Ellipse as a Section of a Cone

An ellipse is produced by a cutting plane which passes through both sides of a cone or will cut both sides of the cone when extended.

CONSTRUCTION

(1) Draw the plane and elevation of a cone.
(2) Divide the plan into sections using generators and project these onto the elevation.
(3) In elevation, draw in the cutting plane so that it cuts both sides of the cone or will cut both sides of the cone if the plane and cone are extended.
(4) The points where the generators are cut in elevation are projected down to the plan producing a curve as shown.
(5) The true shape of this curve is an ellipse and is produced by projecting perpendicularly from the cutting plane.

The widths are taken from the plan.

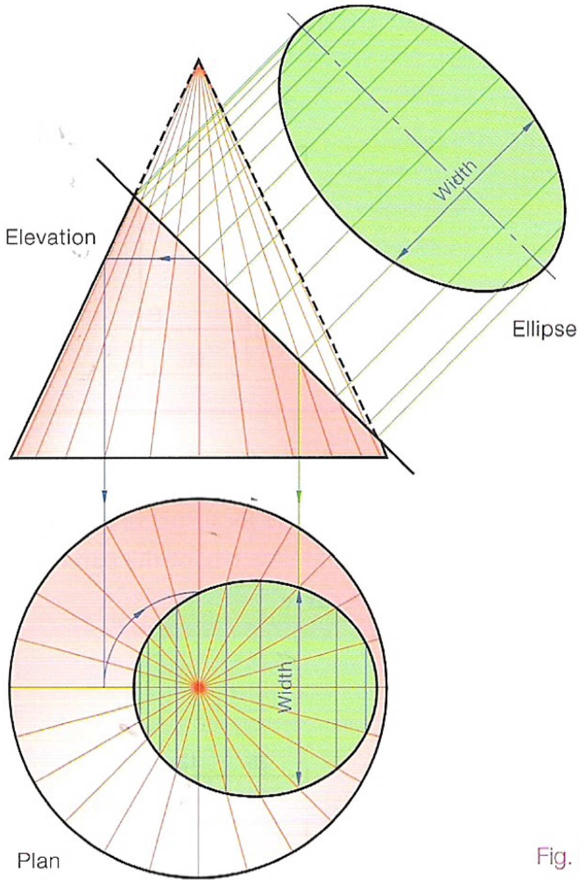

Fig. 8.34

Focal Spheres for an Ellipse

Similar to the construction for a parabola a sphere is inserted into the space between the cutting plane and the cone's vertex. The sphere is to touch the side of the cone and will touch the cutting plane at one point, the focus. The directrix is found by creating a plane at the level where the sphere makes contact with the cone, and extending it to intersect with the cutting plane. The line of intersection thus formed is the directrix, Fig. 8.35.

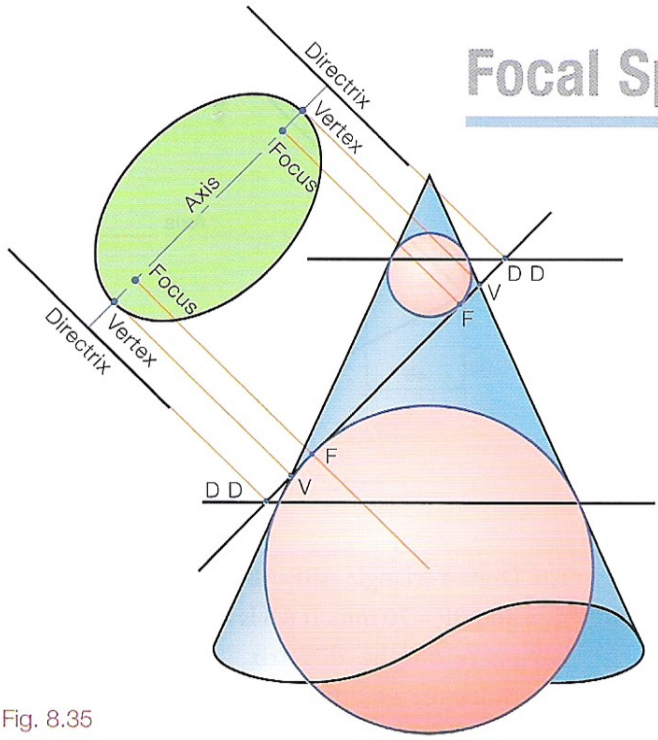

Fig. 8.35

HIGHER LEVEL

A second focal sphere is found underneath the cutting plane giving the second focus and the second directrix, Fig. 8.36.

