Cube and Tetrahedron

The Cube

This is by far the most familiar of the regular polyhedra of which there are five:

- Tetrahedron
- Cube
- Octahedron
- Dodecahedron
- Icosahedron.

A cube has six faces, eight vertices and twelve edges. Each face is a square, three of which come together at each vertex. It is the only regular polyhedron that can be tiled by itself to fill three-dimensional space. A cube of unit edge is defined as the unit of volume and all other volumes are measured by the number of unit cubes they can contain. Another name for the cube is the **regular hexahedron**.

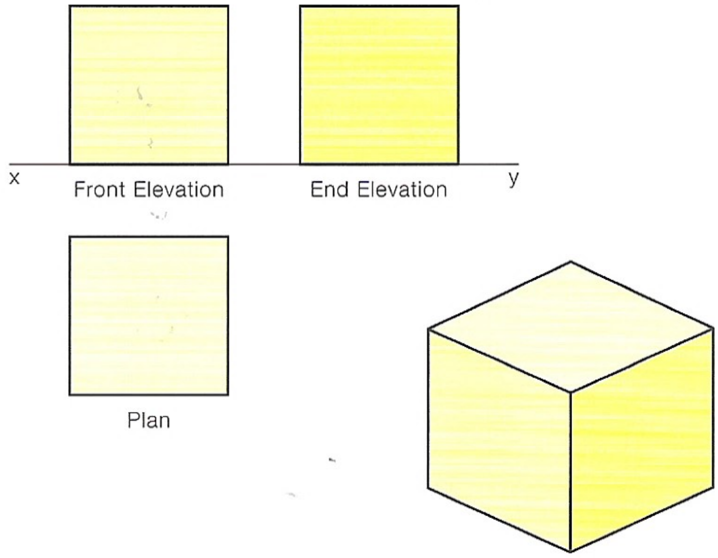


Fig. 3.34

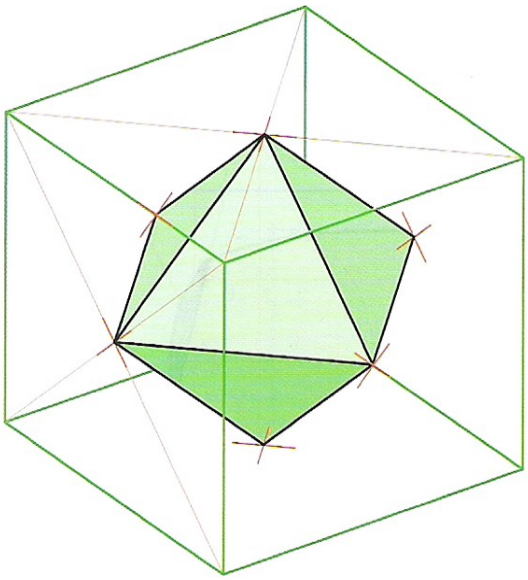


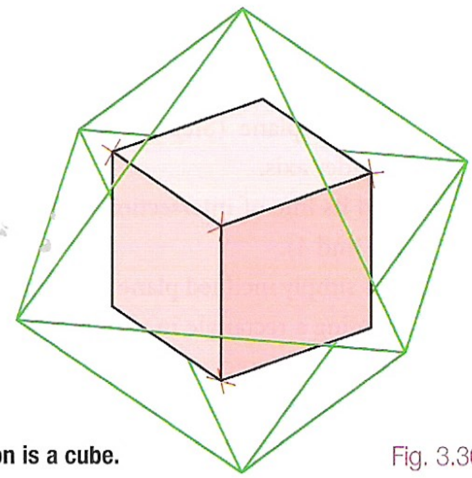
Fig. 3.35 **Dual of a cube is an octahedron.**

What is even more interesting is that the dual of a regular octahedron is a cube. Finding the dual of the dual will give back the original solid.

Duality

For every polyhedron there exists a dual polyhedron. Just looking at the regular polyhedra it will be shown that the tetrahedron is self-dual and the cube and the octahedron are a dual pair. The icosahedron and the dodecahedron are another dual pair but will not be looked at here.

Find the centre of each face by joining the diagonals. This gives the six vertices of the octahedron. Join each corner to its four neighbours, Fig. 3.35.



Dual of an octahedron is a cube. Fig. 3.36

Sectioning a cube to produce a regular hexagon

If a cube is cut by a section plane so that it passes through the centre points of the edges, as shown in Fig. 3.37, a regular hexagon is produced. This section plan cuts the cube into two equal halves.

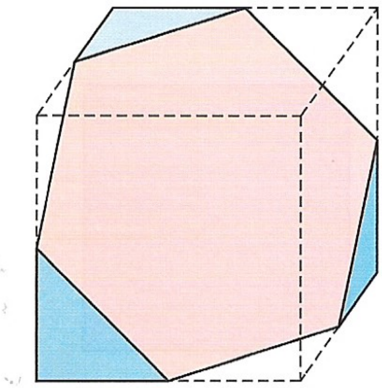


Fig. 3.37

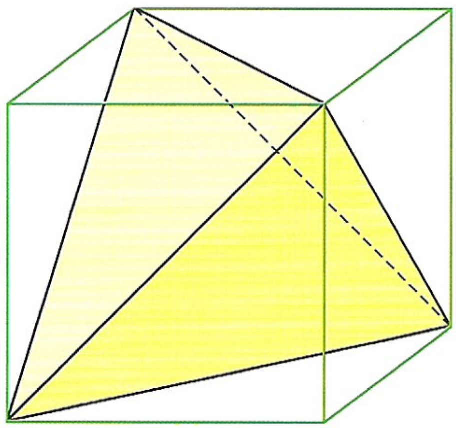


Fig. 3.38

Tetrahedron inside a cube

All the faces of a cube are squares of equal size. The diagonals of each square are therefore also equal. The six face diagonals in the cube in Fig. 3.38 form the edges of a tetrahedron. There are two possible arrangements of such tetrahedrons inside a cube.

