4 Orthographic Projection of Solids in Contact

SYLLABUS OUTLINE

Areas to be studied:

- Right solids in contact.

Learning outcomes

Students should be able to:

Higher and Ordinary levels

- Construct views of up to three solids having curved surfaces and/or plane surfaces in mutual contact.
- Determine point of contact for surfaces in mutual contact.
- Construct views of solids given the point of contact.
- Depict the solutions of two-dimensional problems in three-dimensional format.

Higher Level only

- *Model various problems involving solids in contact, planes of reference and auxiliary planes.*

Solids in Contact

In this chapter you will draw the orthographic views of spheres, cones, cylinders, pyramids and prisms in contact with each other. Before this, we must examine how to find points on the surface of cylinders, cones and spheres.

The elevation of a cylinder is shown in Fig. 4.1 and the position of a point P on its surface. Draw the plan of the cylinder and point P.

By using an end view the location of point P in the plan can be easily found, see Fig. 4.2.

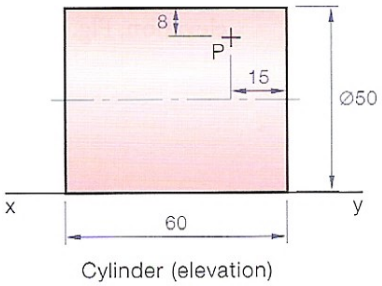

Fig. 4.1 Cylinder (elevation)

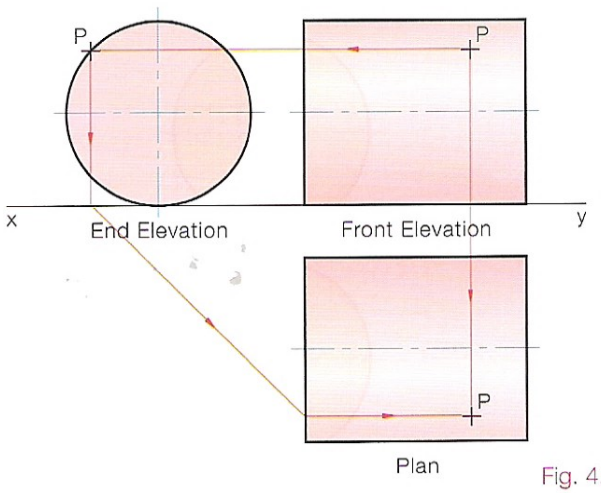

Fig. 4.2

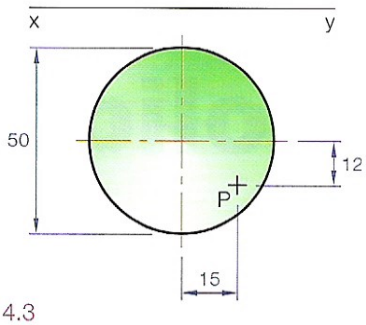

Fig. 4.3

The plan of a cone of altitude 60 mm is shown in Fig. 4.3. Also shown is a point P on its surface. Draw the elevation and locate point P.

There are two methods shown to solve this problem.

Method 1 (Fig. 4.4a)

Rotate point P in plan onto the horizontal axis. Project to the side of the cone in elevation giving the height of point P. Project the height horizontally and bring point P up from plan onto it.

Method 2 (Fig. 4.4b)

Draw the generator through point P in plan. Project this generator to elevation. Point P is projected onto this generator in elevation.

The horizontal section method is the preferred method.

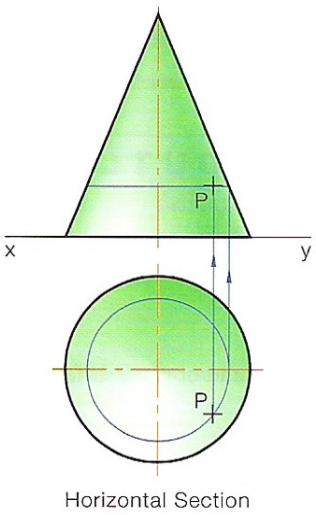

Fig. 4.4a

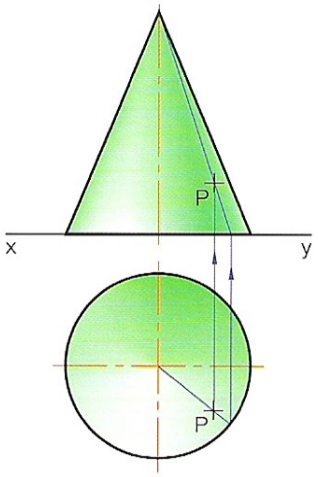

Fig. 4.4b

The elevation of a sphere with a point P on its surface is shown in Fig. 4.5. Draw the plan and locate point P on it.

