3 Sections of Solids

SYLLABUS OUTLINE

Areas to be studied:

• Sectional views. • Projection of cube and tetrahedron, *their inscribed and circumscribed spheres.*

Learning outcomes

Students should be able to:

Higher and Ordinary levels

- Solve problems that involve the intersection of solids by simply inclined planes and obliquely inclined planes, using horizontal and vertical section planes.
- Represent in two dimensions the cube and tetrahedron from given information.

Higher level only

- Solve problems that involve the intersection of solids by simply inclined planes and obliquely inclined planes using simply inclined section planes.
- Determine the incentre and circumcentre of cube and tetrahedron.

Section Planes

A sectional plane is a plane which slices through an object. These planes can be horizontal, vertical, simply inclined or oblique. Sectional planes are used to reveal the inside of an object or can be used to help solve the interpenetration of solids. See Figures 3.1, 3.2, 3.3 and 3.4.

HORIZONTAL SECTION PLANE

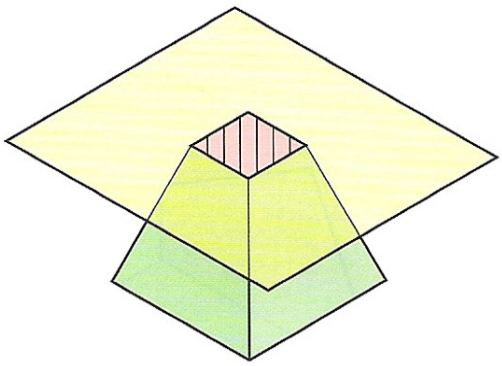


Fig. 3.1a

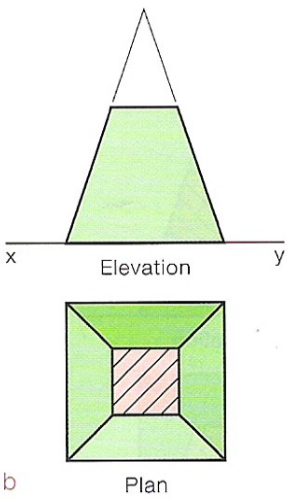


Fig. 3.1b

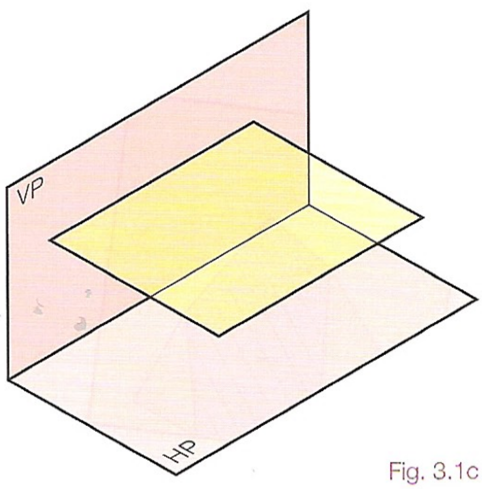


Fig. 3.1c

VERTICAL SECTION PLANE

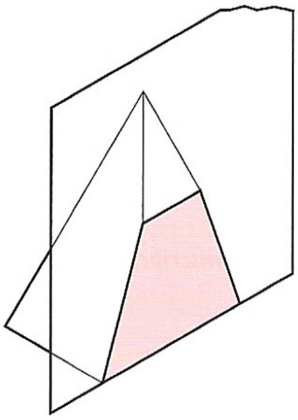


Fig. 3.2a

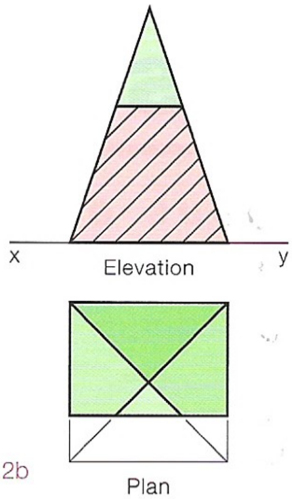


Fig. 3.2b

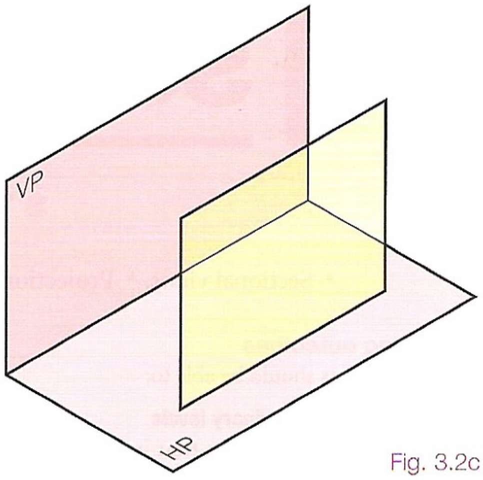


Fig. 3.2c

SIMPLY INCLINED SECTION PLANE

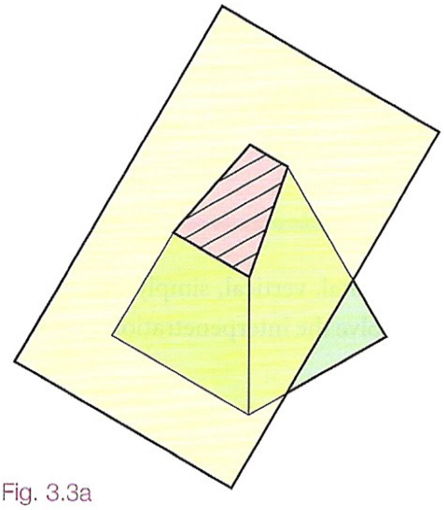


Fig. 3.3a