Diagonals and sides

There obviously is a constant relationship between the sides of a cube, its short diagonals and its long diagonals. This relationship can be seen graphically in Figures 3.40 and 3.41 and can also be worked out quite easily using trigonometry.

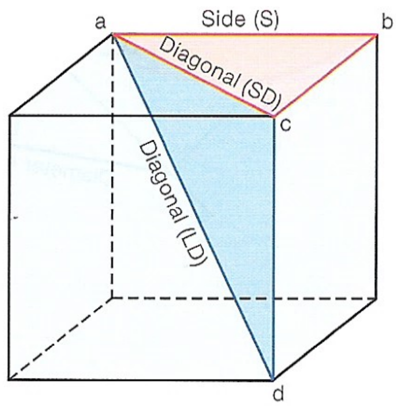


Fig. 3.39

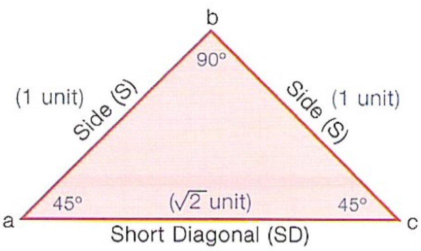


Fig. 3.40

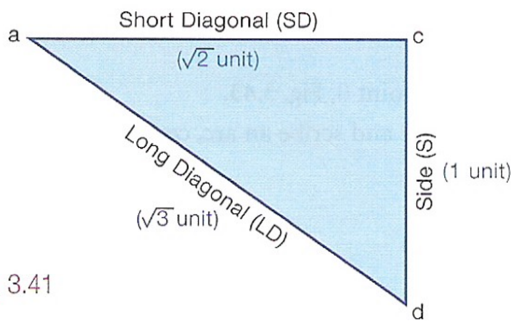


Fig. 3.41

Taking the side of the cube to be one unit in length it can be seen that the short diagonal equals $\sqrt{2}$ units in length (Pythagoras's Theorem). The long diagonal equals $\sqrt{3}$ units in length (Pythagoras's Theorem). Trigonometry can prove that angle dac equals 35° 15′ 51.8″ and angle cda equals 54° 44′ 8.2″.

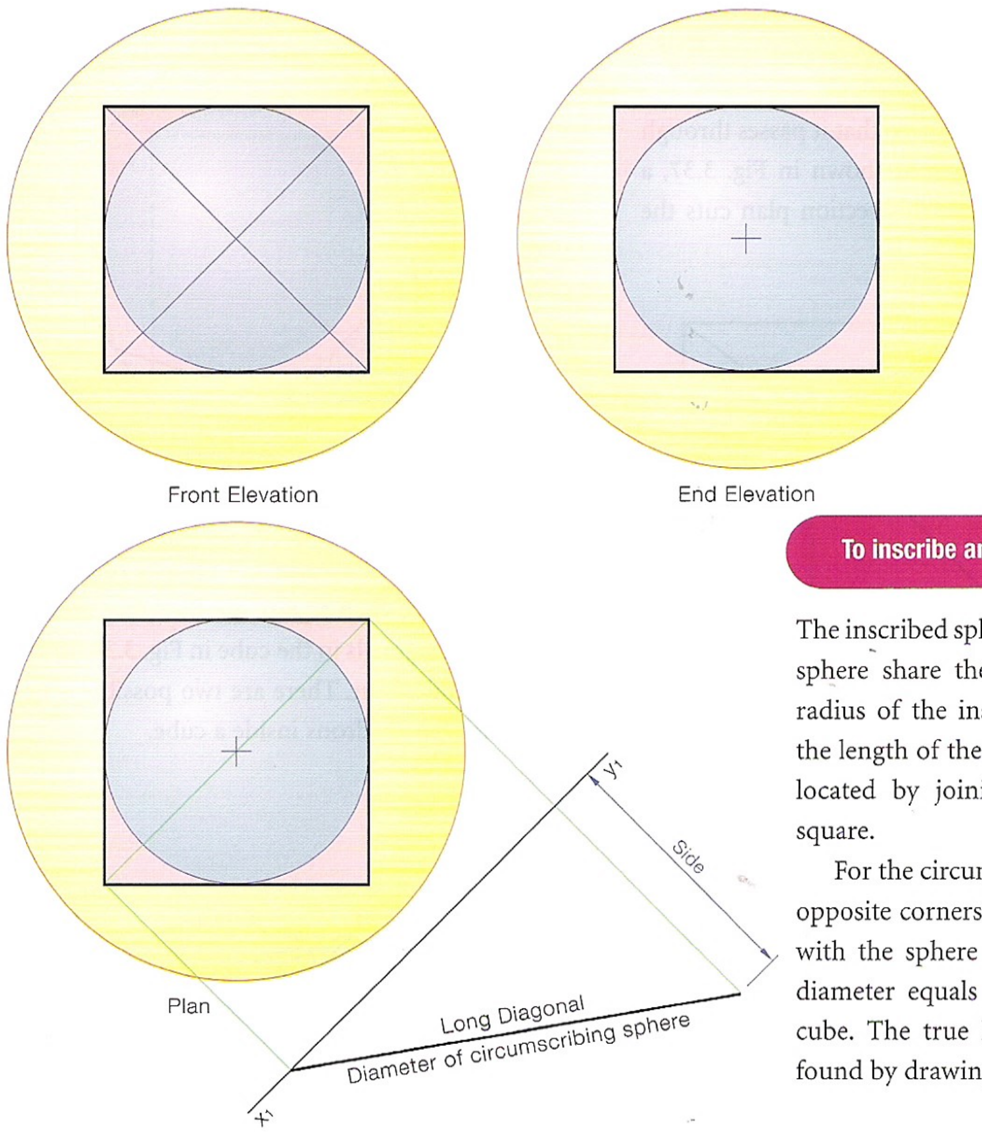


Fig. 3.42

To inscribe and circumscribe a cube

The inscribed sphere and the circumscribed sphere share the same centre point. The radius of the inscribed sphere equals half the length of the square's sides. Its centre is located by joining the diagonals of the square.