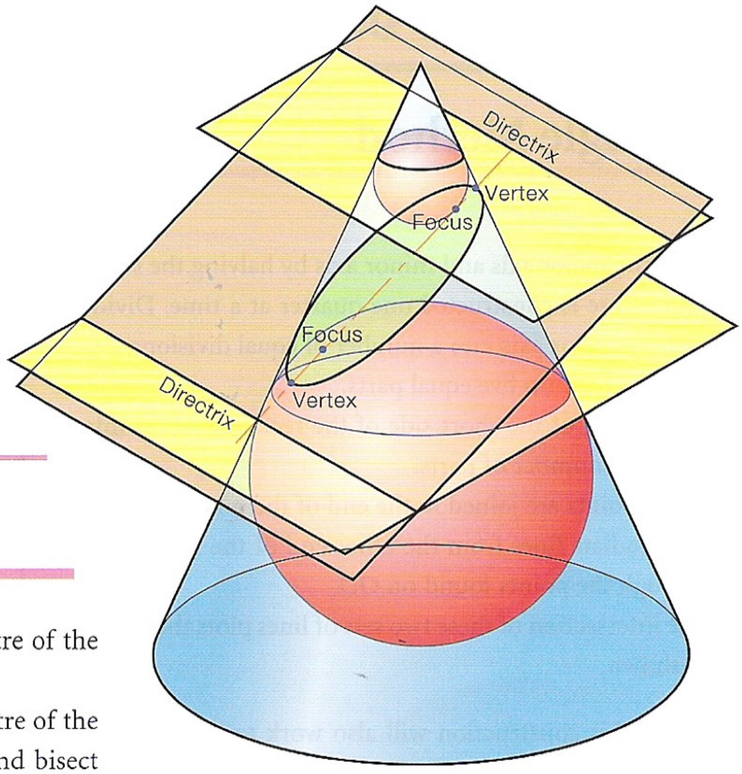

Fig. 8.36

Terminology

Major Axis – The longest axis going through the centre of the ellipse.

Minor Axis – The shortest line going through the centre of the ellipse. The major and minor axes cross at 90° and bisect each other.

Major Auxiliary Circle – The circle passing through both vertices and having its centre on the axis at C.

Minor Auxiliary Circle – The circle passing through both ends of the minor axis and having its centre on the axis at C.

Focal Points – The focal points are two points on the major axis. They are symmetrical about C. They are located a distance of ½ major axis from the ends of the minor axis. Ellipses with focal points near the centre C will be very circular. Ellipses having focal points near the ends of the major axis will be flat ellipses.

Point P – For any point P on the curve the distances F_1P and F_2P added together will equal the length of the major axis. $F_1P + F_2P = V_1V_2$ (Major Axis).