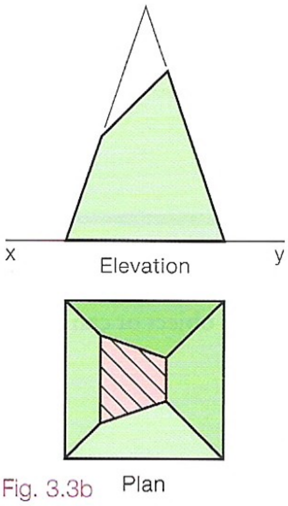


Fig. 3.3b

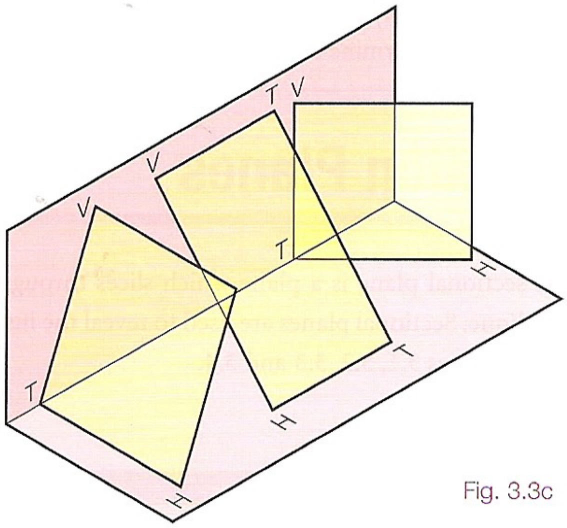
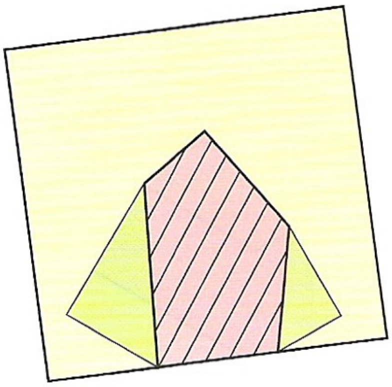


Fig. 3.3c

OBLIQUE SECTION PLANE



Oblique Section Plane

Fig. 3.4a

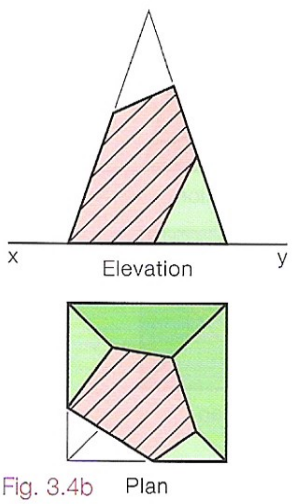


Fig. 3.4b

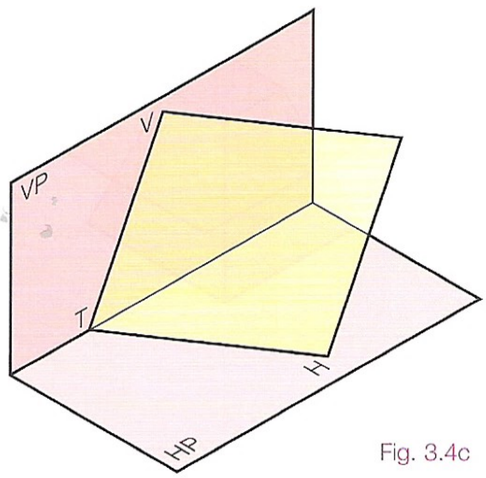


Fig. 3.4c

Traces of a Plane

When a plane meets another plane the line of intersection will always be a straight line. If a plane is extended to meet the horizontal plane the straight line produced is called the **horizontal trace**. Similarly, if a plane is extended to meet the vertical plane the straight line produced is called the **vertical trace**. When the plane intersects both planes simultaneously it will have both a horizontal trace and a vertical trace. These traces will always meet on the xy line except for one type of plane shown in Fig. 3.5, a doubly inclined plane. In this case the traces run parallel to the xy line.

Fig. 3.5

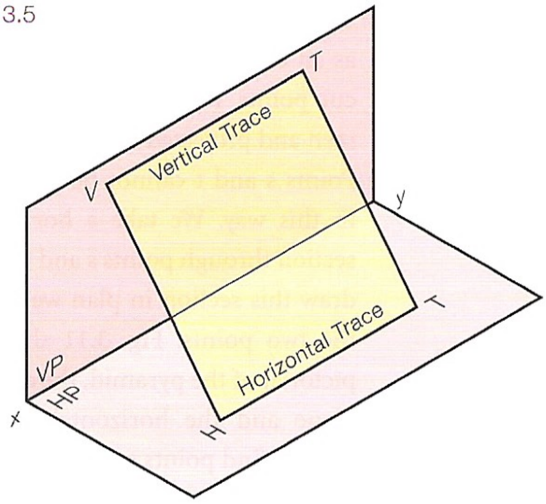


Fig. 3.5 shows the plane pictorially while Fig. 3.6 shows the representation of the same plane in orthographic projection.

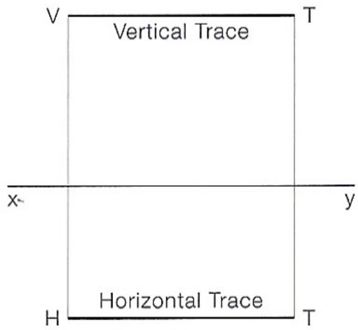
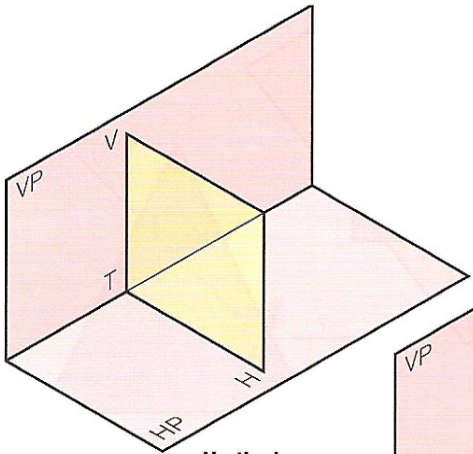


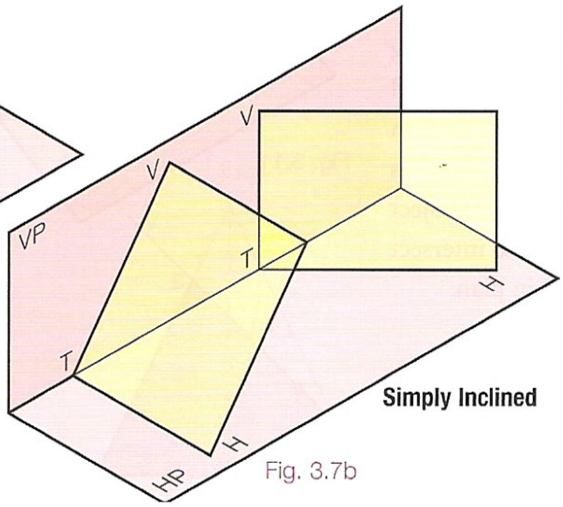
Fig. 3.6

All planes can be represented, in orthographic, by their traces. Fig. 3.7 shows a number of plane types in a pictorial view while Fig. 3.8 shows the corresponding orthographic representation of the same planes.