For the circumscribed sphere diagonally opposite corners of the cube are in contact with the sphere so therefore this sphere's diameter equals the long diagonal of the cube. The true length of this diagonal is found by drawing an auxiliary.

The Tetrahedron

A tetrahedron is the simplest possible polyhedron. It has four faces, four vertices and six edges. Each face is an equilateral triangle for a regular tetrahedron.

Construction

(1) Draw the plan which is an equilateral triangle and bisect each angle to locate point 0, Fig. 3.43.

(2) Edge 2,0 will be seen as a true length in end view. Project vertex 2 to the xy line and scribe an arc, centre at vertex 2, to intersect the line projected for the apex 0.

(3) Complete the end view and front elevation.

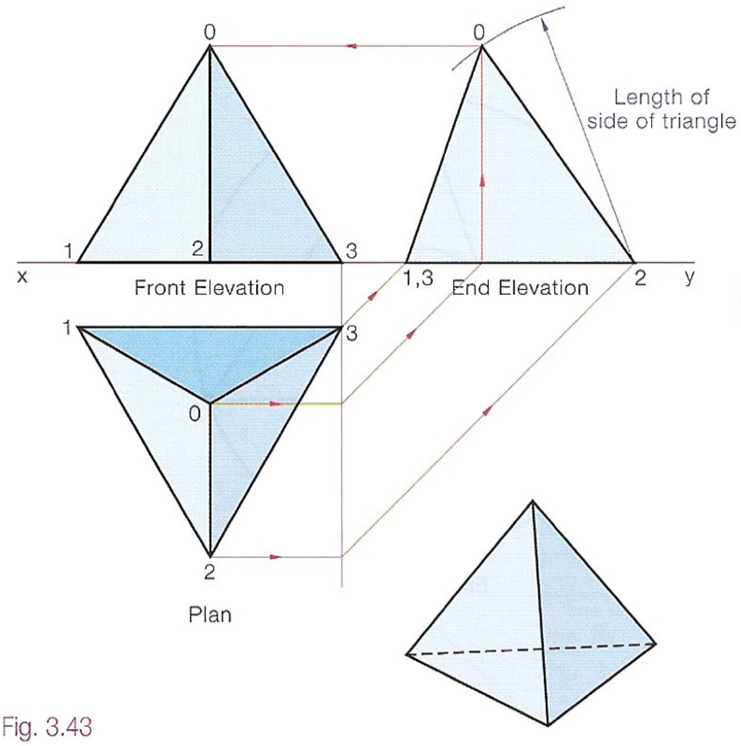


Fig. 3.43

Sectioning a tetrahedron

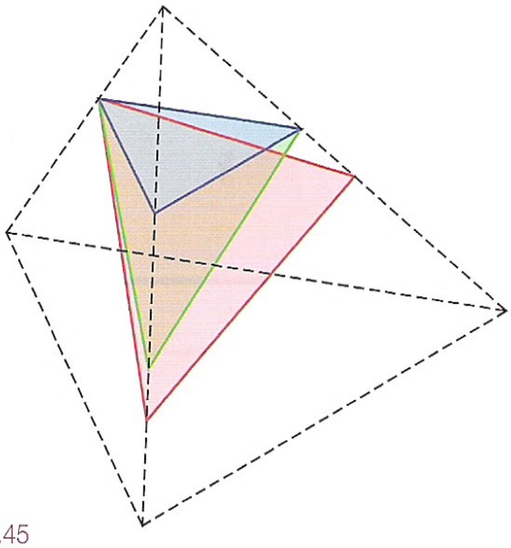


Fig. 3.45

The tetrahedron can be sectioned to give an equilateral triangle, an isosceles triangle or a scalene triangle as shown in Fig. 3.45. It can also be sectioned to produce rectangles and squares as well as other quadrilaterals, Fig. 3.46.

Duality

The tetrahedron stands alone as a self-dual polyhedron. When the centroid of each face is found and joined to its neighbours another tetrahedron is formed, Fig. 3.44.

