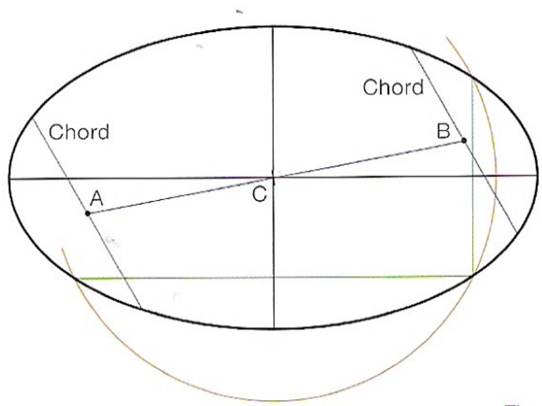

Fig. 8.49

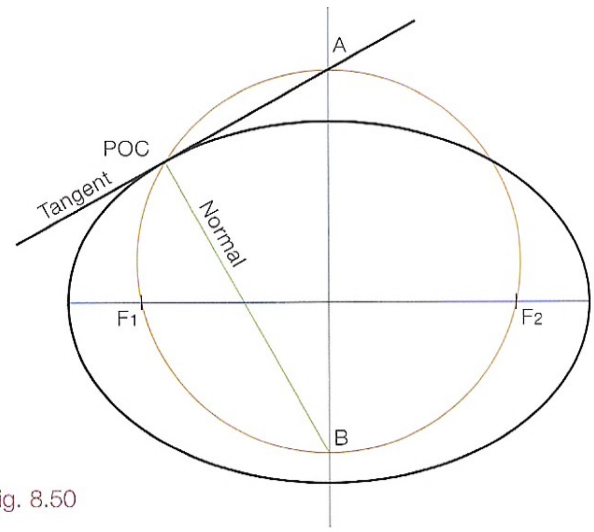

Fig. 8.50

Given a tangent to an ellipse to find its point of contact (POC).

Alternative Method

Fig. 8.50
(1) Extend the minor axis to intersect the tangent at point A.
(2) Construct a circle to contain points A, F_1 and F_2.
(3) This circle locates the point of contact where it crosses the ellipse.

It should be noted that where the circle intersects the minor axis for the second time at B, it locates a point on the normal. (The angle in a semicircle is always a right angle.)

Activities

Q1. Draw the given plan and elevation of the cone and find the true shape of the section (ellipse), Fig. 8.51.

Q2. Construct an ellipse in a rectangle of side 140 mm by 80 mm using the rectangle method.

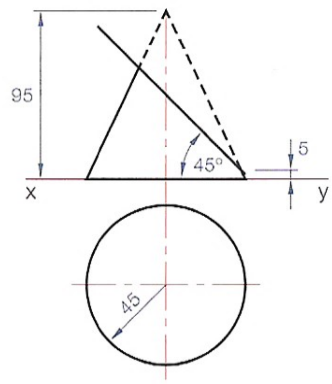

Fig. 8.51

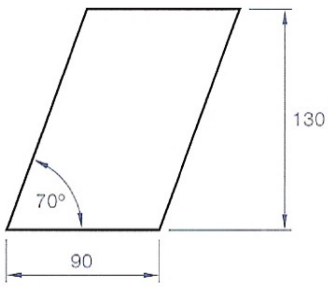

Fig. 8.52

Q3. Construct an ellipse in the given parallelogram such that the sides of the parallelogram form tangents to the ellipse, Fig. 8.52.

Q4. Draw an ellipse having a major axis of 120 mm and a minor axis of 80 mm using the auxiliary circle method. Locate the two focal points.

Q5. Using a trammel like that shown in Fig. 8.53, construct an ellipse and find its foci.

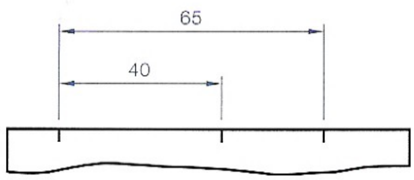

Fig. 8.53

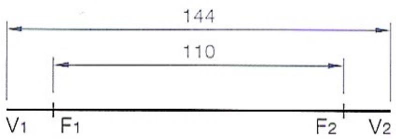

Fig. 8.54

Q6. Given the major axis and the foci, construct the ellipse, Fig. 8.54.