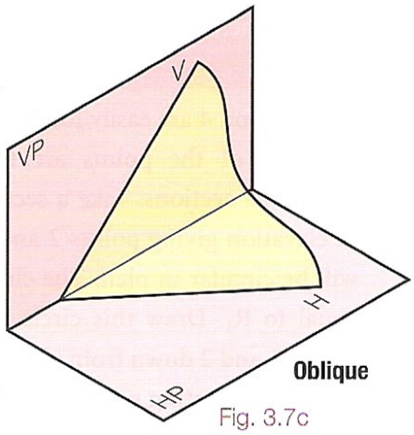
Vertical

Fig. 3.7a



Simply Inclined

Fig. 3.7b



Oblique

Fig. 3.7c

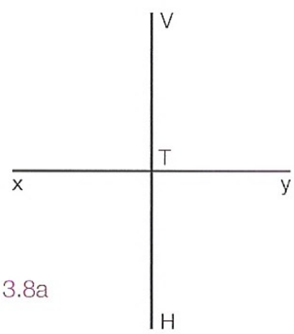


Fig. 3.8a

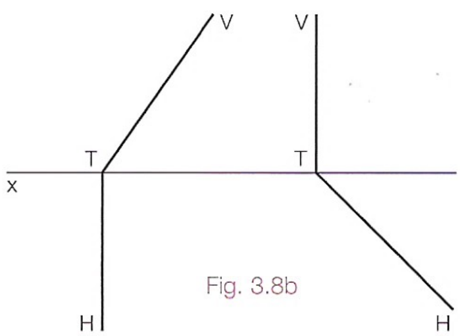


Fig. 3.8b

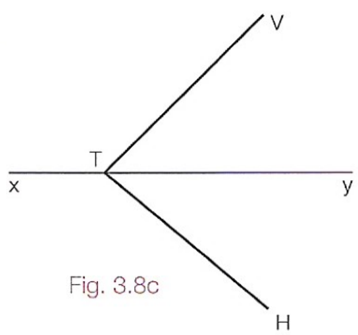


Fig. 3.8c

Projection of Solids Cut by Simply Inclined Planes

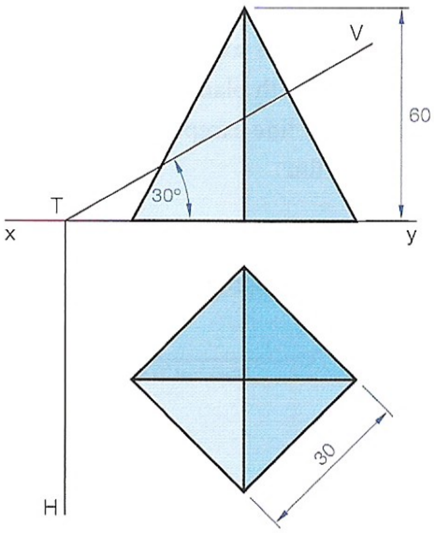


Fig. 3.9

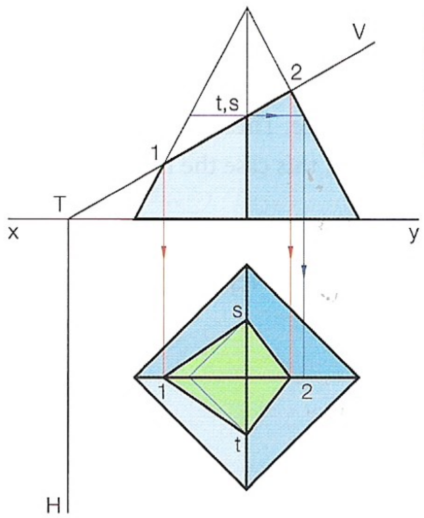


Fig. 3.10

Fig. 3.9 shows the plan and elevation of a pyramid that is to be cut by the simply inclined plane VTH. Draw the plan and elevation of the cut solid.

(1) The simply inclined plane appears as an edge view in elevation so the cut points 1 and 2 can be clearly seen and projected to plan.

(2) Points s and t cannot be projected in this way. We take a horizontal section through points s and t. If we draw this section in plan we locate the two points. Fig. 3.11 shows a pictorial of the pyramid, the cutting plane and the horizontal section used to find points s and t.

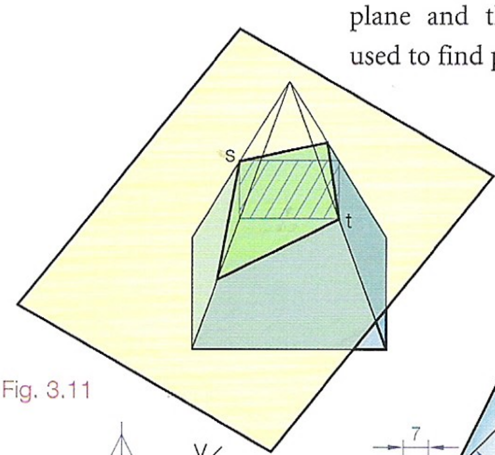


Fig. 3.11

Fig. 3.12 shows the plan and elevation of a cone that is to be cut by the simply inclined plane VTH. Draw the plan and elevation of the cut solid.

(1) Points 3 and 4 are easily found.

(2) The rest of the points are found by using horizontal sections. Take a section at any level in elevation giving points 2 and 1. The section will be circular in plan. The circle has a radius equal to R_1. Draw this circle in plan. Project points 1 and 2 down from elevation to intersect this circle, locating points 1 and 2 in plan.