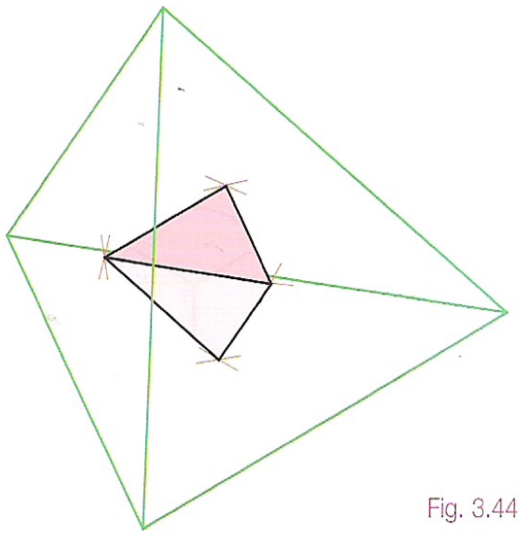


Fig. 3.44

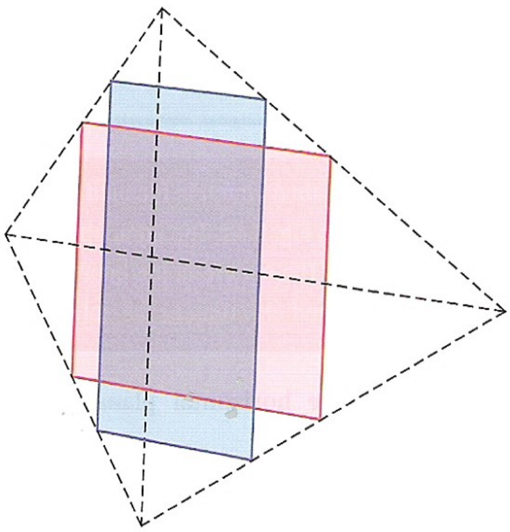


Fig. 3.46

HIGHER LEVEL

To inscribe and circumscribe a tetrahedron

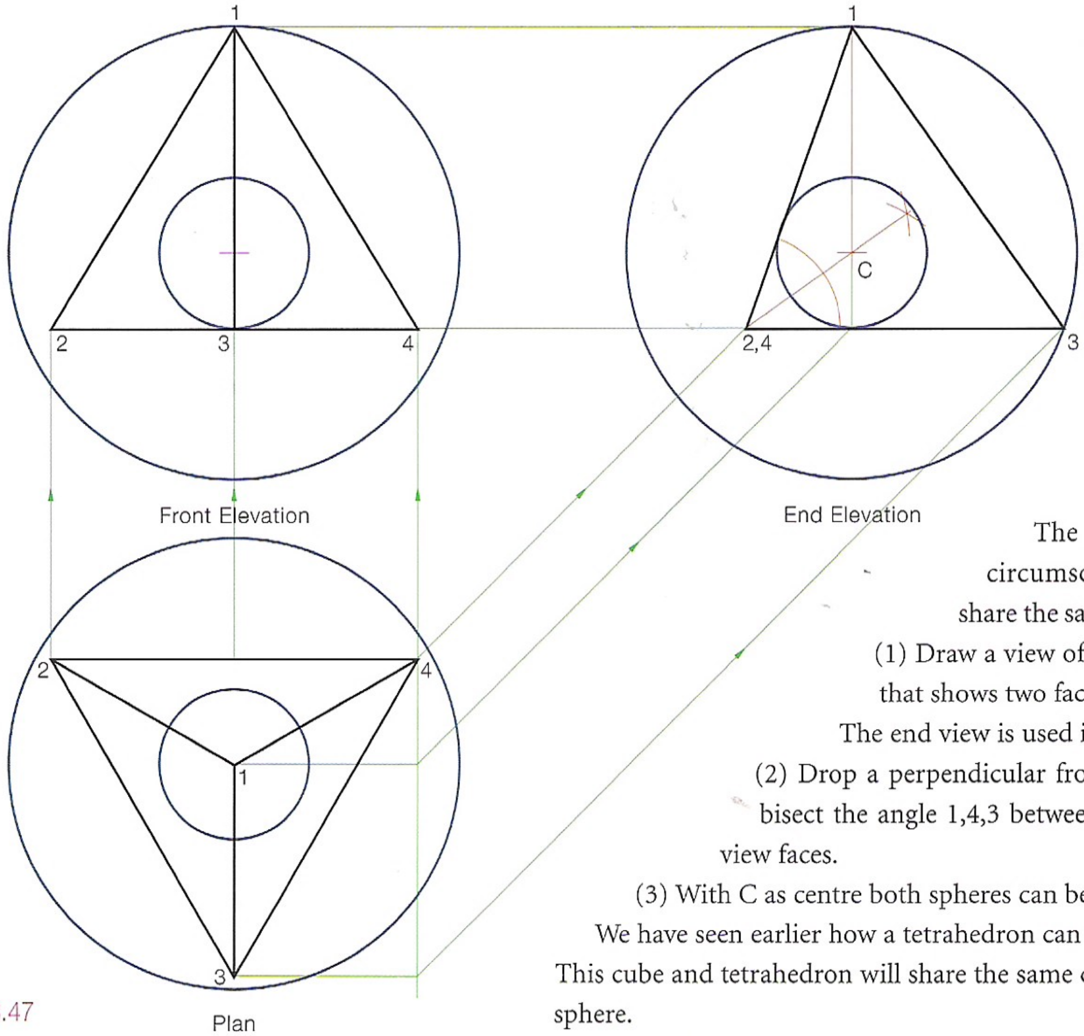


Fig. 3.47

The inscribed and circumscribed spheres share the same centre point.

(1) Draw a view of the tetrahedron that shows two faces as edge views. The end view is used in this example.

(2) Drop a perpendicular from the apex and bisect the angle 1,4,3 between the two edge-view faces.

(3) With C as centre both spheres can be drawn.

We have seen earlier how a tetrahedron can fit into a cube. This cube and tetrahedron will share the same circumscribing sphere.

Worked Problems

The plan of a square abcd which is inclined at 40° to the HP is shown in Fig. 3.48. The edge ab rests on the horizontal plane. The square is the base of a cube. Draw the plan and elevation of the solid.

(1) Line ab rests on the horizontal plane and is therefore a true length.

(2) Complete the square abc_1d_1. This represents the cube base resting on the horizontal plane.

(3) Project an auxiliary view showing ab as a point view. Project d_1c.

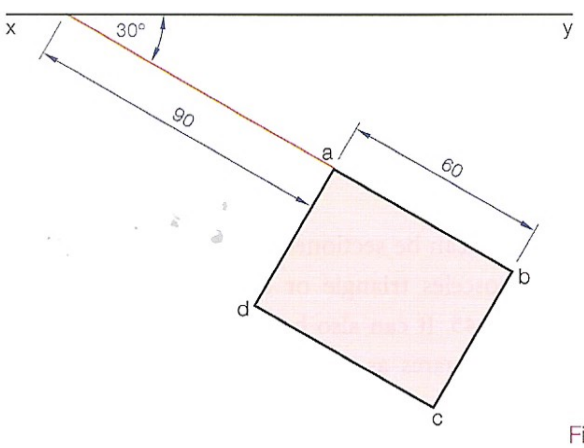


Fig. 3.48

(4) Rotate the base in the auxiliary, about ab to make a 40° angle with the x_1y_1.