Q7. Given the minor axis and the foci, construct the ellipse, Fig. 8.55.

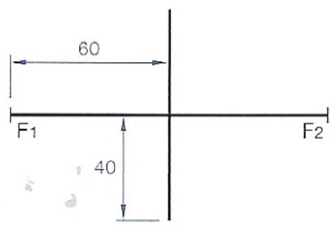

Fig. 8.55

Q8. Given the triangle ABC where A and B are focal points of an ellipse and C is a point on the curve. Draw the ellipse, Fig. 8.56.

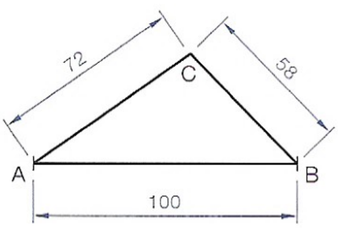

Fig. 8.56

Q9. Given the major axis and a point P on the curve. Draw the ellipse, Fig. 8.57.

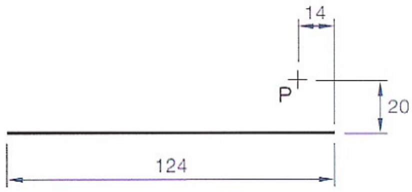

Fig. 8.57

Q10. Given the minor axis of an ellipse and a point P on the curve. Construct the ellipse, Fig. 8.58.

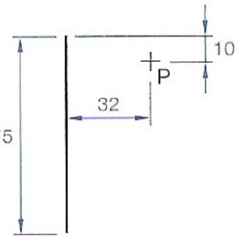

Fig. 8.58

Q11. Given the directrix, axis, focus of an ellipse and an eccentricity of 0.8. Draw a portion of the curve, Fig. 8.59.

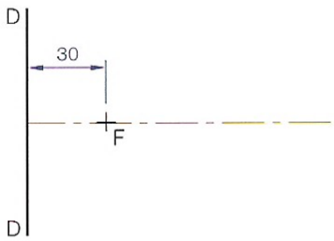

Fig. 8.59

Q12. Given the axis, focus, eccentricity of 3/5 and a point P on an ellipse. Draw the curve and construct a tangent at point P, Fig. 8.60.

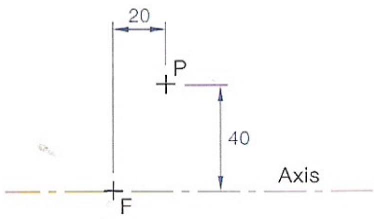

Fig. 8.60

Q13. Construct the triangle shown in Fig. 8.13. DD is the directrix of an ellipse. Points P_1 and P_2 are points on the curve and the eccentricity of the ellipse is 3/4. F shows the approximate position of the focus. Locate the focus and draw the curve, Fig. 8.61.

$$\text{Eccentricity} = \frac{\text{Distance from focus to a point}}{\text{Point to directrix}}$$

$$= \frac{\text{F to P}}{\text{P to DD}}$$

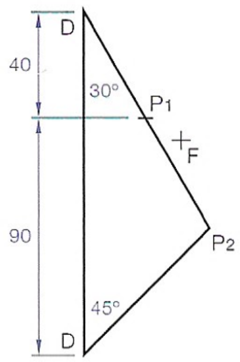

Fig. 8.61

Q14. Given the focus and vertex of an ellipse having an eccentricity of 0.8. Draw a portion of the curve, Fig. 8.62.

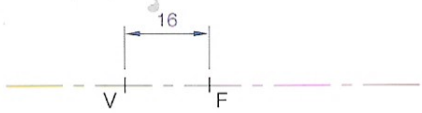

Fig. 8.62

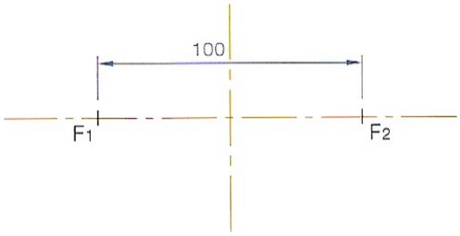

Fig. 8.63

Q15. In an ellipse the distance between the focal points is 100 mm. The length of the major axis and the minor axis are in the ratio 3:2. Draw the ellipse, Fig. 8.63.