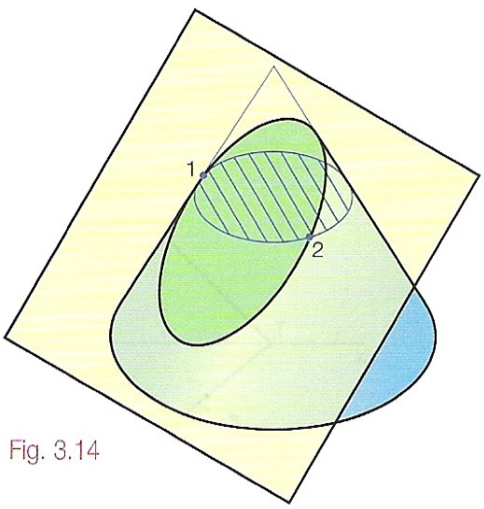


Fig. 3.14

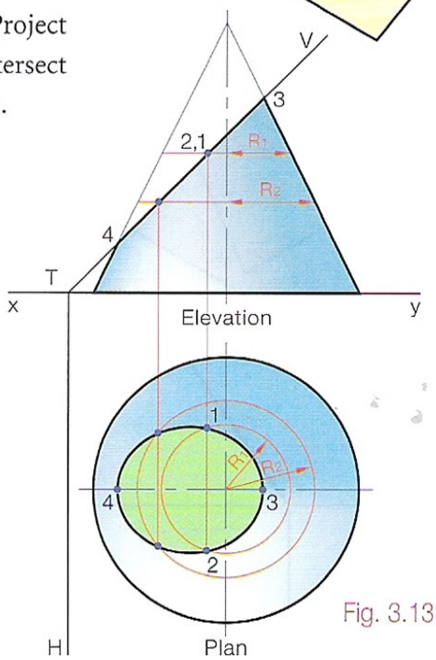


Fig. 3.13

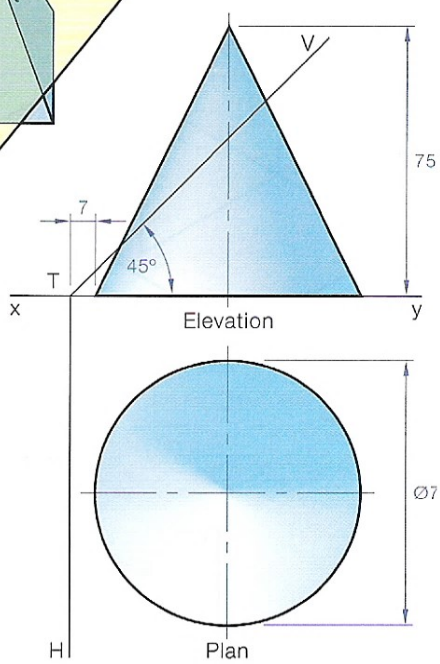


Fig. 3.12

The plan of a sphere resting on the horizontal plane is shown in Fig. 3.15. The sphere is to be cut by the simply inclined plane S–S. Find the plan and elevation of the cut solid using vertical section planes.

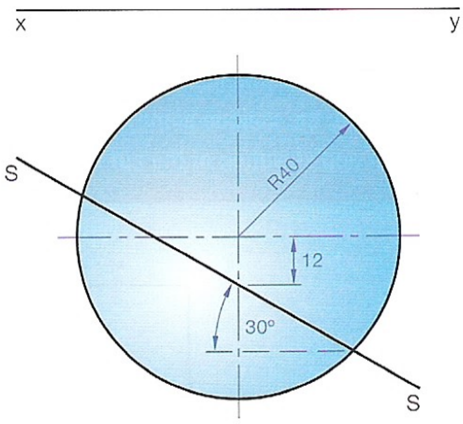


Fig. 3.15

(1) Each point on the cut surface is found by taking vertical sections. A vertical section parallel to the vertical plane is taken in plan. This section will be circular in elevation, Fig. 3.16.

(2) The intersection between the vertical section plane and section plane S–S in plan is projected to elevation, giving p and q, Fig 3.17.

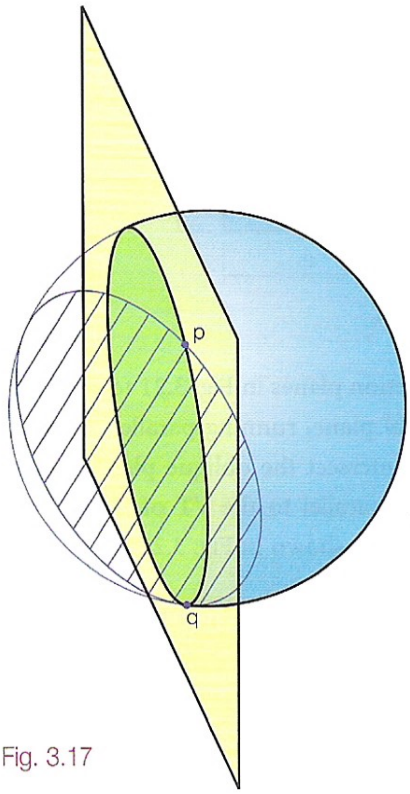


Fig. 3.17

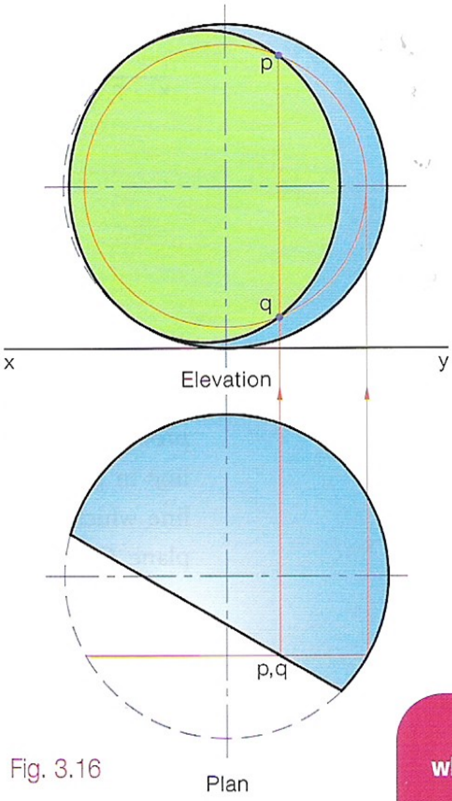


Fig. 3.16

An oblique cylinder is shown, Fig. 3.18, which is to be cut by the plane VTH. Draw the plan and elevation of the cut solid.