(5) Complete the cube in auxiliary by rotating d_1c_1 onto the 40° line and completing a square.

(6) Project the corners back to plan. The points in plan move perpendicular to the hinge line ab.

(7) Heights for the elevation are found from the auxiliary.

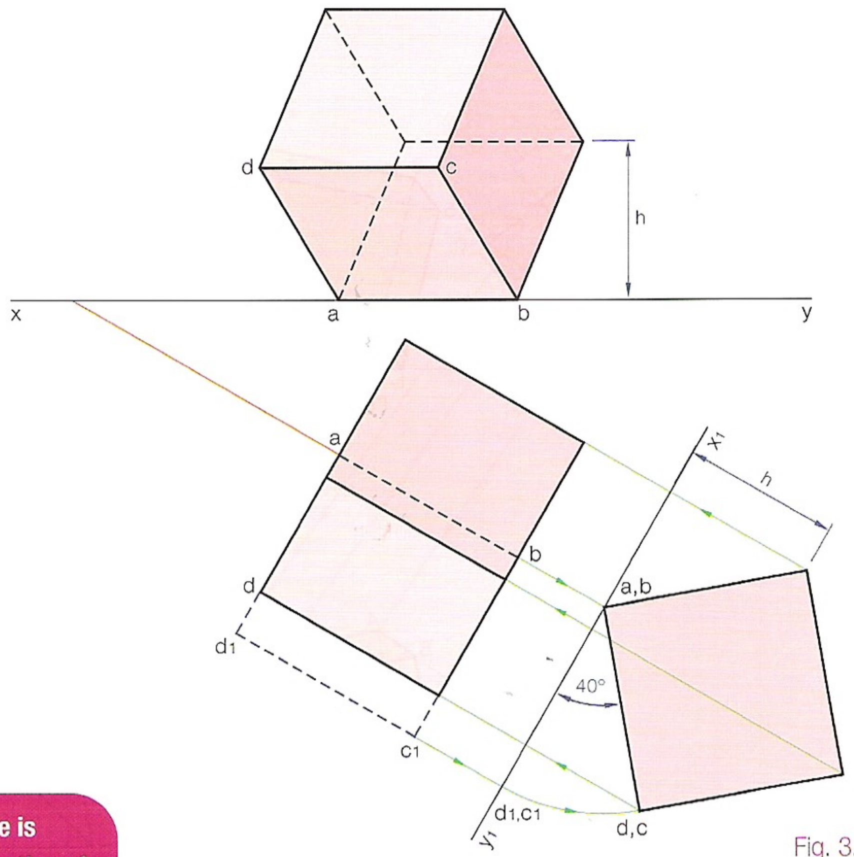


Fig. 3.49

The elevation of a cube of 50 mm side is shown in Fig. 3.50. Draw the plan and elevation of the cube.

(1) Set up line ac in elevation. Project an auxiliary plan from this with x_1y_1 parallel to ac. This view will show the true shape of abcd.

(2) Construct a square of correct size, and find the length of its diagonal. Start with a point c in auxiliary plan. Swing an arc, centre c, radius equal to the diagonal, to locate point a. Complete the square in auxiliary.

(3) Project back to the front elevation and plan similar to the previous example.

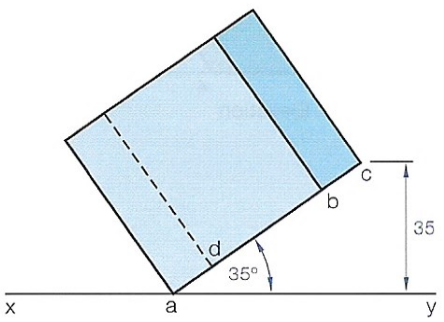


Fig. 3.50

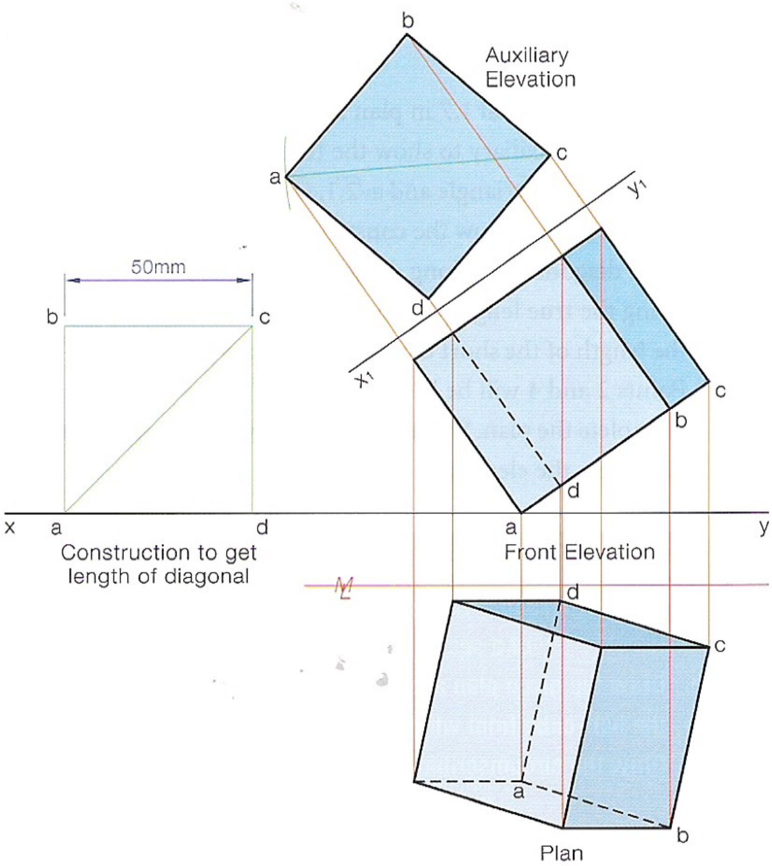


Fig. 3.51

Given the plan of a cube. Line 1,7 is the long diagonal. Draw the plan and elevation of the cube when corner 1 rests on the HP and corner 7 is 60 mm above the HP.