Q16. Given the directrix, axis, eccentricity of 0.75 and a point P on the curve. Determine the position of the focus and draw a portion of the curve. Construct a tangent to the curve from point A, Fig. 8.64.

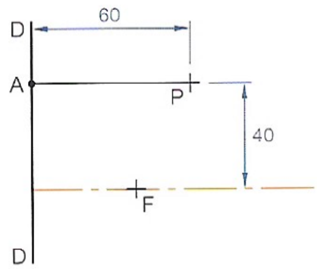

Fig. 8.64

Q17. Given the major axis of 110 mm, focus and a point P on an ellipse. Draw the curve, Fig. 8.65.

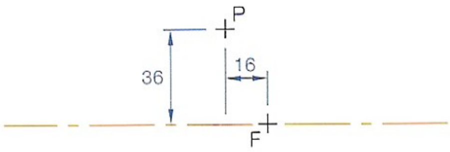

Fig. 8.65

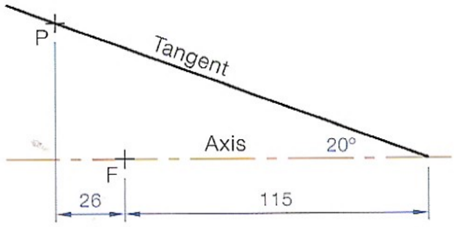

Fig. 8.66

Q18. Given the axis, focus, tangent and point of contact P to an ellipse. Construct a portion of the curve, Fig. 8.66.

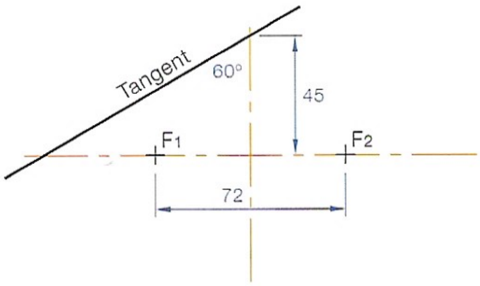

Fig. 8.67

Q19. Given F_1, F_2 and a tangent to an ellipse. Find the point of contact and draw the curve, Fig. 8.67.
Hint: See Figures 8.48 and 8.50.

Q20. Given the latus rectum of an ellipse and the vertex. Construct a portion of the curve, Fig. 8.68.

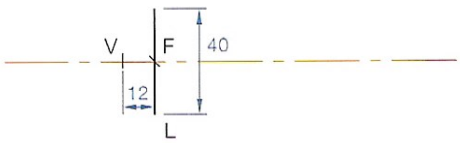

Fig. 8.68

HIGHER LEVEL

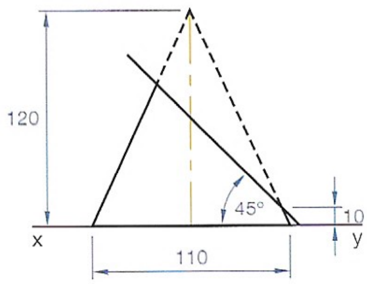

Fig. 8.69

Q21. Given the cone in Fig. 8.69 which is sectioned as shown. Construct the focal spheres and hence find the focus, vertex and directrix of the ellipse. Draw the ellipse.

Q22. Given the triangle APF. AP is a tangent to an ellipse with P as the point of contact. F is a focal point and the major axis is 120 mm long, Fig. 8.70. Draw the ellipse.