In this example vertical sections were used. The construction should be clear from Fig. 3.19.

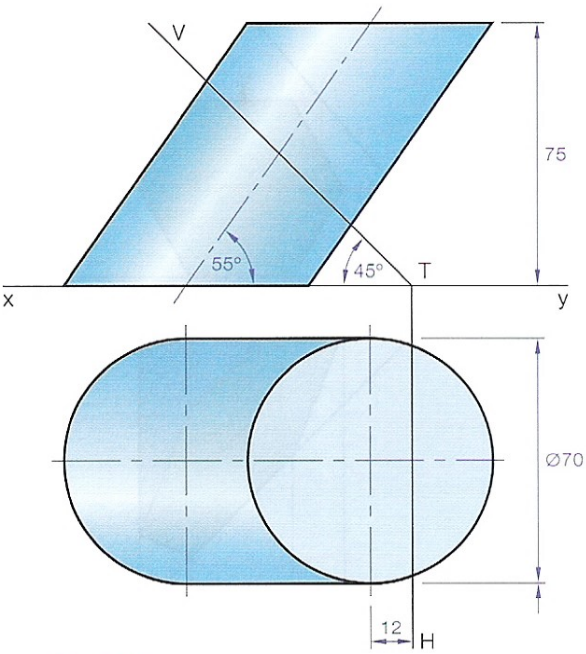


Fig. 3.18

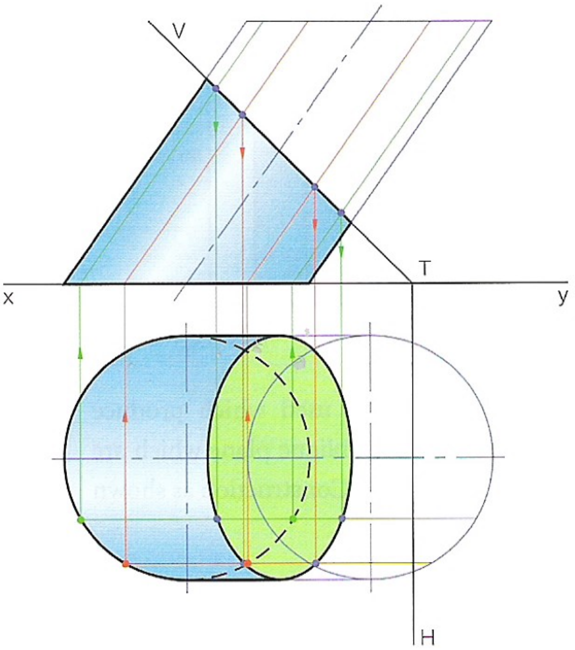


Fig. 3.19

Projection of Solids Cut by Oblique Planes

The rectangular prism shown in Fig. 3.20 is to be cut by the oblique plane VTH. Draw the plan and elevation of the cut solid.

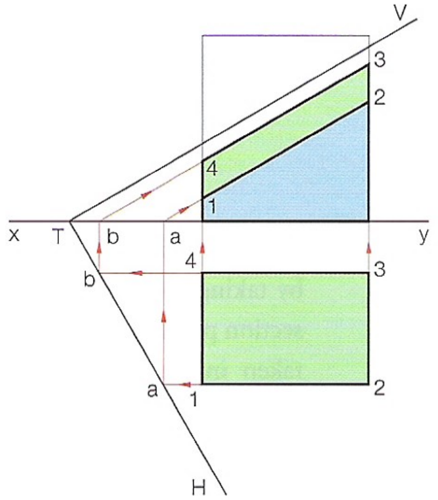


Fig. 3.21

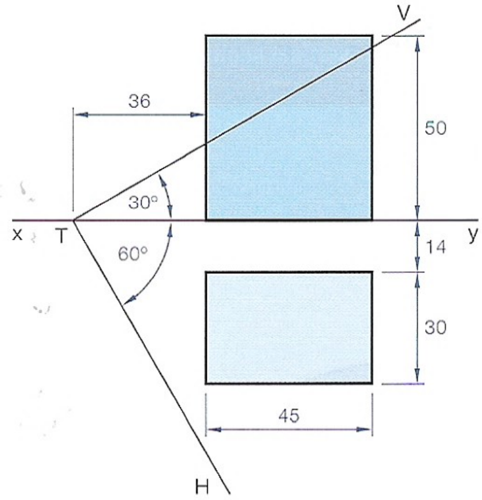


Fig. 3.20

We use vertical section planes in Fig. 3.21 to solve this problem. A vertical plane, running parallel to the xy line in plan, will intersect the oblique plane along a line which will be parallel to the VT of the oblique plane. Construction as shown in Fig. 3.21.

A regular rectangular prism is to be cut by the oblique plane VTH, Fig. 3.22. Project the plan and elevation of the cut solid.

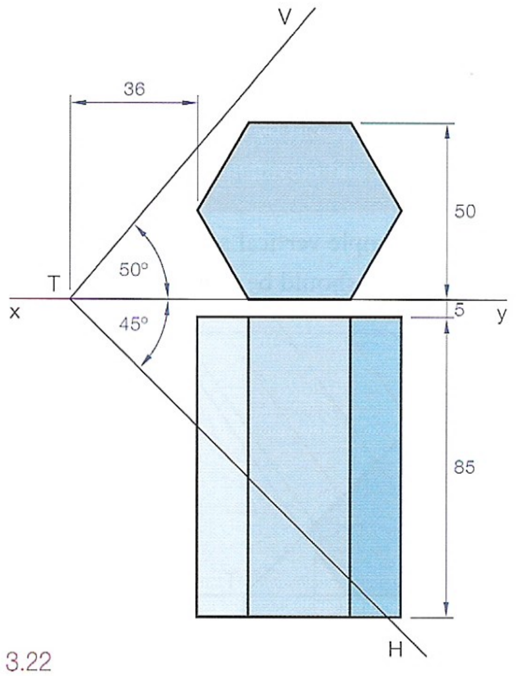


Fig. 3.22

Horizontal section planes are used which produce lines of intersection with the oblique plane which are parallel to the horizontal trace. Construction as shown in Fig. 3.23.