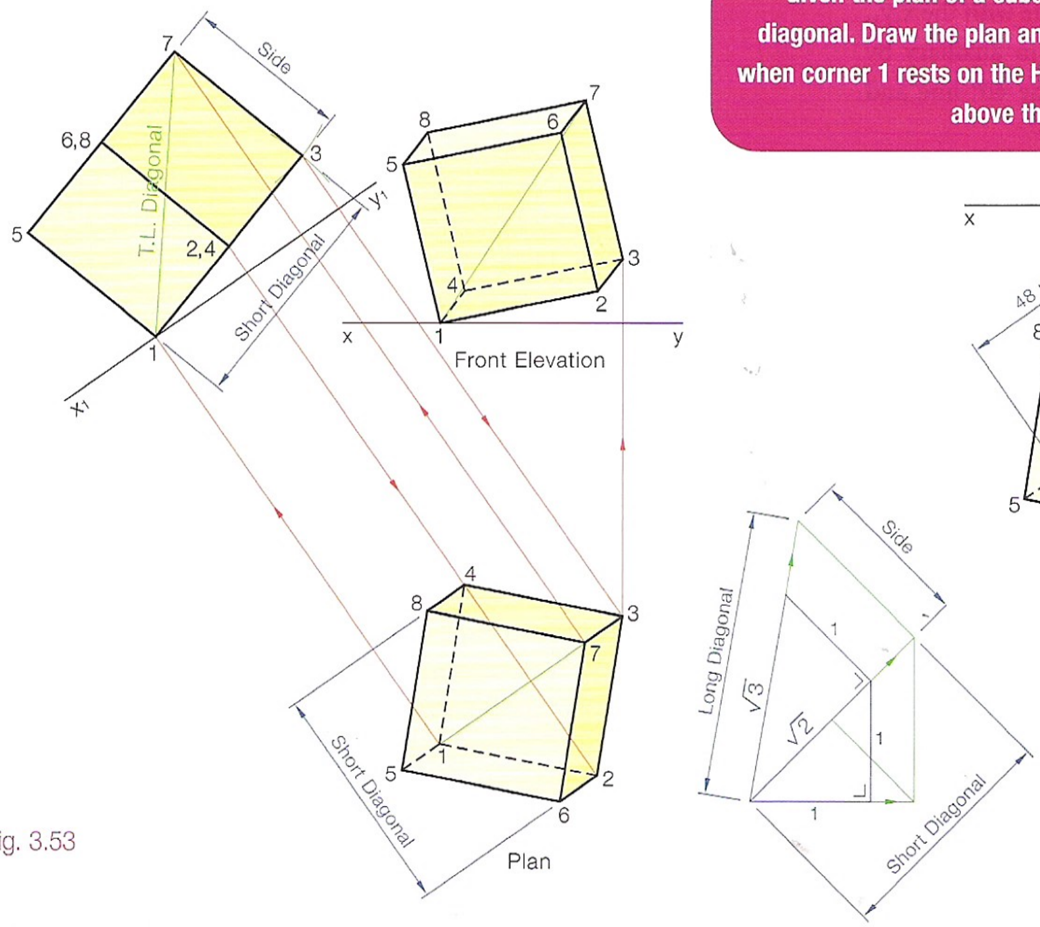


Fig. 3.53

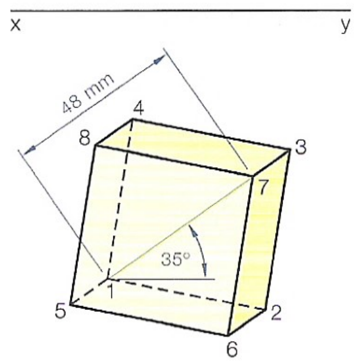


Fig. 3.52

(1) Draw the diagonal 1,7 in plan and elevation.
(2) Project an auxiliary to show the true length of diagonal 1,7.
(3) Set up a 1,1,$\sqrt{2}$ triangle and a $\sqrt{2}$,1,$\sqrt{3}$ triangle using any unit length. These triangles show the constant relationship between the edges, short diagonals and long diagonals of a cube.
(4) Using the true length of the long diagonal, found in the auxiliary, the length of the short diagonals and sides can be found as shown.
(5) Points 2 and 4 will be halfway between 1 and 3.
(6) Complete the plan. Short diagonal 2,4 will be a true length in plan.
(7) Complete the elevation.

Shown in the diagram is the plan and elevation of a regular hexagon. This hexagon is the cut surface of a sectioned cube.
(i) Draw the given plan and elevation and construct the half-cube from which the section was found.
(ii) Draw the circumscribing sphere.

