20 Surface Geometry

SYLLABUS OUTLINE

Areas to be studied (in an applied context):

• Dihedral angles between surfaces. • Surface developments of containers and structures such as plane intersecting roof surfaces, sheet metal containers, hoppers and transition pieces. • Projections and developments of intersecting prismatic, right cylindrical transition and ducting details. • *Projections and developments of intersecting prismatic, oblique cylindrical, oblique conical transition and ducting details.* • Projection and developments of transition pieces connecting rectilinear to rectilinear and circular to circular cross-section. • *Projection and developments of transition pieces connecting circular to rectilinear cross-section.*

Learning outcomes

Students should be able to:

Higher and Ordinary levels

- Determine the dihedral angles between adjacent plane surfaces forming solid objects.
- Prepare surface developments of surface containers, intersecting roof surfaces, and sheet metal fabrications.
- Determine the lines and points of intersection between two intersecting surfaces or objects.
- Develop intersecting ductwork involving prismatic and right cylindrical surfaces.
- Determine the developments of transition pieces between ducts of circular/circular and rectilinear/rectilinear cross-section.

Higher level only

- *Develop intersecting ductwork involving oblique prismatic and oblique cylindrical surfaces.*
- *Determine the developments of transition pieces between ducts of circular/rectilinear cross-section.*

Dihedral Angle

The angle between two planes is a dihedral angle. We have already looked at the finding of dihedral angles in chapter 9, The Oblique Plane. The dihedral angle is the smallest angle that can be measured between the planes and is measured perpendicular to each plane and perpendicular to the line of intersection. The dihedral angle is often needed in roof construction, hopper construction and other fabrications involving plane surfaces.

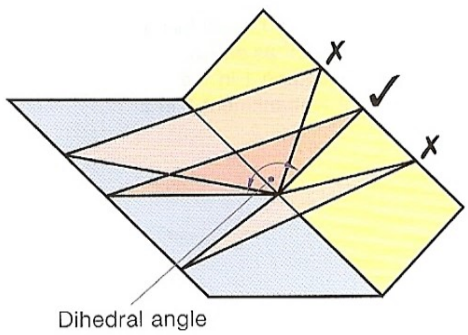
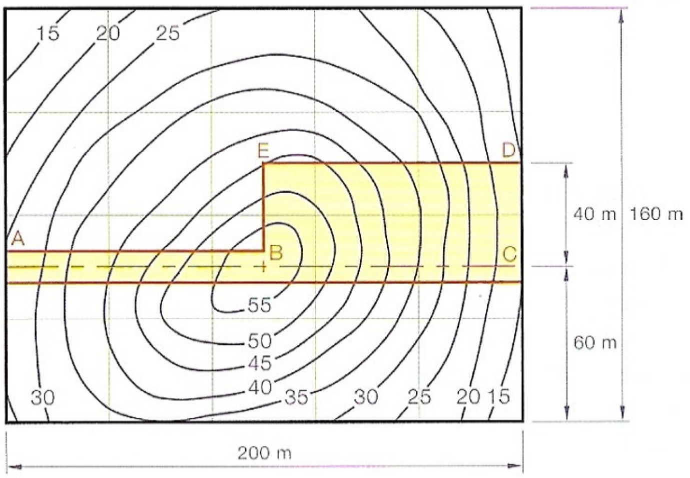


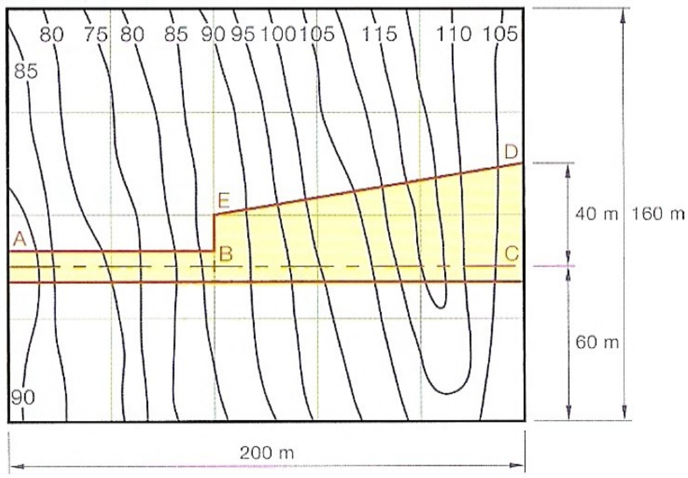
Fig. 20.1

Q25. Figures 19.65a, 19.65b, 19.65c and 19.65d, show ground contours at 5 m vertical intervals. ABC is a roadway which widens into a car parking area. Redraw each map to a scale of 1:1,000 and show the earthworks necessary to accommodate the road and car park.