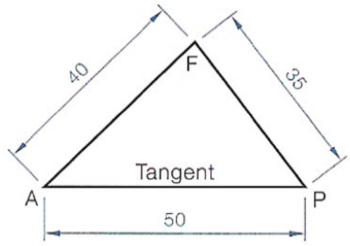

Fig. 8.70

Q23. Given the major axis V_1V_2 of an ellipse and a tangent to it. Draw a portion of the curve to include the point of contact, Fig. 8.71

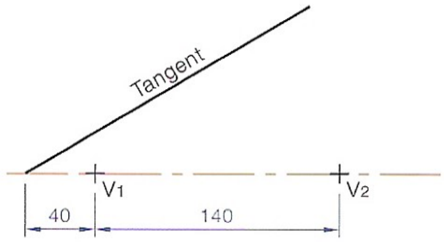

Fig. 8.71

Q24. Given the line P_1FP_2 as shown in Fig. 8.42. F is the focal point of an ellipse and both P_1 and P_2 are points on the curve. The directrix is 50 mm from point P_1. Draw a portion of the curve. Construct a tangent at P_2, Fig. 8.72.

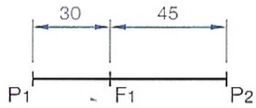

Fig. 8.72

Q25. Given the directrix focus and vertex construct the ellipse. Construct a tangent to the ellipse from point P, Fig. 8.73.

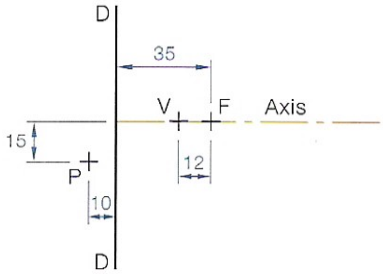

Fig. 8.73

Q26. In Fig. 8.74, AB is a tangent to an ellipse. V is the vertex and F is the focus. Construct the ellipse.

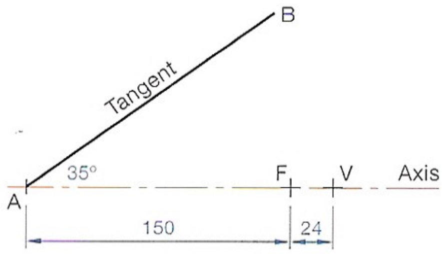

Fig. 8.74

Q27. Given the directrix, axis, tangent and the point of contact P. The focus is closer to the point P than the directrix. Draw the curve, Fig. 8.75.

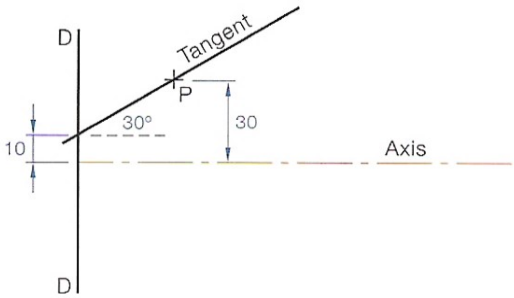

Fig. 8.75

Q28. Given the focus, axis, directrix and tangent to an ellipse. Construct the ellipse and find the point of contact, Fig. 8.76.