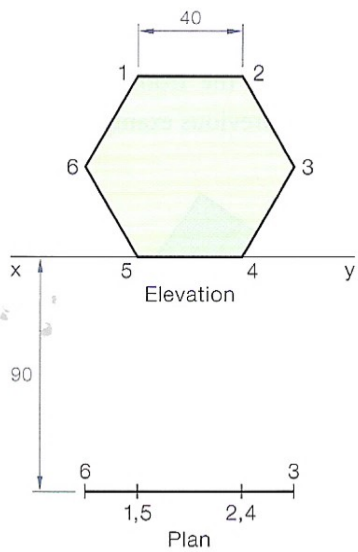


Fig. 3.54

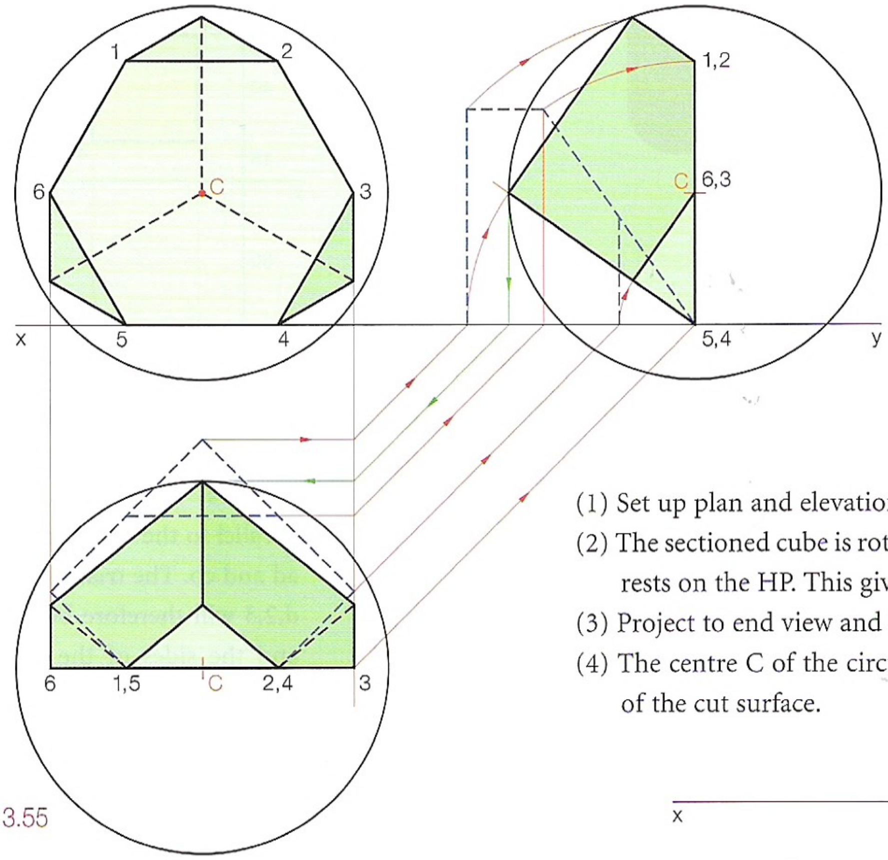


Fig. 3.55

From looking back at Fig. 3.37 it can be seen that each corner of the hexagon is a midpoint of one of the cube edges. By using one of the hexagon's edges, we can easily find the size of the cube's faces. The cut cube will be drawn with its base resting on the HP and then rotated about edge 5,4 into the correct position.

(1) Set up plan and elevation and project an end view.

(2) The sectioned cube is rotated about edge 4,5 so that the base rests on the HP. This gives the length of the cube edges.

(3) Project to end view and rotate back into position.

(4) The centre C of the circumscribed sphere lies at the centre of the cut surface.

The plan of the base of a tetrahedron is shown in Fig. 3.56. Edge ab rests on the horizontal plan. Draw the plan and elevation of the solids.

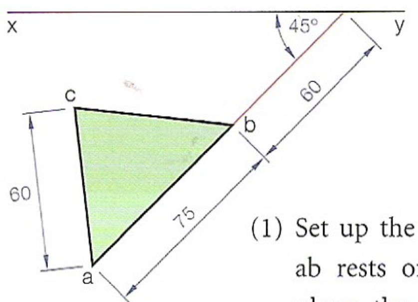


Fig. 3.56

(1) Set up the plan. Since edge ab rests on the horizontal plane the triangle will be isosceles.

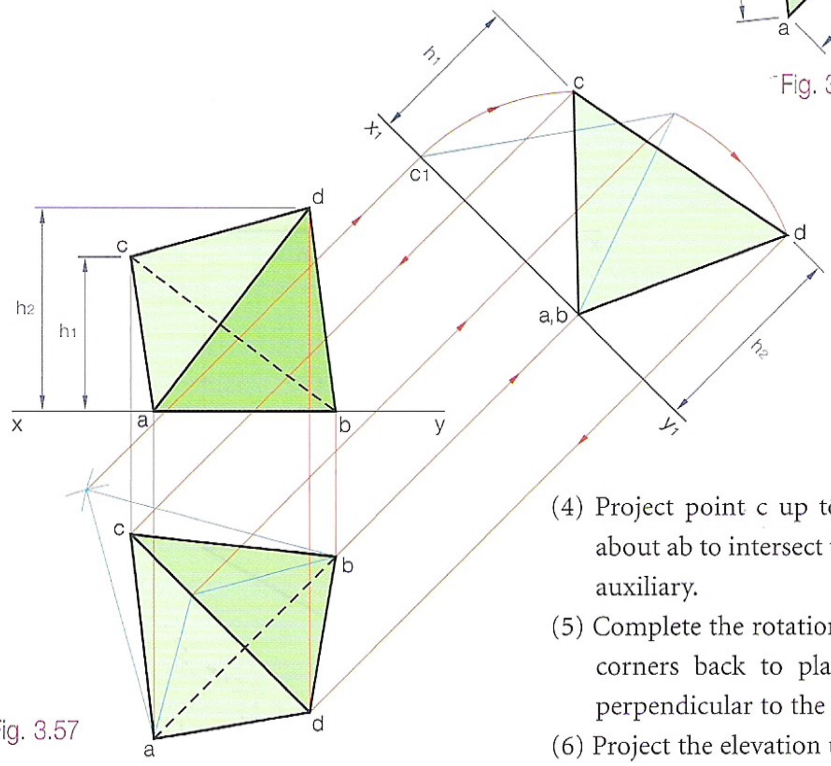


Fig. 3.57

(2) Draw a plan of the tetrahedron when its base is resting on the horizontal plane. Edge ab is horizontal and therefore seen as a true length in plan.