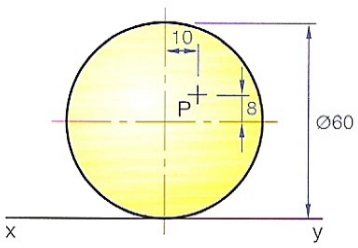

Fig. 4.5

As in the previous example, a horizontal section is used. Project P horizontally to touch the side of the sphere. Project down to the horizontal axis of the plan and rotate round. Drop point P from elevation, Fig. 4.6.

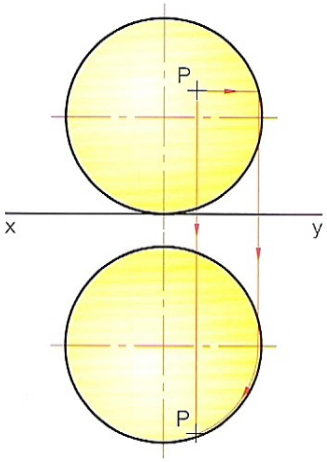

Fig. 4.6

The plan of a cone of altitude 70 mm is shown in Fig. 4.7. Also shown is a point P on the cone's surface. A sphere which rests on the horizontal plane touches the cone at point P. Draw the plan and elevation of the two solids in contact.

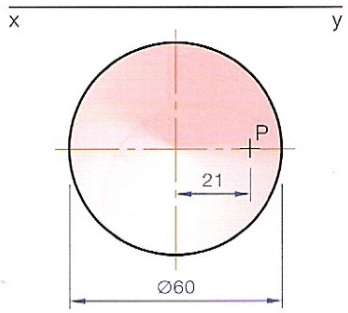

Fig. 4.7

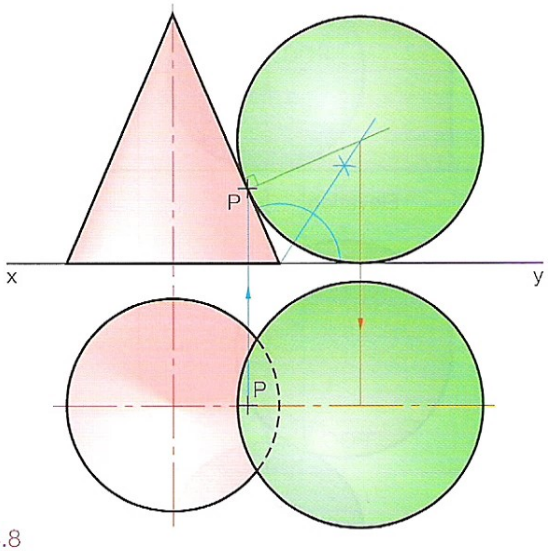

Fig. 4.8

(1) Draw the plan and project the elevation. Locate point P in elevation.

(2) Since the cone is tangential to the sphere, its edge will form a tangent to the sphere in elevation. A perpendicular to the side of the cone from point P will therefore pass through the sphere's centre.

(3) Since the sphere touches the horizontal plane and the cone edge, its centre will be on the bisector of the angle between the two, in elevation, Fig. 4.8.

(4) Draw the sphere in elevation and plan.

The plan of a cone of 70 mm altitude is shown in Fig. 4.9. Also shown is a point P on the cone's surface.

(i) Draw the plan and elevation of the cone and the point P.

(ii) Draw the plan and elevation of a sphere that rests on the horizontal plane and touches the cone at point P.