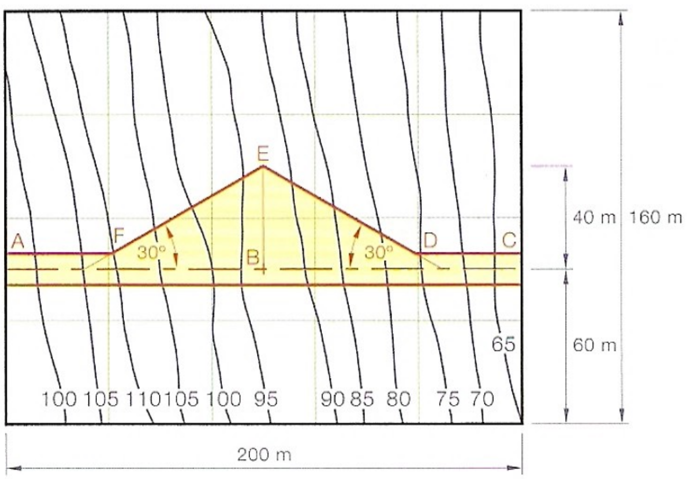
Formation width A to B 12 m
Formation level at A 40 m
Gradient from A to C 1 in 20 rising
From B to C the road widens as shown
Side slopes for cutting 1 in 1.5
Side slopes for embankment 1 in 2

Fig. 19.65a



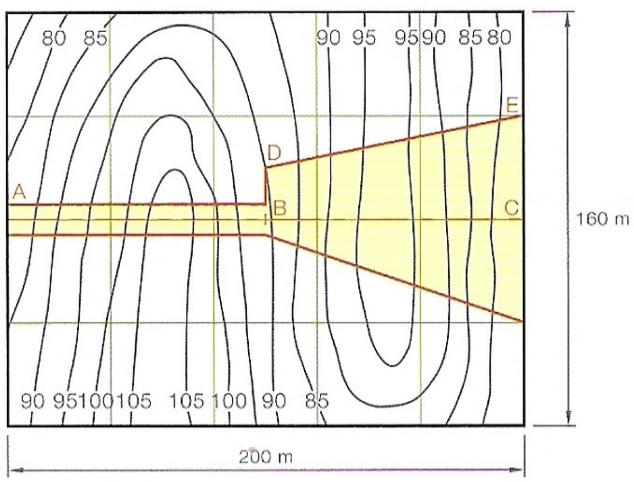
Formation width A to B 12 m
Formation level at B 100 m
Gradient from A to C 1 in 15 rising
From B to C the road widens as shown
Side slopes for cutting 1 in 1.5
Side slopes for embankment 1 in 2

Fig. 19.65b



Formation width 12 m
Formation level at A 90 m
Gradient from A to C 1 in 20 falling
Road widens at DEF as shown
Side slopes for cutting 1 in 1.5
Side slopes for embankment 1 in 2

Fig. 19.65c



Formation width A to B 12 m
Formation level at A 100 m
Gradient from A to C 1 in 15 rising
Road widens at DEF as shown
Side slopes for cutting 1 in 1.5
Side slopes for embankment 1 in 1

Fig. 19.65d

Finding the Dihedral Angle

1. Point View Method

Draw a line on a plane, e.g. a sheet of paper, and view along the length of the line so that we see the line as a point. When the line is seen as a point, the plane that it is resting on appears as an edge view.

To get the edge view of a plane you must find the point view of any straight line on that plane.

When two planes intersect, the line of intersection will be a straight line and this straight line rests on both planes at the same time. A point view of the line of intersection will show both planes as edge views and will therefore show the dihedral angle between the two planes.

To get the dihedral angle between two planes, get the point view of the line of intersection of the planes.