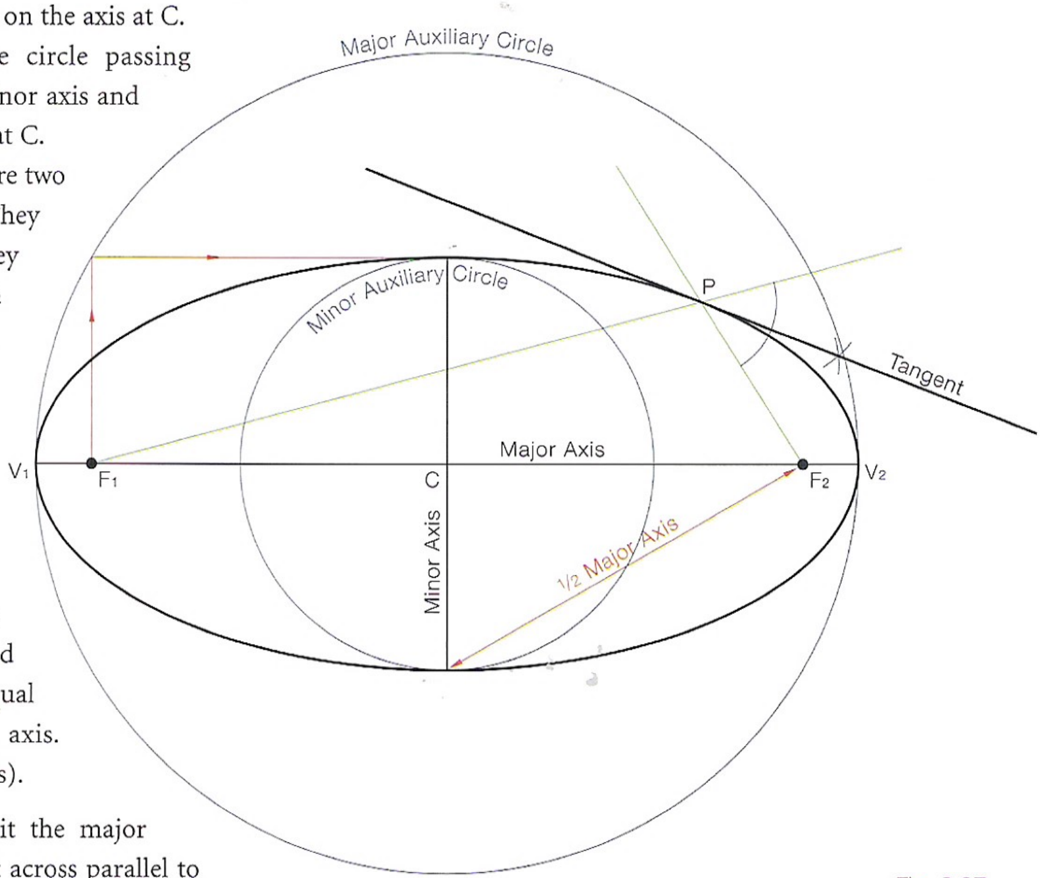

Fig. 8.37

A vertical from the focus to hit the major auxiliary circle and then brought across parallel to the major axis will give the top of the minor axis.

Five Methods of Constructing an Ellipse

Rectangle Method

(1) Find the major axis and minor axis by halving the sides.
(2) The ellipse is constructed one quarter at a time. Divide half the major axis into a number of equal divisions, e.g. OD divided into five equal parts.
(3) Divide half of the short side of the rectangle (CD) into the same number of parts.
(4) These points are joined to the end of the minor axis.
(5) Now radiate lines from the other end of the minor axis through the points found on OD.
(6) The intersection of these two sets of lines plots the ellipse as shown.

Note: This construction will also work to construct an ellipse in a parallelogram.