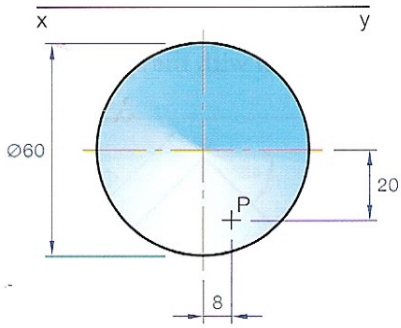

Fig. 4.9

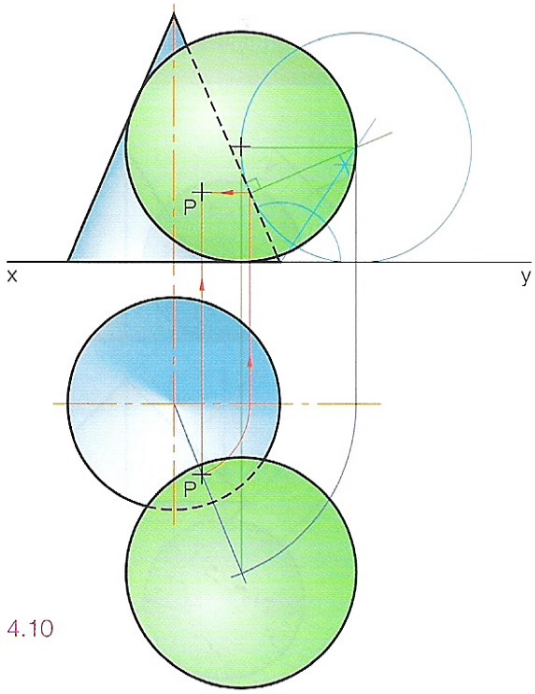

Fig. 4.10

(1) Draw the plan and elevation of the cone.

(2) Rotate point P in plan about the cone centre onto the axis. Project up to the side of the cone in elevation and then project across horizontally. Locate point P on this horizontal.

(3) The sphere required to touch the cone at point P and touch the horizontal plane is constructed at the side of the cone. The construction is the same as in the previous example.

(4) Once the sphere centre is located, it is dropped to plan and rotated onto a line drawn from the cone centre through point P.

The centre of the cone, the point of contact and the sphere centre will form a straight line.

(5) Draw the sphere in position in plan. Project the sphere centre to elevation and draw the sphere, Fig. 4.10.

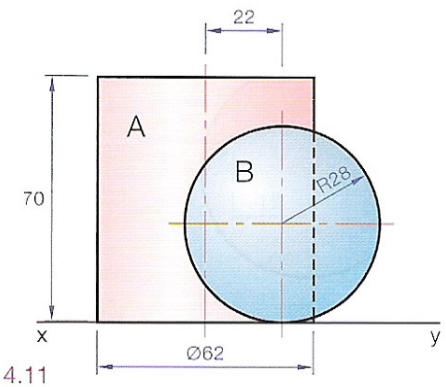

Fig. 4.11

Shown in Fig. 4.11 is the elevation of a cylinder A and a sphere B. Both solids are in contact with each other and rest on the horizontal plane. Draw the elevation and plan of the solids and the point of contact.

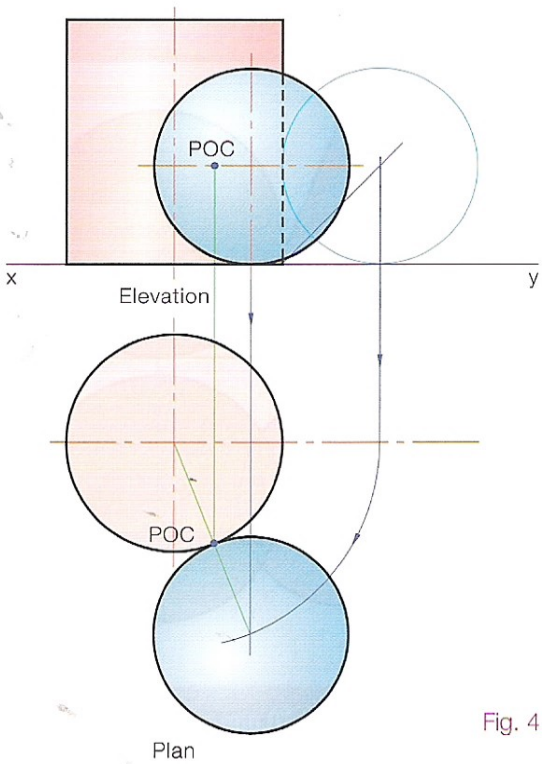

Fig. 4.12

(1) Draw the given elevation of both solids and project the plan of the cylinder.
(2) The sphere is drawn at the side of the cylinder. The point of contact (POC) can be clearly seen.
(3) Drop the centre of the sphere to the plan's horizontal axis and rotate it into the correct position. Draw the sphere in plan.
(4) Join the centres of the solid in plan thus locating the point of contact.
(5) Project the point of contact up from the plan and across from the constructional sphere, Fig. 4.12.