Fig. 20.2 shows the plan and elevation of a solid with an equilateral triangular base. Find the dihedral angle between surfaces P and Q.

Using the theory outlined above we need to get a point view of line bO.

(1) Identify the line of intersection between the planes, line bO.

(2) Find the true length of bO. This is done by projecting an auxiliary viewing perpendicular to bO in plan. x_1y_1 will therefore be parallel to the line of intersection in plan. The view projected from the plan will be an auxiliary elevation and as such will have heights equal to the front elevation.

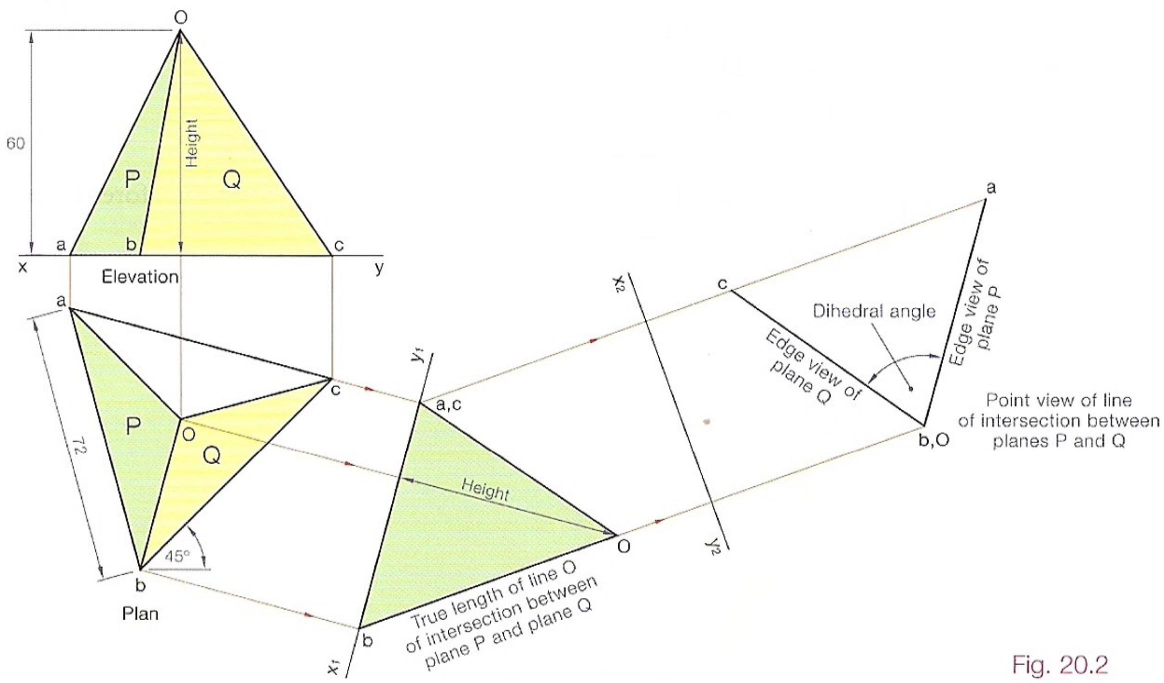


Fig. 20.2

(3) The final stage is to view along the true length found in step (2). Extend on line bO and project the other points in this direction. Draw x_2y_2 perpendicular to this line of sight, i.e. perpendicular to the true length. Being a second auxiliary plan the distances are taken from x_1y_1 back to the plan. The line of intersection appears as a point, the planes appear as lines and the dihedral angle is displayed clearly.