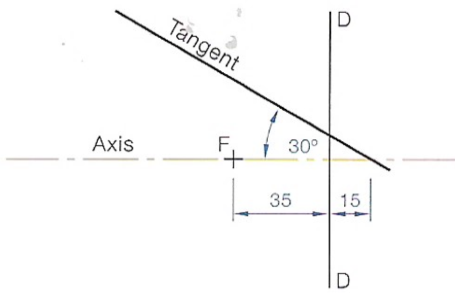

Fig. 8.76

Hyperbola as a Section of a Cone

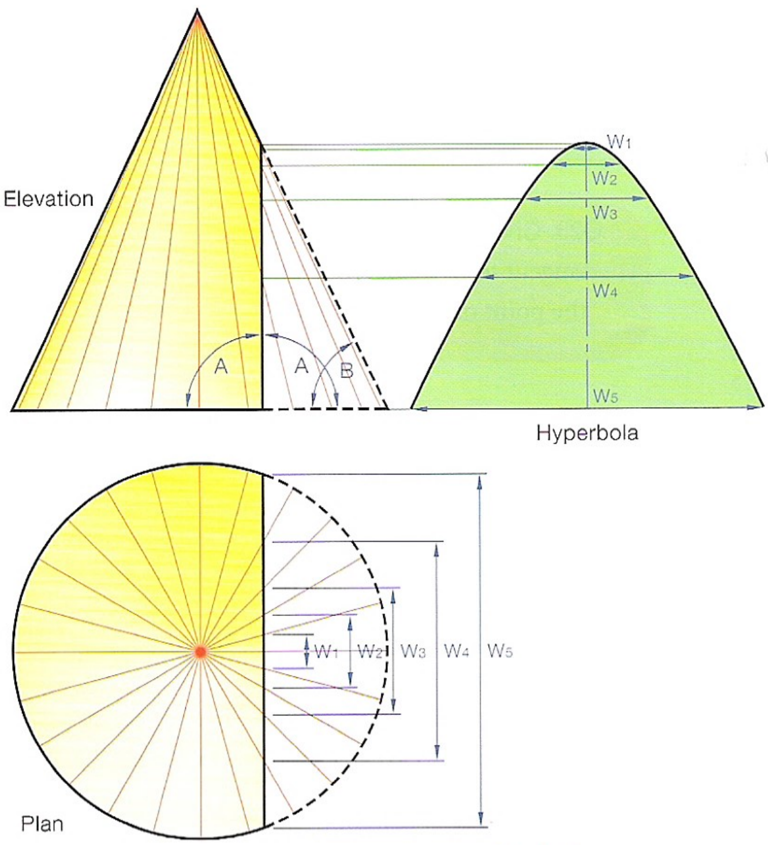

Fig. 8.77

If a cone is cut by a cutting plane such that either angle at the base (angle A) is equal to 90°, or is between 90° and the angle of the side of the cone (angle B), then a hyperbola is produced. The cutting plane only cuts one side of the cone no matter how far the cone and plane are extended.

CONSTRUCTION

Fig. 8.77

The construction is the same as that for the parabola and ellipse.

(1) Draw the plan, elevation, elements and cutting plane.

(2) The auxiliary view is projected perpendicularly from the cutting plane in elevation.

(3) Points where the cutting plane crosses elements are projected out.

(4) Widths are taken from the plan as shown.

Focal Sphere

The hyperbola has one focal sphere. The position and size of the sphere is such that it is tangential to both the sides of the cone and the cutting plane, i.e. it fits neatly into the top of the cone and also touches the cutting plane at one spot, the focus. The directrix is found by finding the intersection between the plane through the sphere containing all the points of contact between the sphere and the cone and the cutting plane.