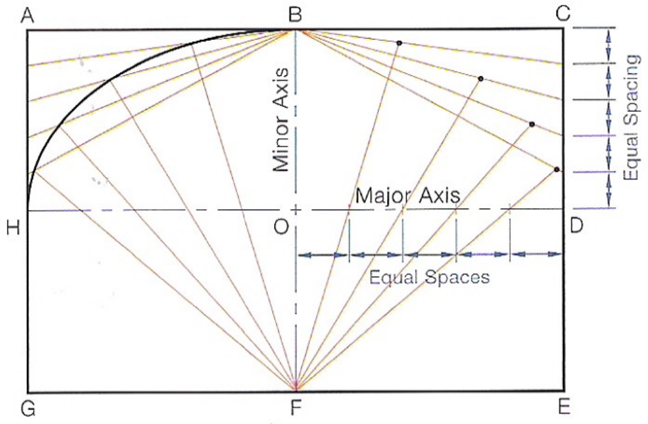

Fig. 8.38

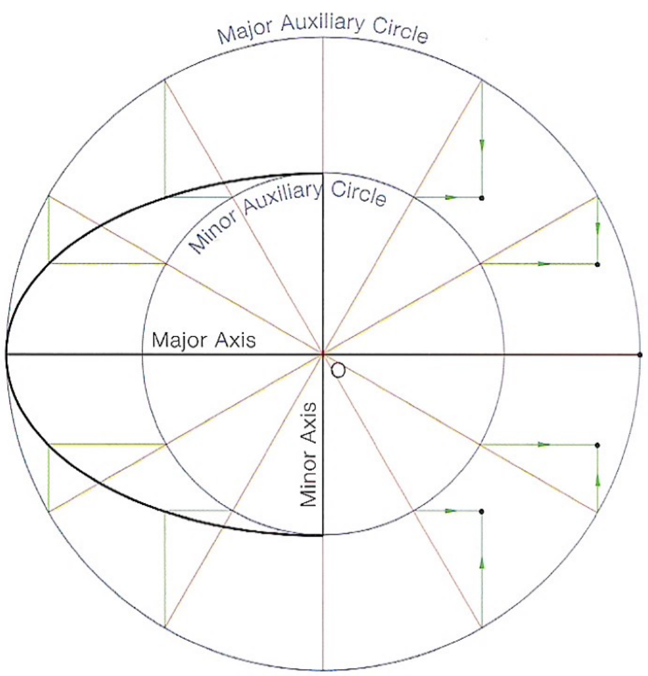

Fig. 8.39

Circle Method

(1) Given the major and minor axes, construct the two circles having O as centre and radii equal to half of the major axis and the minor axis.
(2) Divide the circles up from the centre, usually with the 30°/60° set-square.
(3) Where each radial line hits the major circle, draw lines toward the major axis and parallel to the minor axis.
(4) Where each radial line crosses the minor circle, draw lines away from the minor axis and parallel to the major axis.
(5) Where these lines intersect plots points on the curve.

Eccentricity Method

The eccentricity for an ellipse is always **less than 1**, e.g. 3/4, 2/3, 0.7, 0.61 etc. but is a constant for that particular ellipse.

As in the parabola, it is an expression of the relationship between the two distances:

(i) From the focal point to a point P on the curve.

(ii) From the same point on the curve to the directrix.

$$\text{Eccentricity} = \frac{\text{Distance from focus to a point}}{\text{Point to directrix}} = \frac{\text{F to P}}{\text{P to DD}}$$

The eccentricity line for an ellipse will always be at an angle of less than 45° to the axis.

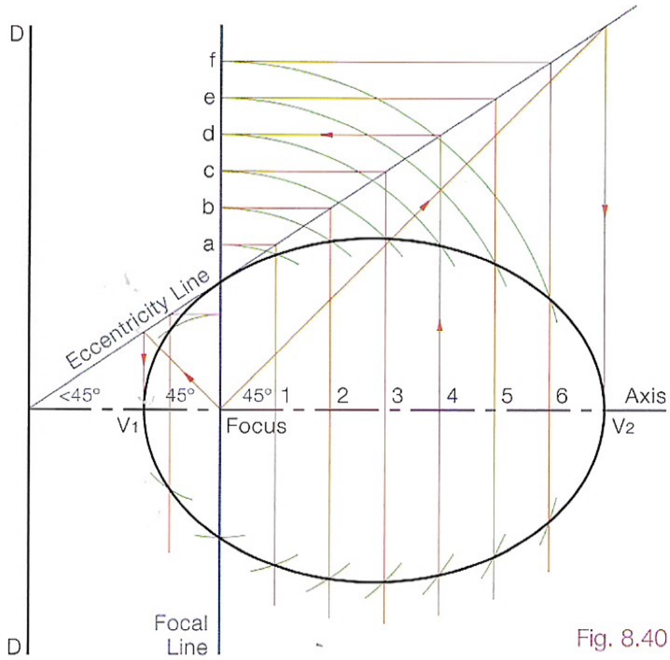

Fig. 8.40

CONSTRUCTION

Given the directrix, axis, focal point and eccentricity of 2/3.