2. Rebatment Method

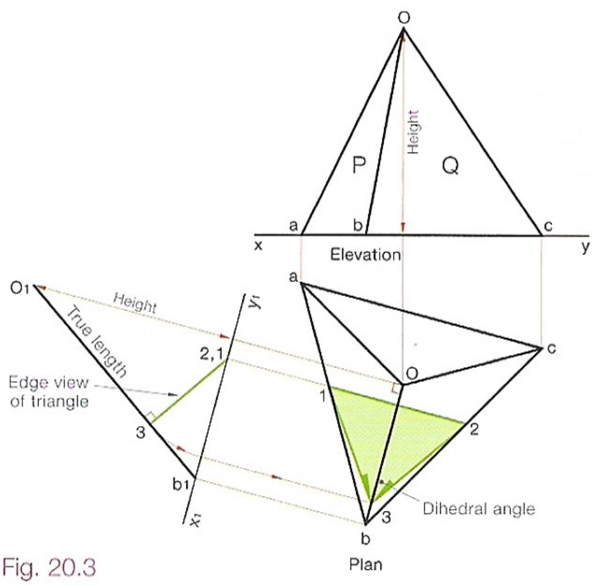


Fig. 20.3

This method is perhaps more difficult to understand but is much neater. If we introduce a cutting plane to cut through the line of intersection, bO, perpendicularly, the triangle that it produces underneath the planes contains the dihedral angle. If we can rebat this plane onto the horizontal plane we will see its true shape and angles in plan.

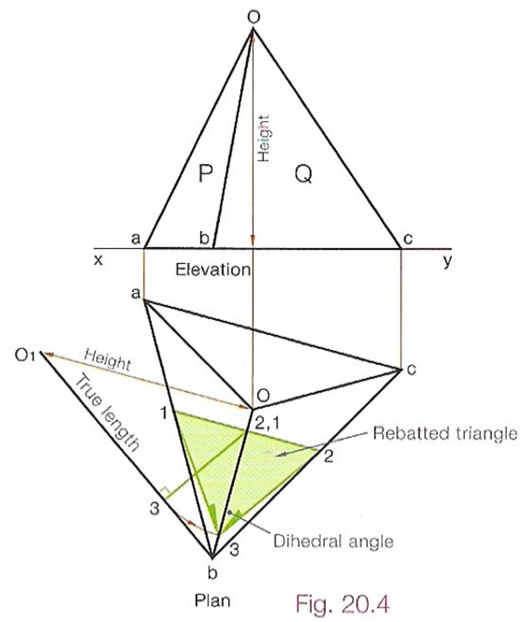


Fig. 20.4

(1) Ob is the plan of the line of intersection. Find the true length of this line by using an auxiliary elevation. Draw x_1y_1 parallel to the line of intersection in plan. In the auxiliary, point b rests on x_1y_1 and point O_1 is the 'height' above x_1y_1.

(2) In the auxiliary, draw the edge view of the triangle 1,2,3 perpendicular to the true length.

(3) Rebat or hinge the triangle about edge 1,2 so that it rests on the x_1y_1 and project back to the line of intersection in plan.

(4) Join up the triangle 1,2,3 in plan. This is the true shape of the triangle and therefore shows true angles. Angle 3 is the dihedral angle between planes P and Q.

(5) Fig. 20.4 shows exactly the same method except that the auxiliary is drawn in on the plan and gives a neater result.