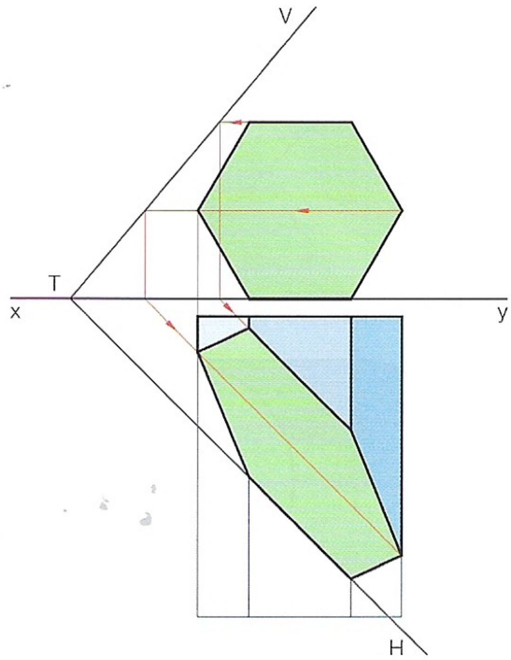


Fig. 3.23

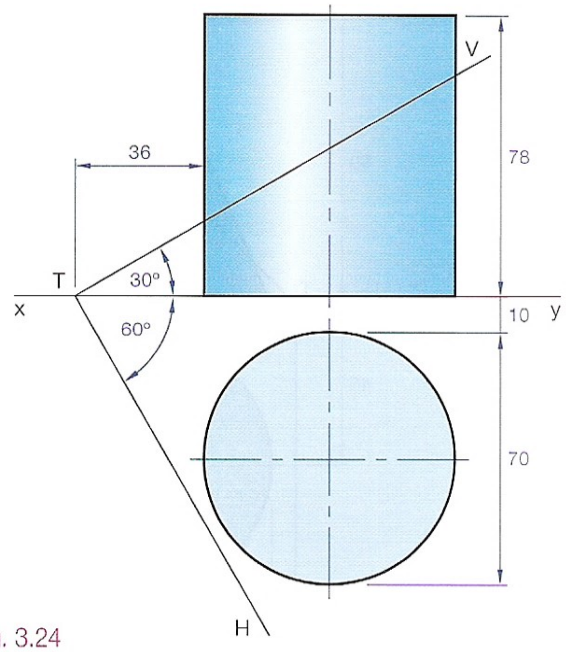


Fig. 3.24

The plan and elevation of a cylinder are shown in Fig. 3.24. Show the projections of this cylinder when it has been cut by the oblique plane VTH.

In this example the problem is solved using a series of vertical planes.

(1) A vertical plane is drawn to cut the cylinder in plan giving points 1 and 7.

(2) This vertical plane will intersect the oblique plane along a straight line which will run on the cut surface (see Fig. 3.25).

(3) This line of intersection is parallel to the vertical trace.

(4) Draw the line in elevation and project points 1 and 7 onto it. Repeat as necessary.

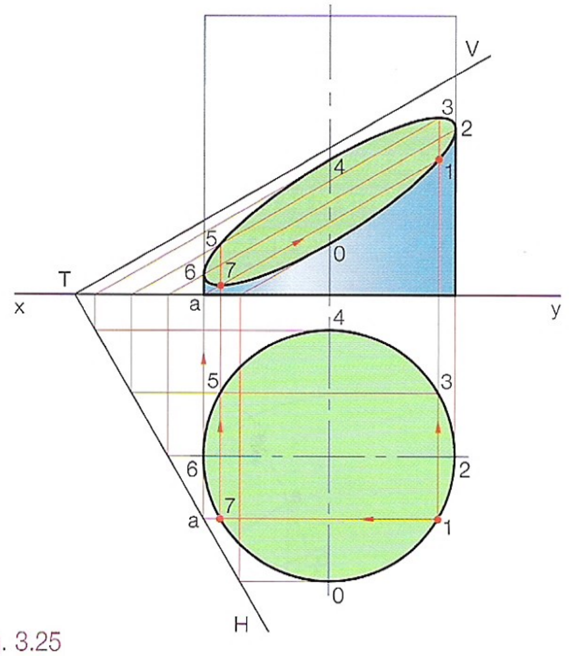


Fig. 3.25

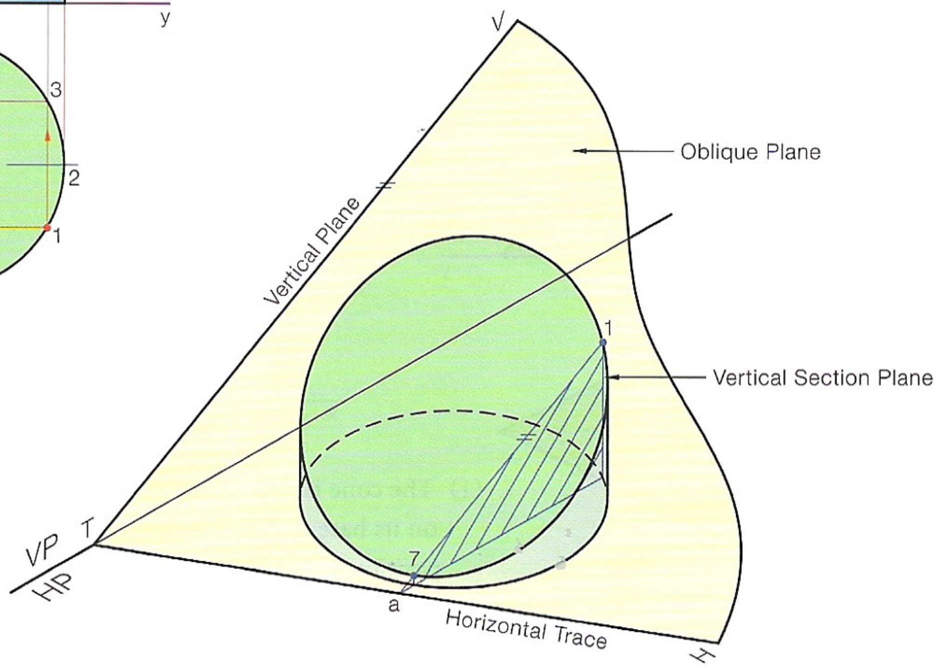


Fig. 3.26

Fig. 3.27 shows a cone which is to be cut by a simply inclined plane. Using simply inclined section planes determine the cut surface.