(3) Project an auxiliary elevation viewing along edge ab. Construct the auxiliary elevation with the tetrahedron resting on the x_1y_1 line.

(4) Project point c up to the auxiliary. Now rotate the corner c_1, about ab to intersect this projection line, locating corner c in the auxiliary.

(5) Complete the rotation in the auxiliary elevation and project the corners back to plan. The points in plan will have moved perpendicular to the hinge line ab.

(6) Project the elevation using heights from the auxiliary view.

HIGHER LEVEL

The cut surface of a tetrahedron is shown in Fig. 3.58. It is a square, 40 mm side. Construct the remaining part of the solid.

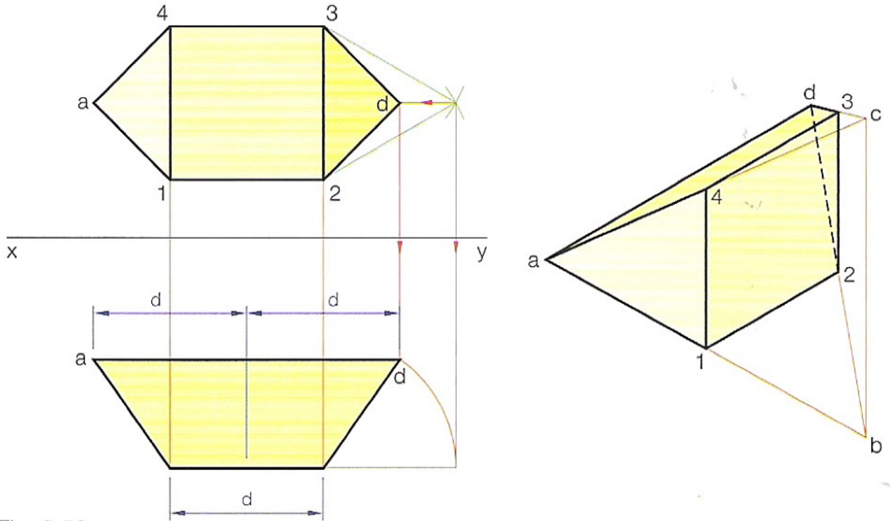


Fig. 3.59

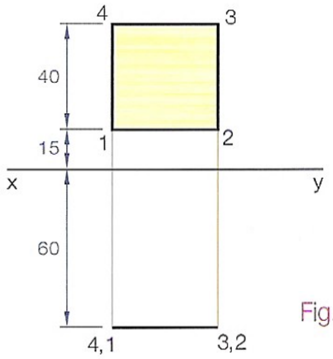


Fig. 3.58

The cut surface 1,2,3,4 divides the edges ab, ac, cd and bd in half. The square's edges will also run parallel to the tetrahedron's edges ad and cb. The triangles a,1,4 and d,2,3 will therefore be equilateral and the sides of the square will equal half of the tetrahedron's edges. Construction as shown in Fig. 3.59.

The plan of a sphere resting on the horizontal plane is shown, Fig. 3.60. The sphere is inscribed in a regular tetrahedron. Draw the plan and elevation of the tetrahedron.

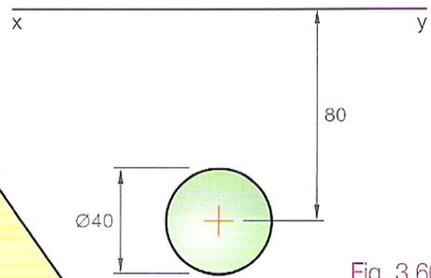


Fig. 3.60

There is only one size of tetrahedron that will fit around this sphere, but it can be positioned in an infinite number of aspects. It is simplest to place it with one face resting on the HP and another appearing edge on in elevation.

(1) Draw the sphere in plan and elevation.
(2) Draw a tetrahedron, of any size, such that the solid's apex is above the sphere centre in plan and that two edges appear parallel in elevation.
(3) Enlarge this solid to the required size so that the two edges, seen edge on in elevation, will be tangential to the sphere.

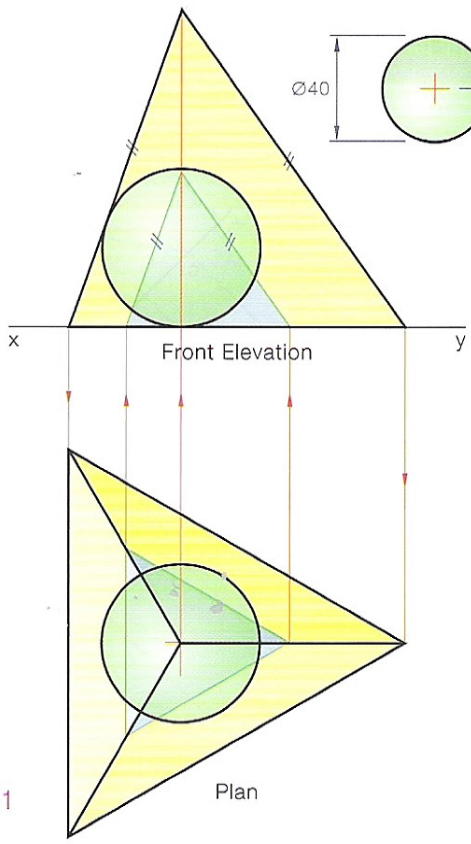


Fig. 3.61