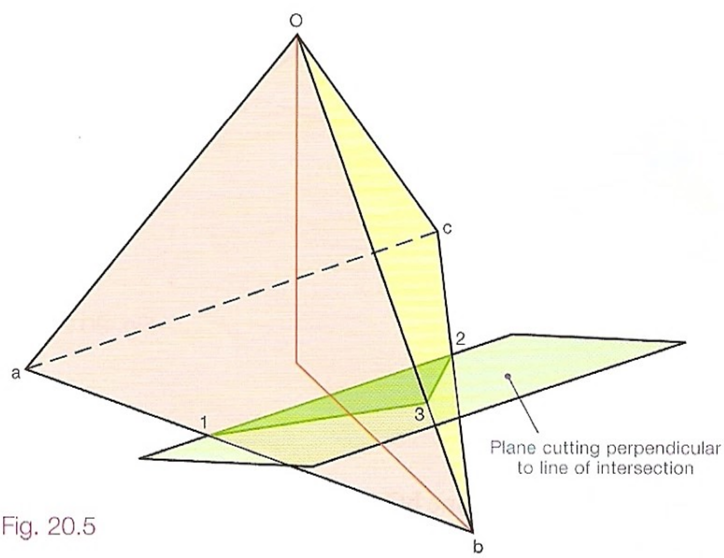


Fig. 20.5

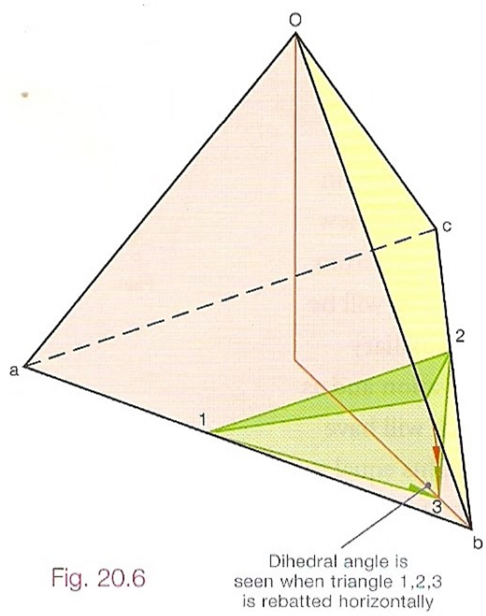


Fig. 20.6

Roof Geometry

Roof geometry provides a very practical application of plane geometry, development of surfaces and dihedral angles. There is nothing new in this section, rather it is the application of what has already been learned.

Fig. 20.7 shows the outline plan of a roof. Surfaces A and B have a pitch of 45°. Surface C has a pitch of 40° and surface D has a pitch of 60°.

(i) Draw the plan and elevation.

(ii) Develop surfaces B, C and D.

(iii) Find the dihedral angle between surfaces B and D.