When a sphere and cylinder are in contact the point of contact will be level with the sphere centre.

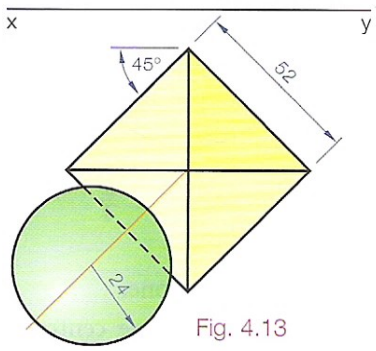

Fig. 4.13

The plan of a square-based pyramid and a sphere, resting on the horizontal plane, are shown in Fig. 4.13. They are in contact with each other. Draw the plan and elevation of the solids showing the point of contact. The pyramid has an altitude of 58 mm.

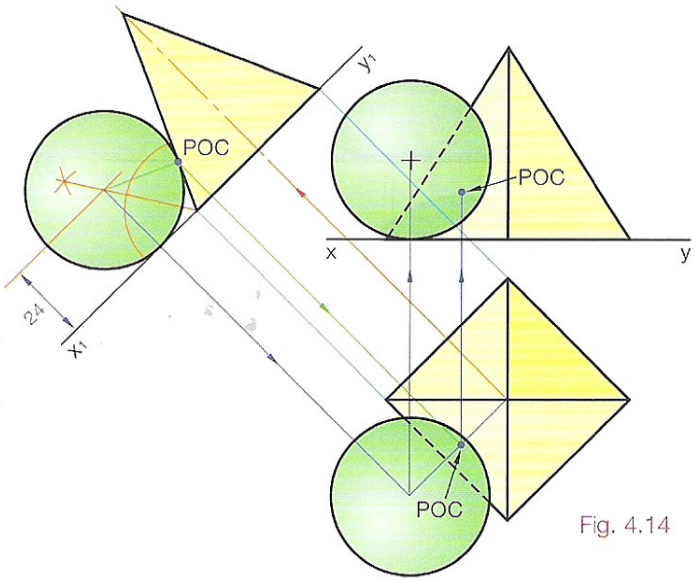

Fig. 4.14

(1) Draw the plan and elevation of the pyramid.
(2) Project an auxiliary elevation to show the face that is in contact with the sphere as an edge view.
(3) Construct the sphere in the auxiliary. The radius is 24 mm so the centre is 24 mm above the x_1y_1 line. Also bisect the angle between the x_1y_1 and the side of the pyramid.
(4) Once the sphere is located in auxiliary it is projected back to plan and elevation.
(5) The point of contact is first found in the auxiliary by drawing a line from the sphere centre, perpendicular to the face of the pyramid. It is projected to plan and then elevation. The height of the POC in elevation equals the height of the POC in the auxiliary elevation, Fig. 4.14.

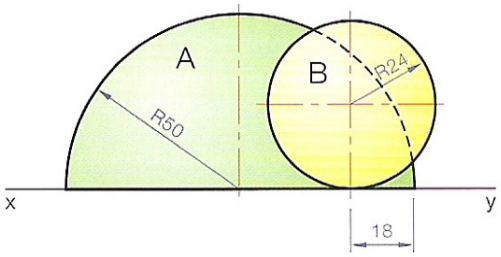

Fig. 4.15

The elevation of a hemisphere A and a sphere B resting on the horizontal plane is shown in Fig. 4.15. The solids are in contact. Draw the plan and elevation of the solids and show the point of contact in both views.

(1) Draw the given elevation and draw the plan of the hemisphere A.

(2) Sphere B is drawn to the side of the hemisphere in elevation. The centre is located by projecting the centre of sphere B horizontally. This horizontal line is then cut by an arc drawn from the centre of the hemisphere and equal to the two radii added together, i.e. 50 mm + 24 mm = 74 mm.

(3) The point of contact is located between the hemisphere and the constructional sphere by joining their centres.

(4) Both point of contact and sphere centre are dropped to plan and rotated into position.

(5) The point of contact is located in elevation, Fig. 4.16.