(1) Set up the eccentricity line by measuring out on the axis a set distance from the directrix, e.g. 30 mm. Construct a perpendicular to the axis at this point. Measure up from the axis on this line a distance equal to 2/3 of the previous distance. 20 mm in this example.

(2) This gives a point on the eccentricity line which now can be drawn.

(3) Now follow the normal procedure as explained for the parabola in Fig. 8.7.

(4) The ellipse has two vertices and these are found by constructing 45° lines to the axis from the focus to hit the eccentricity line. These points are projected to give V_1 and V_2.

(5) Points on the curve are found by drawing ordinates. Where these intersect the eccentricity line projects across, parallel to the axis, to the focal line.

(6) With the focus as centre, rotate the points found on the focal line back to each ordinate, above and below the axis.

(7) The points found in this way can be joined giving an ellipse.

Trammel Method

This is a very useful method of constructing an ellipse as it is both quick and accurate.

(1) Cut a piece of paper to use as a trammel. It should be slightly longer than half the major axis.

(2) Mark the length of half the minor axis on the trammel AB.

(3) Using A as an end point, now mark half the major axis on the paper, AC.

(4) The trammel can now be placed on the major and minor axis so that point B rests on the major axis and point C on the minor axis.

(5) Plot the location of point A which is a point on the curve.

(6) Rotate the trammel, keeping the points B and C on their appropriate axes. Continue to plot points at A.

(7) Join these plotted points to form an ellipse.

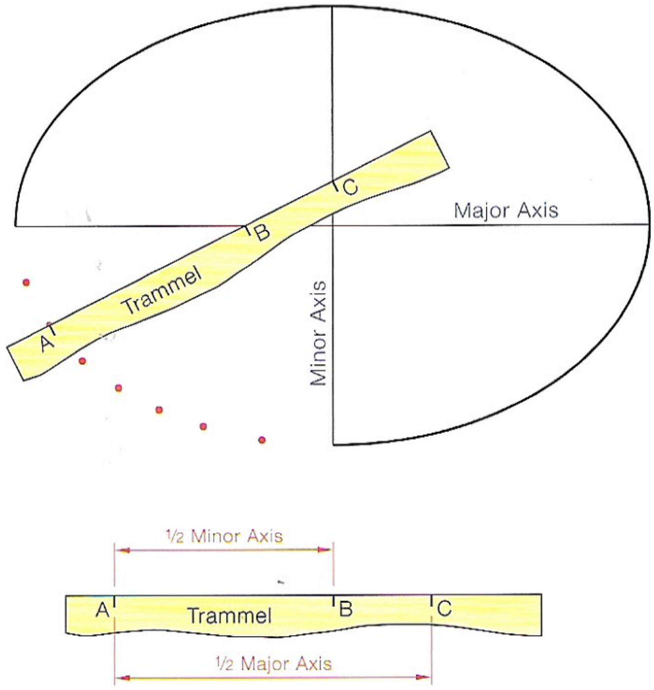

Fig. 8.41

Pin and String Method

This is a good method for drawing large scale ellipses. The method of construction is based on the fact that the distance from F_1P added to F_2P will remain constant and equals the length of the major axis.

$F_1P + F_2P =$ Major Axis

(1) The pins are fixed at the focal points.

(2) String is tied to the pins so that the amount of string left between the pins equals the length of the major axis of the required ellipse.

(3) By keeping the string stretched with a pencil and moving it around, an ellipse will be plotted.