Scale 1:100

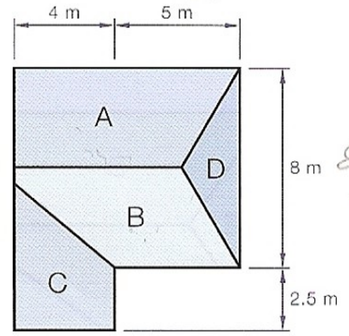


Fig. 20.7

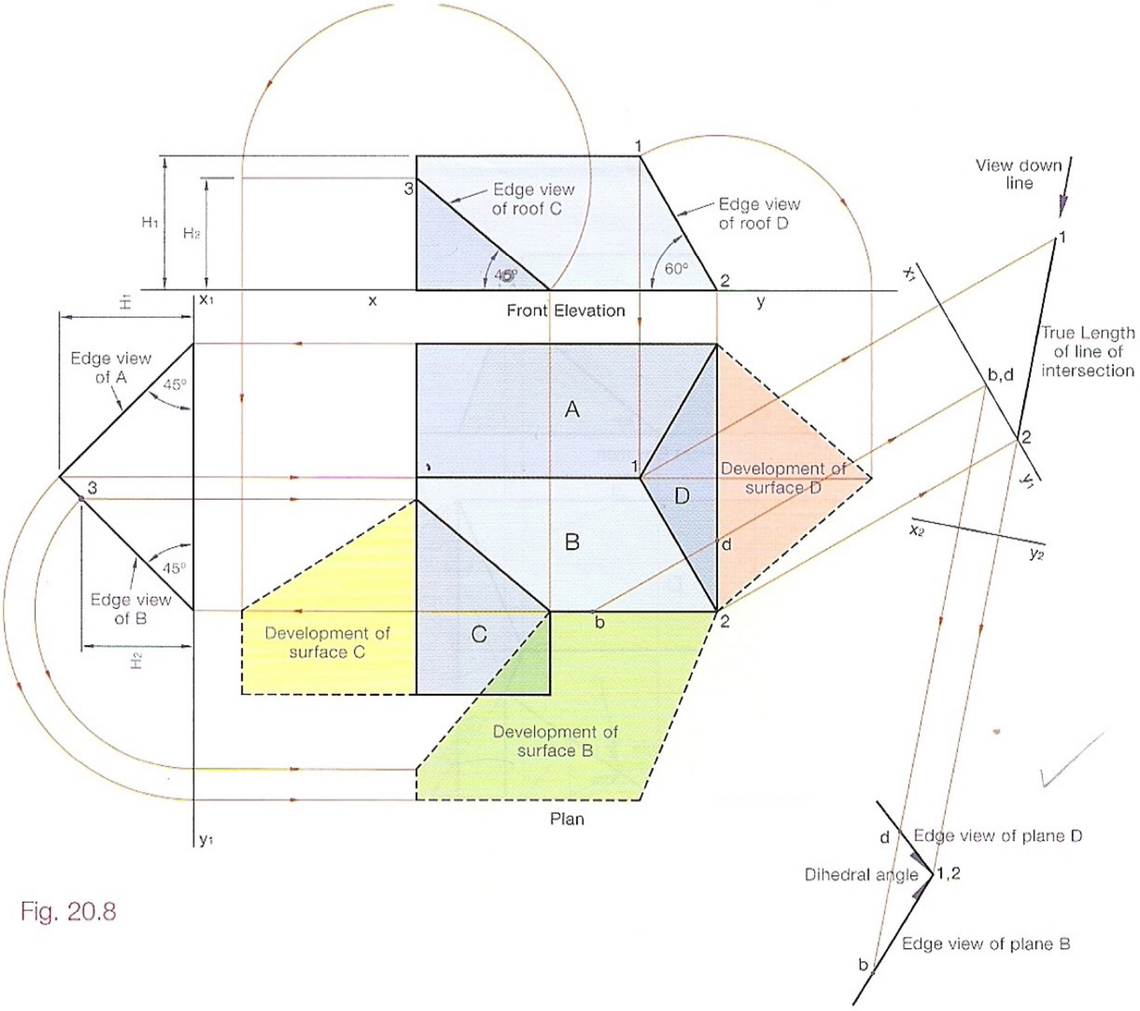


Fig. 20.8

(1) Auxiliary views are used extensively in solving roof geometry questions. Edge views of planes A and B are found by using an auxiliary elevation. The auxiliary shows the true pitch of these planes, 45° (from the question). The height, H_1, for the elevation is found here.

(2) Draw the elevation which will show planes C and D as edge views and again their true pitch will be seen, 40° and 60° respectively. H_2, the height of roof C, is found here. Point 1 is projected to plan and surface D is drawn in plan.

(3) H_2, the height of roof C, is stepped up on the auxiliary which shows roof A and B as edge views. Point 3 is found and projected to plan.

(4) Develop the surfaces by rebatting the planes where they are seen as edge views. Planes C and D are rebatted in the front elevation and surface B is rebatted in the auxiliary elevation. The planes in each case are folded until they are horizontal and the developments are found in plan.

(5) The dihedral angle is constructed using the point view method and is found as explained earlier.

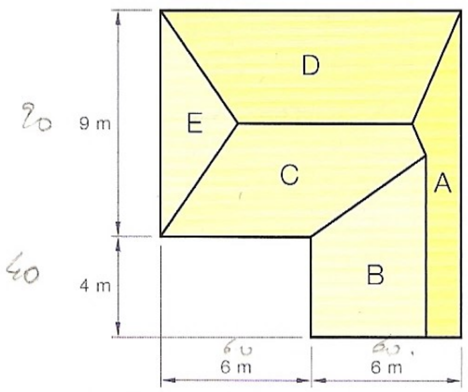


Fig. 20.9

Fig. 20.9 shows the plan of a roof. Surfaces C and D have a pitch of 50°. Surface A has a pitch of 70°, surface B has a pitch of 40° and surface E has a pitch of 60°.

(i) Draw the plan and elevation of the roof.

(ii) Develop the surfaces A and C.

(iii) Find the dihedral angle between surfaces A and D.

(iv) Find the dihedral angle between surfaces B and C.

Scale 1:100

The problem is solved using edge views of the roof surfaces, either using auxiliary views or the front elevation. As we have learned earlier, **an edge view of a plane is found by viewing along a line which is seen as a true length on the plane.** In most cases we use a horizontal line and view along its length in plan. When we see the planes as edge views we can use the pitch angle given in the question. The two dihedral angles are found using the two methods outlined earlier, point view method and rebatted triangle method.