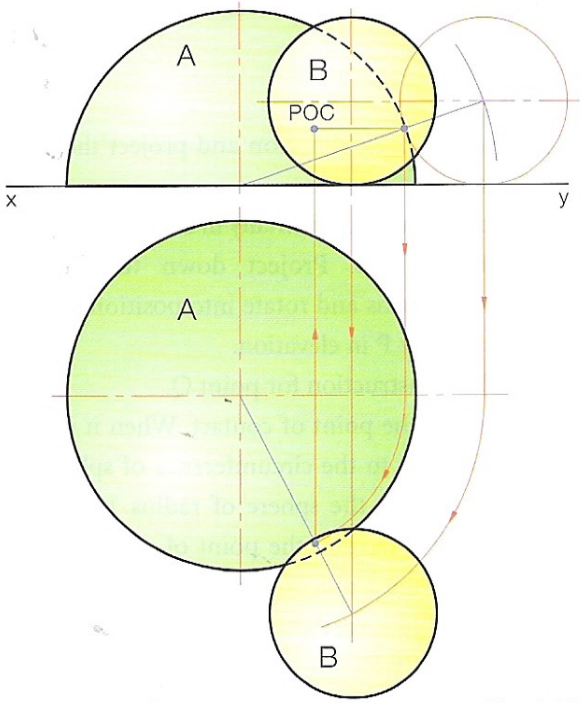

Fig. 4.16

Fig. 4.17 shows the plan of a cone, resting on the horizontal plane, having an altitude of 60 mm. Also shown is a point P on the cone's surface. Draw the plan and elevation of the cone. Draw the plan and elevation of a sphere that will touch point P and also rest on the horizontal plane.

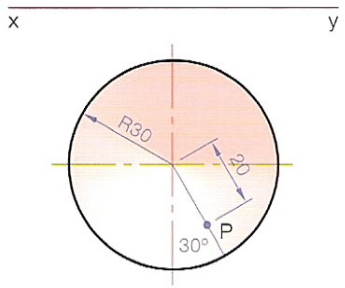

Fig. 4.17

(1) Draw the given plan and project the elevation. Point P is rotated in plan onto the horizontal axis, projected to the side of the cone and then horizontally.

(2) The sphere is constructed to the side of the cone in elevation. Point P is brought horizontally to the side of the cone and a perpendicular constructed to the cone edge. The angle between the xy and the cone side is bisected. The intersection between the perpendicular and the bisector gives the sphere centre.

(3) The sphere is rolled into position and drawn in both views, Fig. 4.18.

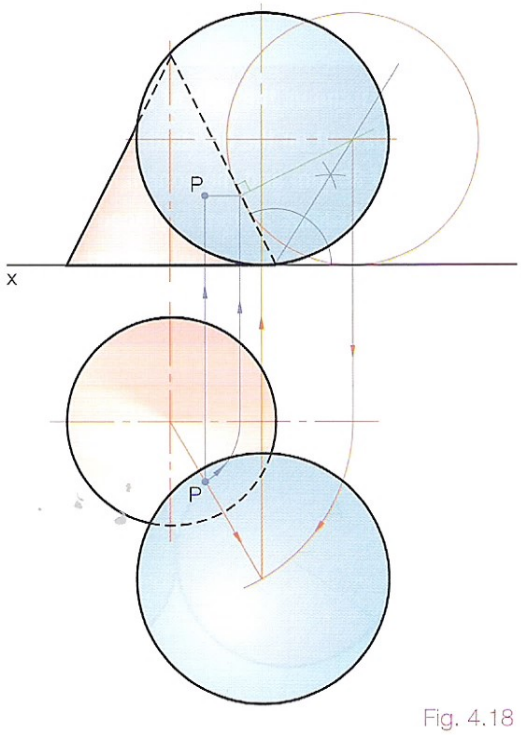

Fig. 4.18

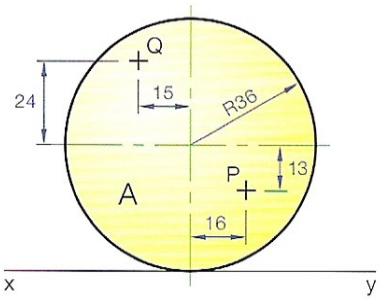

Fig. 4.19

Fig. 4.19 shows the elevation of a sphere A with two points, P and Q on its surface.