If we use simply inclined section planes with each containing the apex of the cone then the sections produced will be triangles (see Fig. 3.29). Construction of solution is shown in Fig. 3.28.

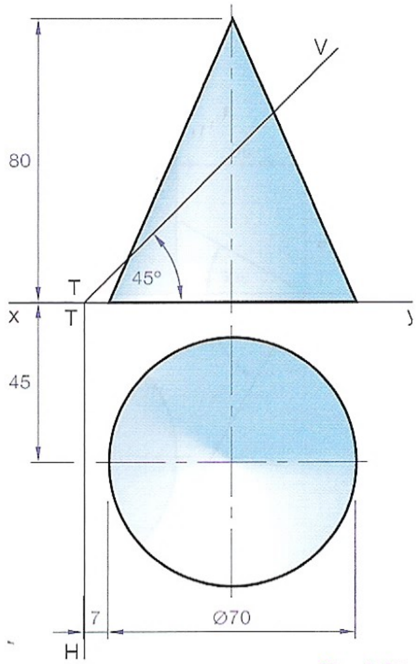


Fig. 3.27

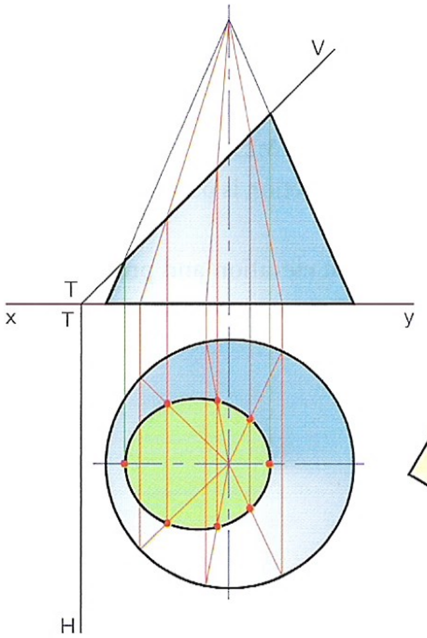


Fig. 3.28

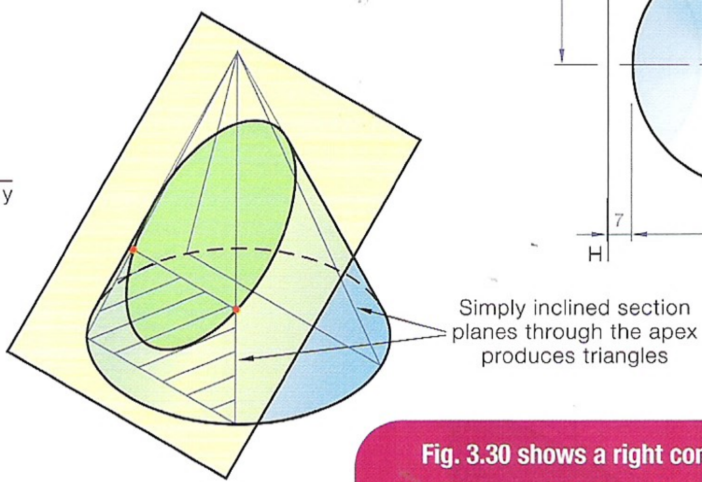


Fig. 3.29

Fig. 3.30 shows a right cone resting on its side. The cone is to be cut by the simply inclined plane VTH. Determine the cut surface of the solid.

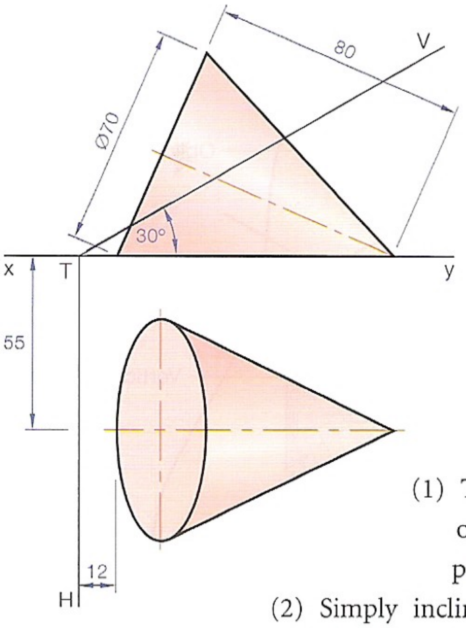


Fig. 3.30

(1) The cone is first drawn standing on its base and then rotated into position.

(2) Simply inclined section planes originating from the cone apex are used.

(3) Each section plane Oaa, Obb, Occ, produce triangles in plan (Fig. 3.31).

(4) The intersection between these triangles and plane VTH can be easily found.