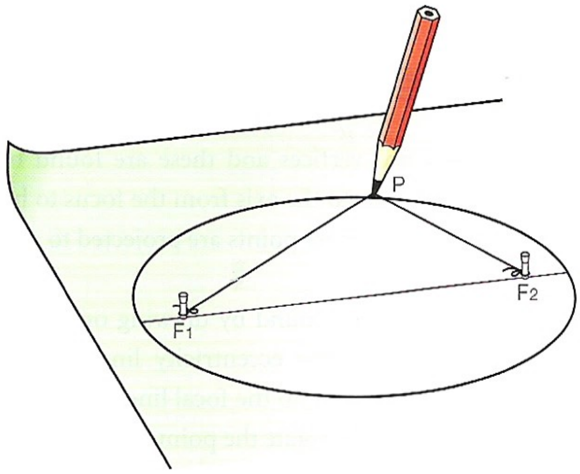

Fig. 8.42

Tangent to an Ellipse from a Point on the Curve

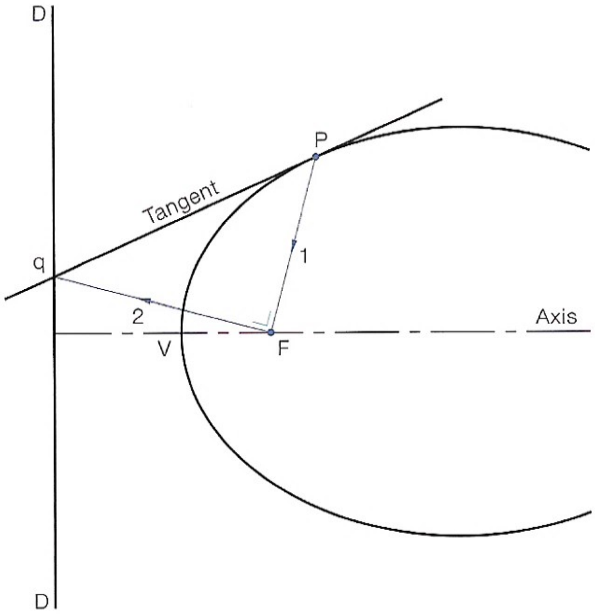

Fig. 8.43

Method 1

Fig. 8.43

This method works for all conics and has already been shown in Fig. 8.8 for the parabola.

(1) Draw a line joining point P back to the focus.
(2) At the focus, draw a new line perpendicular to PF and extend to hit the directrix DD at q.
(3) Point q is a second point on the tangent.
(4) Join P to q, forming the tangent.

Method 2

Fig. 8.44

(1) Join F_1 to P and extent to q.
(2) Join F_2 to P and extent to r.
(3) The line that bisects the angle F_2Pq will be the required tangent.
(4) Alternatively F_1Pr could be bisected.
(5) The normal could be found by bisecting the angle F_1PF_2. See the similarity to that used for the parabola in Method 3, Fig. 8.10.

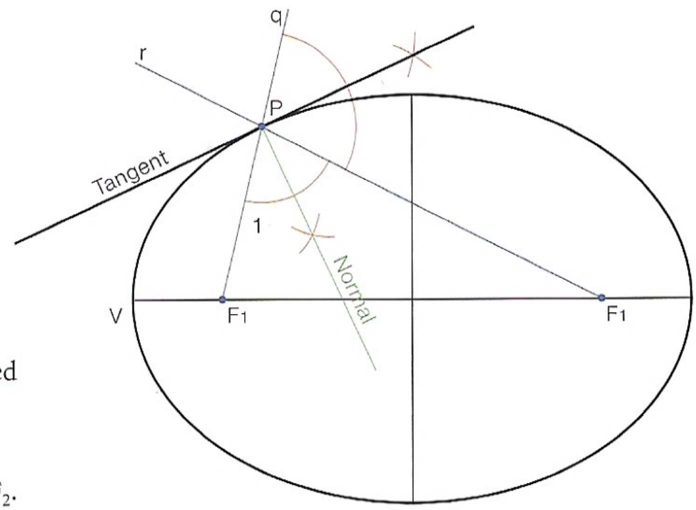

Fig. 8.44