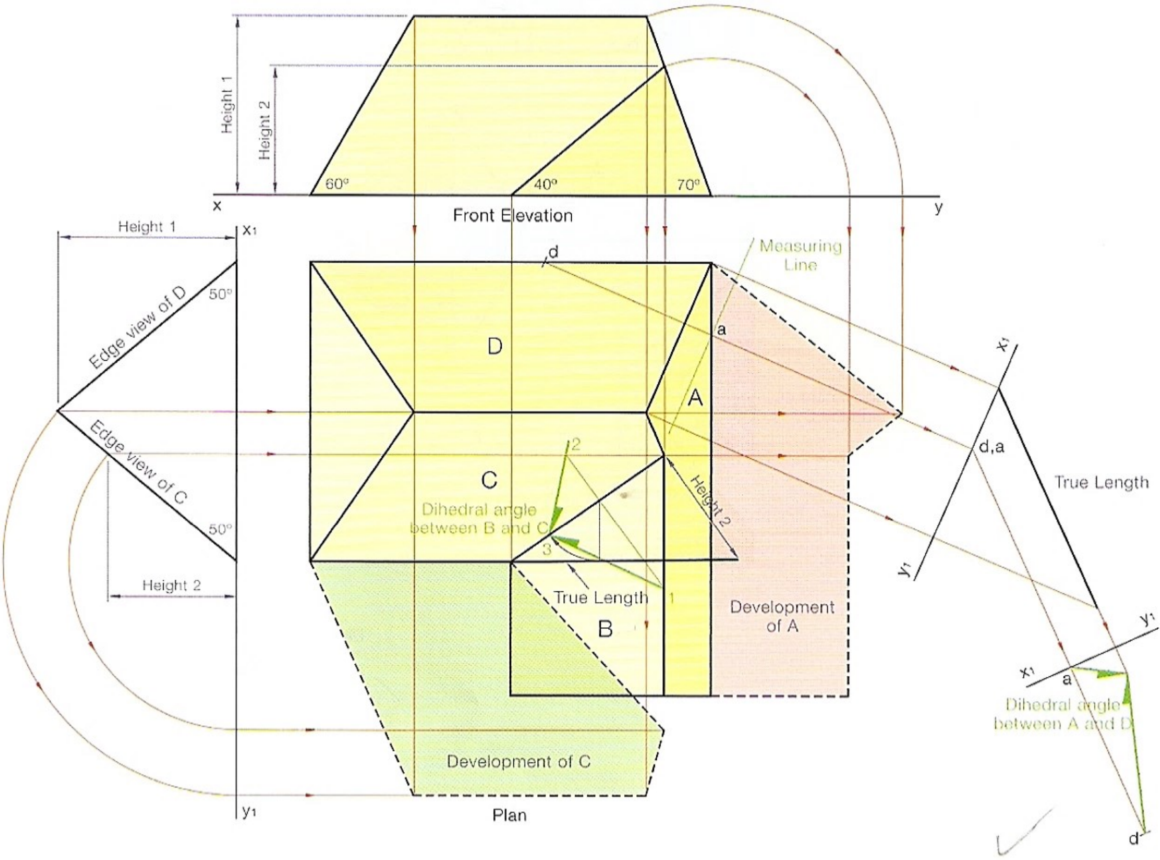


Fig. 20.10

The surfaces C and B form a valley. When forming the triangle that will define the dihedral angle, triangle 1,2,3, it is essential that corners 1 and 2 are level with each other.

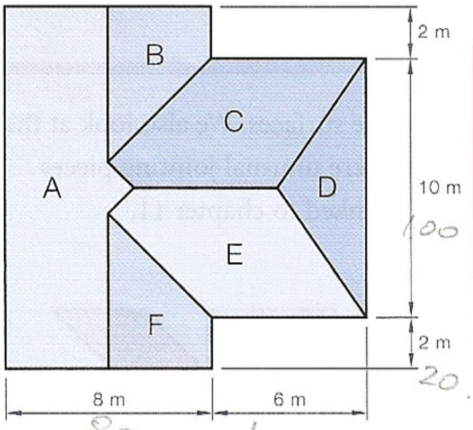


Fig. 20.11

Fig. 20.11 shows the plan of a roof. Surfaces A, B, C, D, E and F have a pitch of 50° and surface D has a pitch of 60°.

(i) Draw the plan and elevation.

(ii) Develop surfaces A and E.

(iii) Find the dihedral angle between surfaces D and E.

(iv) Find the dihedral angle between surfaces A and C.

Scale 1:100

When two roof surfaces of equal pitch intersect, the line of intersection will bisect the angle formed by the roof surfaces. The line of intersection between surfaces B and C will therefore be a 45° line, as will the line of intersection between surfaces E and F. The dihedral angle between A and C is best found using the point view method. Line pq is the line of intersection between them.