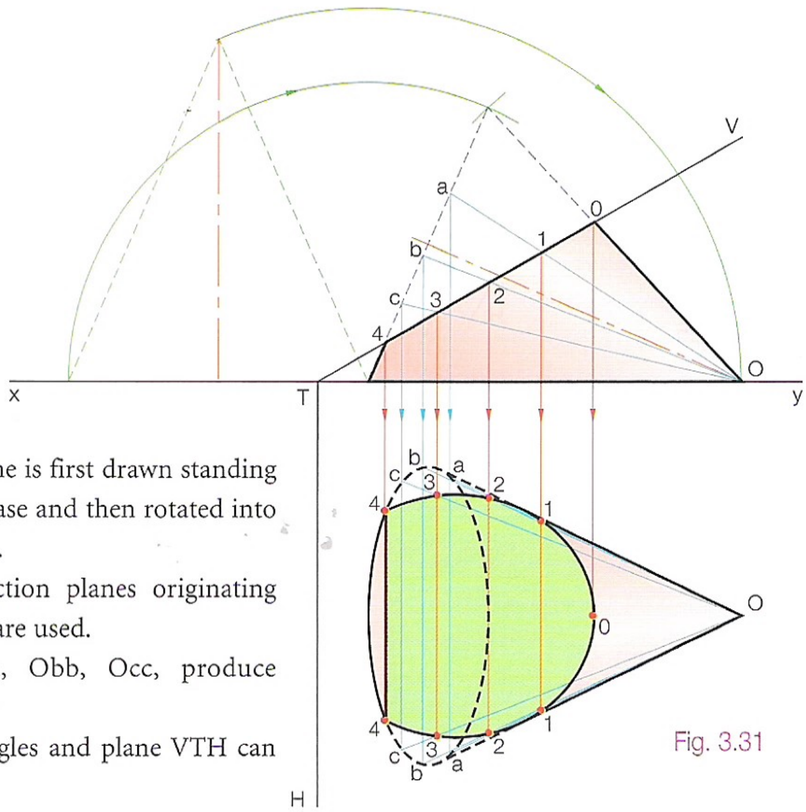


Fig. 3.31

HIGHER LEVEL

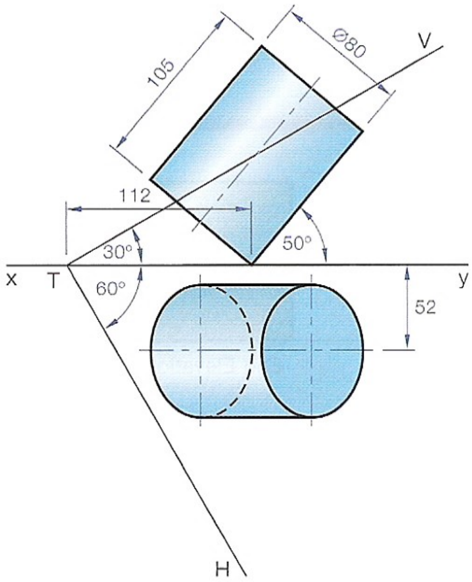


Fig. 3.32

Fig. 3.32 shows a right cylinder with its base inclined to the horizontal plane. Determine the cut surface of the solid.

(1) The line of intersection between the simply inclined plane containing the base of the cylinder and the oblique plane VTH is found. This gives the straight-line cut edge on the base.

(2) The cut line on the top of the cylinder will be parallel to the cut line on the cylinder base.

(3) Points on the curved surface of the cylinder are found by using simply inclined planes parallel to the cylinder axis.

(4) Draw the vertical trace/edge view of a simply inclined plane (Step 1) which is parallel to the cylinder axis.

(5) Find its line of intersection with plane VTH (Steps 2, 3 and 4).

(6) This simply inclined plane cuts through the cylinder forming a rectangle in plan (Steps 5 and 6).

(7) Where the rectangle and the intersection line between the planes cross, gives a point p on the cut surface. Repeat as necessary.

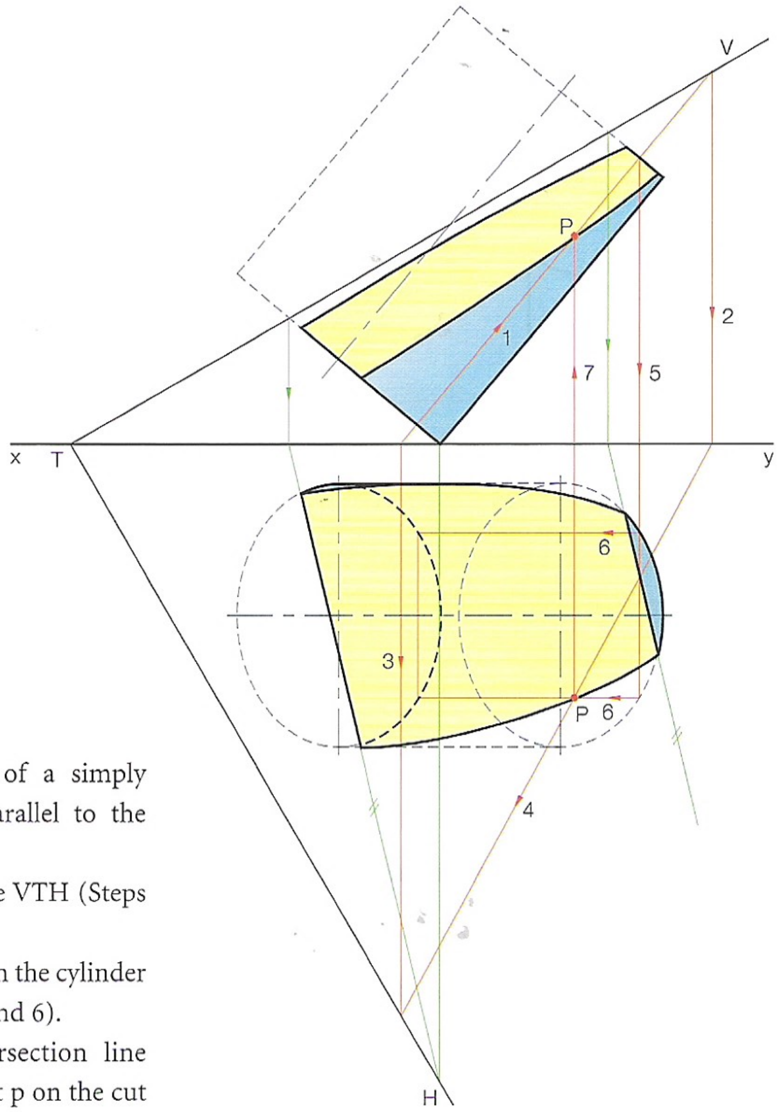


Fig. 3.33