(i) Draw the given elevation and project a plan showing points P and Q in both views.

(ii) Show the projections of a sphere of radius 15 mm that will be in contact with the sphere A at point P.

(iii) Show the projections of a sphere of radius 28 mm that will be in contact with sphere A at point Q.

(1) Draw the given elevation and project the plan of the sphere.

(2) Project point P horizontally in elevation to the side of the sphere. Project down to the plan's horizontal axis and rotate into position, vertically below point P in elevation.

(3) Similar construction for point Q.

(4) Point P is the point of contact. When it is moved horizontally to the circumference of sphere A we can construct the sphere of radius 15 mm. The centre of sphere A, the point of contact and the centre of the required sphere will be in line. The distance between the centres will be equal to the sum of the radii.

(5) Draw the sphere and project through the views.

(6) Similar construction for the sphere touching at point Q, Fig. 4.20.

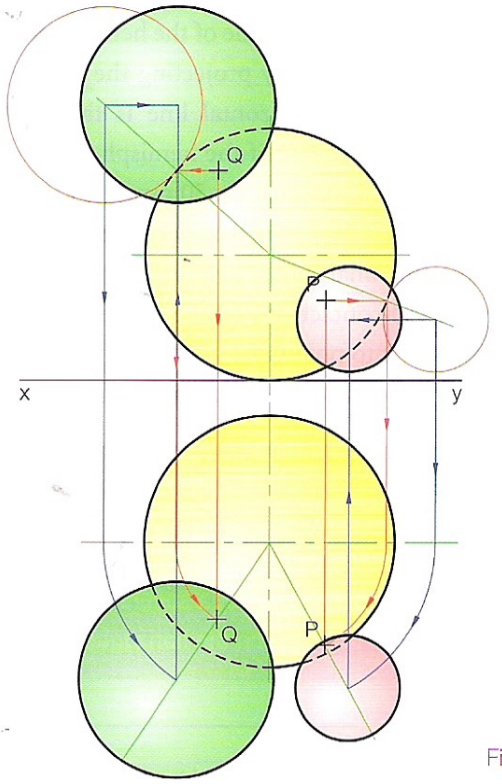

Fig. 4.20

Fig. 4.21 shows the plan of a cylinder A and a cone B. Both solids rest on the horizontal plane. The cylinder has an altitude of 80 mm and the cone has an altitude of 50 mm. A sphere C of radius 20 mm is placed so that it touches both these solids and rests on the horizontal plane. Draw the plan and elevation of the solids and show the points of contact.

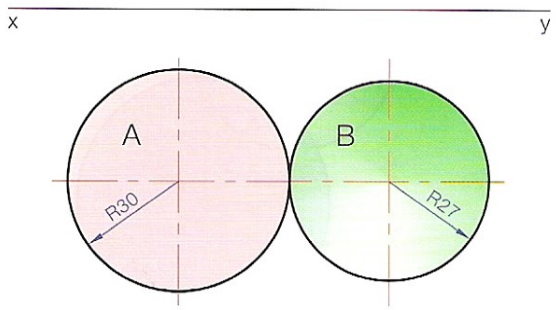

Fig. 4.21

(1) Draw the elevation and plan of the cone and cylinder.

(2) Draw the sphere C touching the side of the cylinder. Project the centre to the plan and rotate it about the cylinder. Similarly for the cone, draw the sphere C touching its side in elevation.