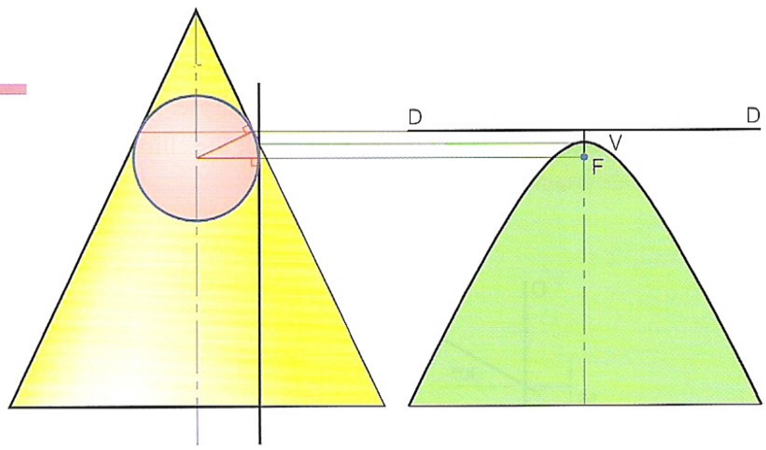

Fig. 8.78

HIGHER LEVEL

Terminology

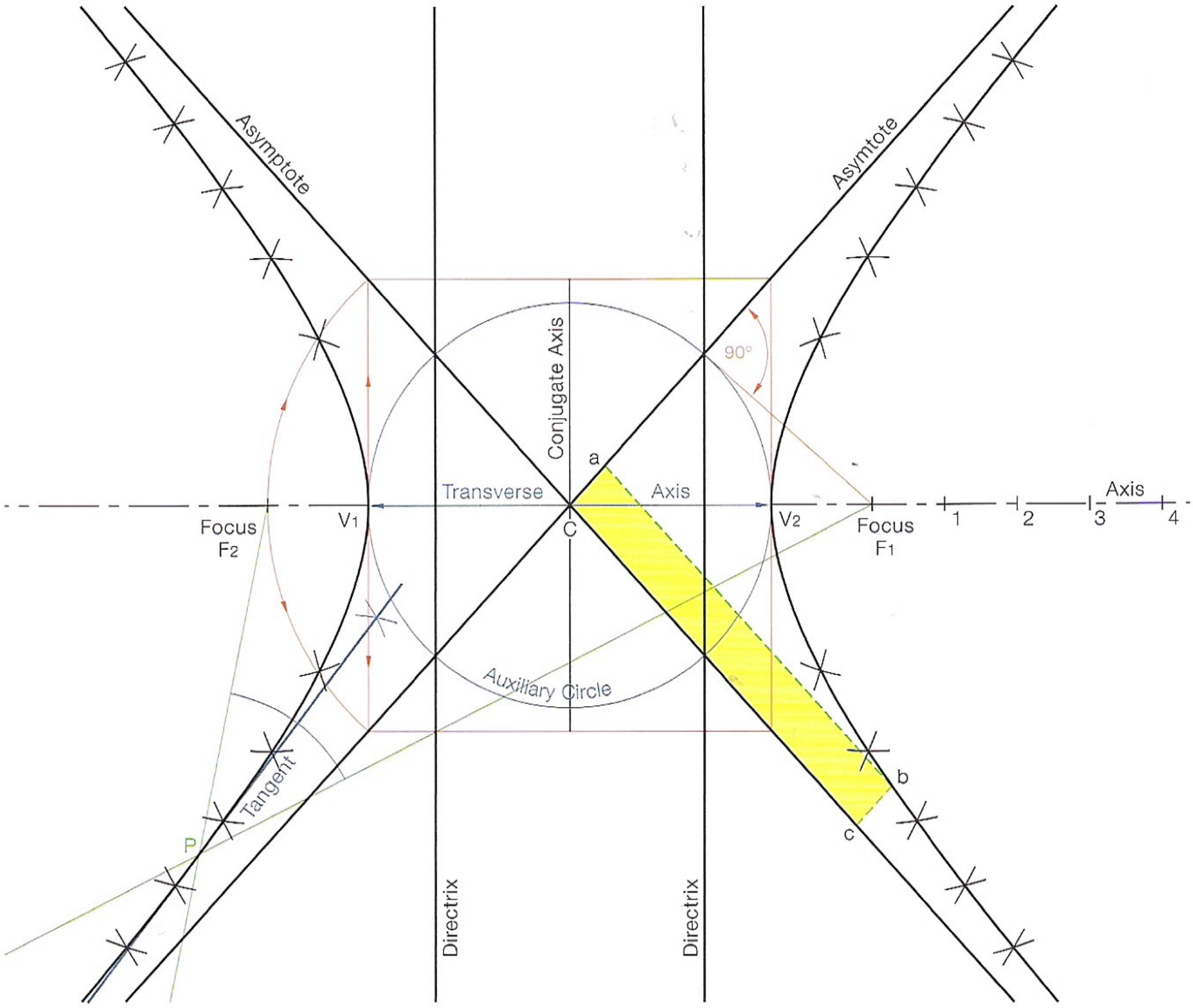

Fig. 8.79

Distance from $Focus_2$ to P − Distance from $Focus_1$ to P = Transverse Axis

$F_2P - F_1P$ = Transverse Axis

$F_2P - F_1P = V_1V_2$

Transverse Axis – The part of the axis between the vertices V_1 to V_2 = Transverse Axis.

Auxiliary Circle – The circle passing through both vertices and having its centre on the axis at C is called the auxiliary circle.

Asymptotes – These are the outer limits of the cones. They are straight lines which cross at C. The hyperbola curve will get closer and closer to the asymptote but will never touch it.

Conjugate Axis – A perpendicular to the axis through the centre C is called the conjugate axis.

Rectangle abcC – Select any point b on the curve. From b draw ba and bc parallel to the asymptotes. The resulting parallelogram has an area which is constant no matter where on the curve point b is selected.