Project an auxiliary to show the true length of pq. Select a point on plane A, point 1, and project to the auxiliary. The height of point 1 is found in the elevation. Similarly, select a point on plane C, point 2, and project to the auxiliary. The height of point 2 is also found in elevation. View along the true length to get the dihedral angle. A measuring line is used to shorten distances in the second auxiliary.

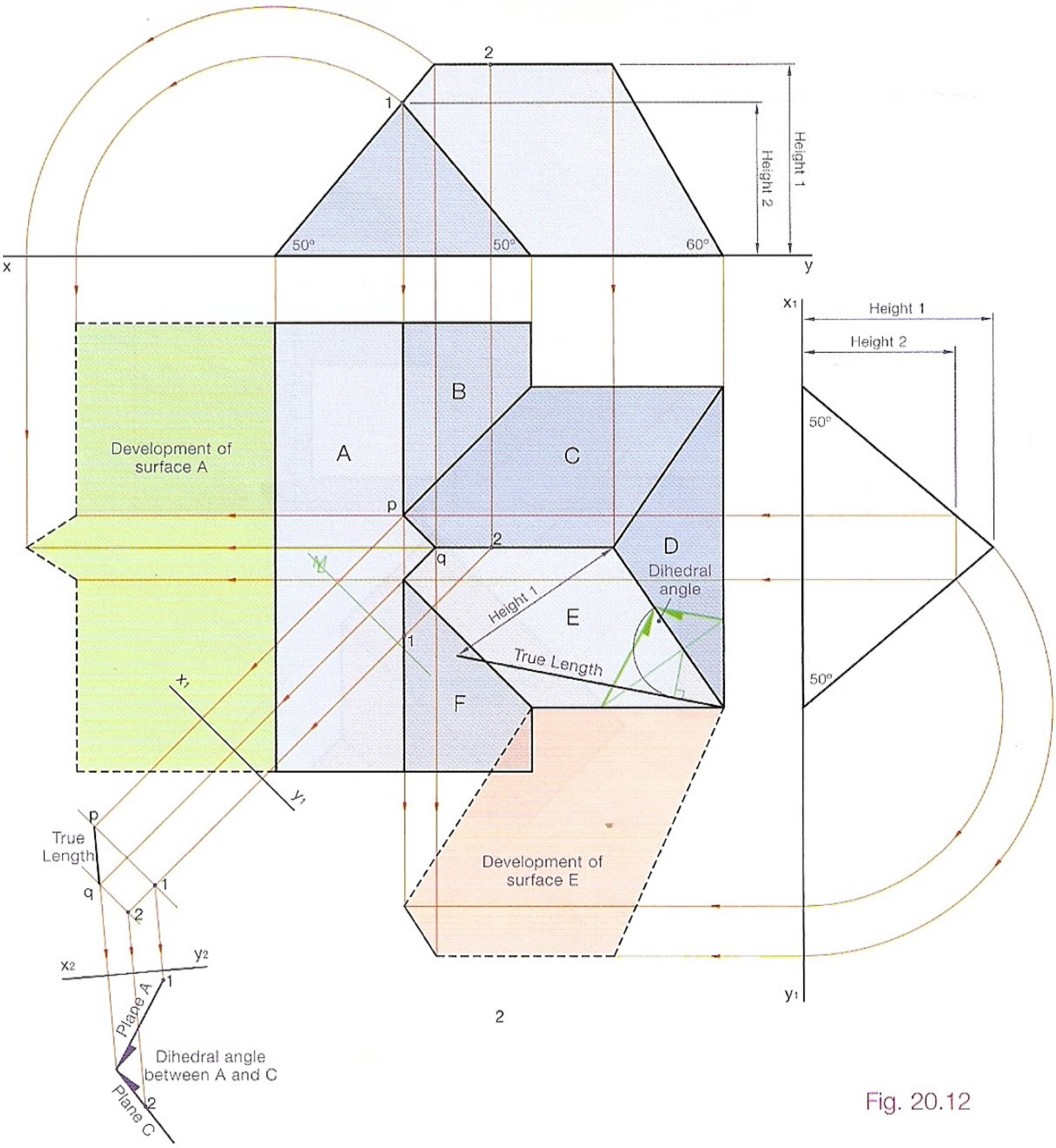


Fig. 20.12