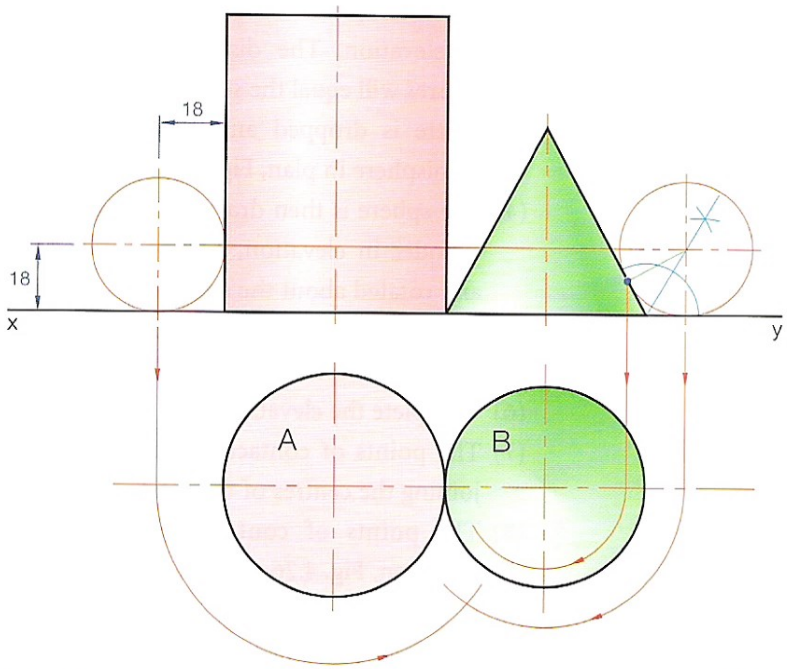

Fig. 4.22

(3) The required sphere has its centre where these two arcs cross in plan, Fig. 4.22.

(4) The point of contact between the sphere and cone is found at the side of the cone in elevation, dropped to the plan and rotated into position. the construction is evident from Fig. 4.23.

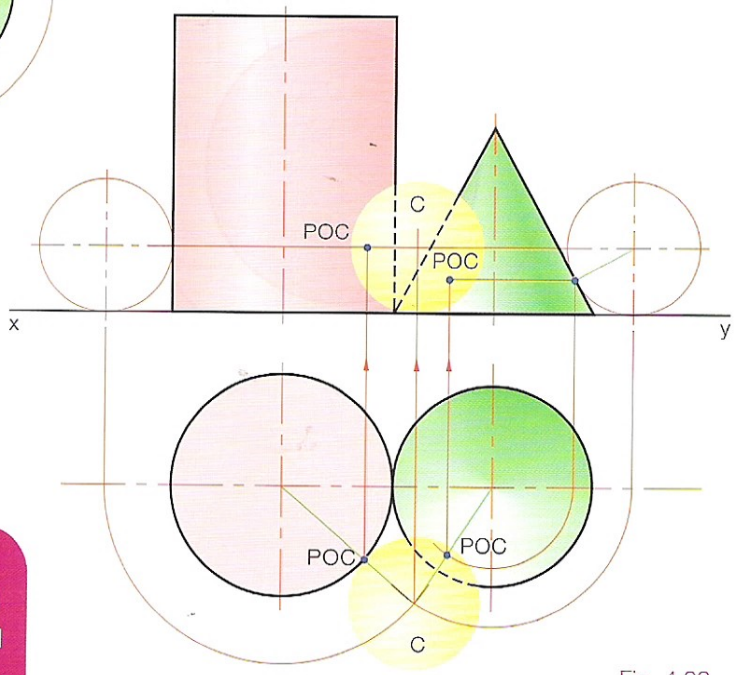

Fig. 4.23

Shown in Fig. 4.24 is the elevation of a cylinder A and a hemisphere B in contact with each other and resting on the horizontal plane. A sphere of 50 mm diameter is placed in position C. The sphere is to be in contact with the other two solids and the horizontal plane. Draw the plan and elevation of the solids showing all points of contact.

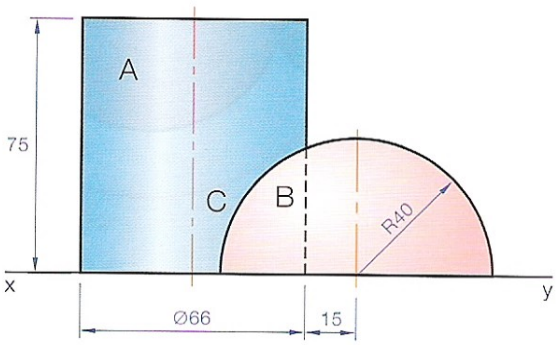

Fig. 4.24

(1) Draw the given elevation and project the plan of the cylinder.

(2) The centre of hemisphere B is projected to plan. The distance between the centres of A and B in plan will equal the sum of their radii.

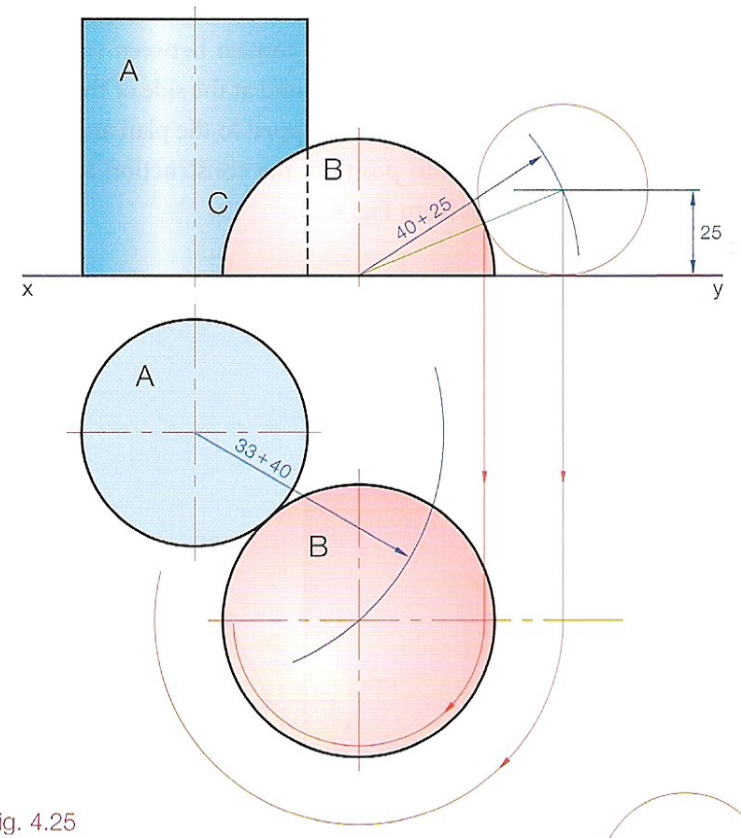

Fig. 4.25

(3) Draw the sphere to the side of hemisphere B in elevation. The distance between their centres will equal the sum of their radii. The centre is dropped and rotated about the hemisphere in plan, Fig. 4.25.
(4) The sphere is then drawn to the side of the cylinder in elevation, dropped to the plan and rotated about the cylinder.
(5) The centre of the sphere is located where the arcs from steps 3 and 4 cross.
(6) Complete the elevation.
(7) The points of contact are located on lines joining the centres of the solids in plan.
(8) The points of contact are projected to elevation, Fig. 4.26.

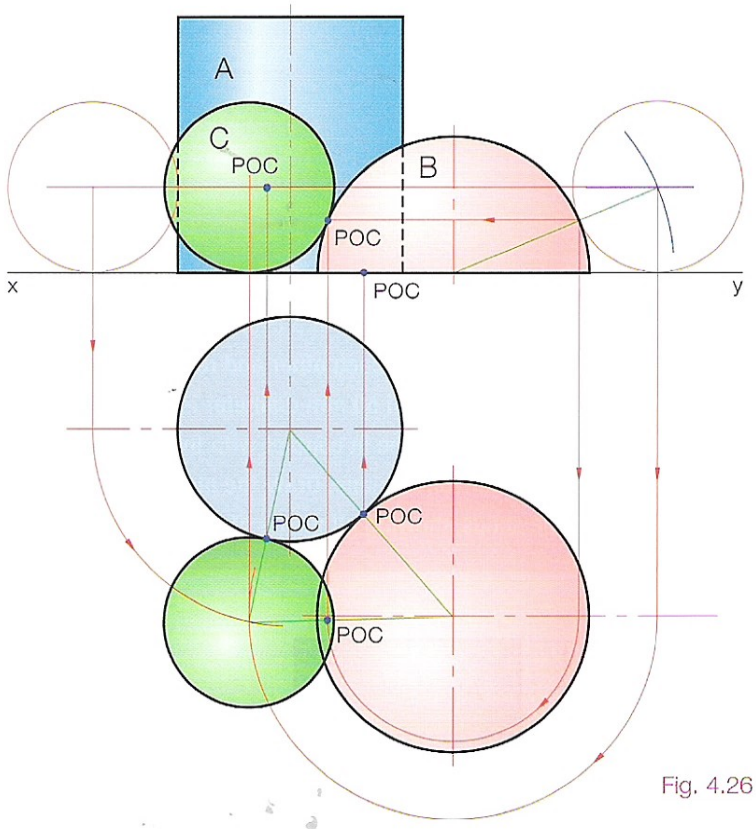

Fig. 4.26