Hyperboloid of Revolution (contd.)

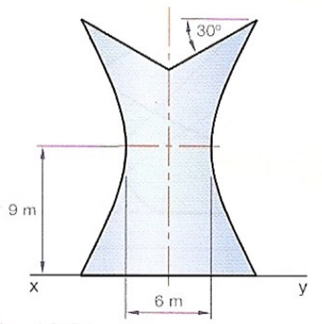


Fig. 18.61

Fig. 18.62 shows the elevation of a hyperboloid of revolution which has been cut at the top. The true length of all full elements on the surface of this structure is 21 m.

(i) Draw the plan and elevation of the building.

(ii) Project an end view.

(iii) Find the true shape of one side of the cut section.

Scale 1:100

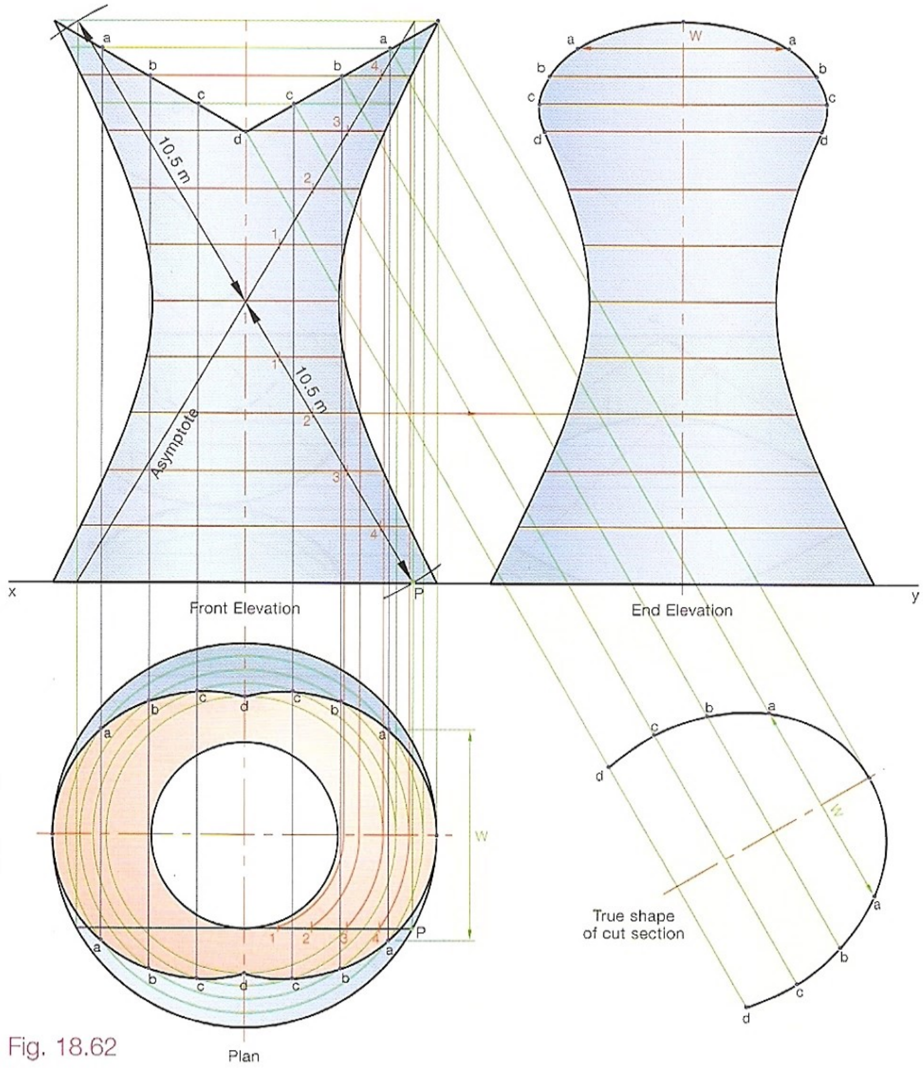


Fig. 18.62

(1) Set up the xy line, axis, throat line and overall height in elevation.

(2) Draw the throat circle in plan.

(3) The true length of all full straight line elements is 21 m. The asymptotes are elements and are seen as true lengths in elevation. The asymptotes always cross in the middle of the throat. Set the compass to half the length of the asymptote, place it at the centre of the throat and draw an arc to hit the xy line at P. Draw a horizontal line tangential to the throat circle in plan. Project P onto this line. Point P is on the base circle.

(4) Draw the hyperboloid as explained earlier.

(5) To find the points on the section in plan we take horizontal sections which produce circles in plan onto which the appropriate points are projected.

(6) The end view and auxiliary are found by taking widths from the front elevation and plan.

The elevation of a piece of sculpture is shown in Fig. 18.63. It is in the form of two solid semi-hyperboloids of revolution. Any straight line element on the full hyperboloid of revolution would measure 17 m.

(i) Draw the front elevation, end elevation and plan of the sculpture.

(ii) Determine the true shape of section S–S.

Scale 1:100

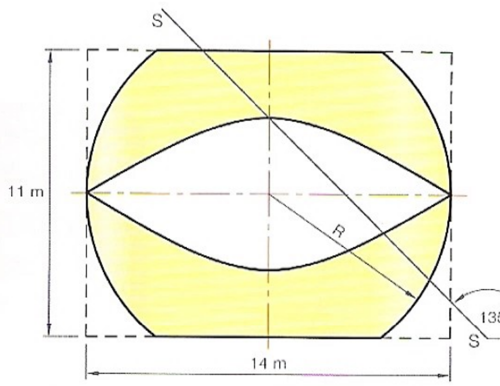


Fig. 18.

(1) The asymptotes will be 17 m long. Since they are only semi-hyperbolic paraboloids we only use half of this distance. Swing an arc from P to cut the side at Q. Point P is projected to the end view and the asymptote is drawn vertically. The throat circle is tangential to the asymptote in end view.

(2) The hyperboloid is constructed in the usual way.

(3) The shaped ends are found in end view and plan by taking sections perpendicular to the axes which appear as circles in end view.

(4) The section S–S is also found by using sections (construction not shown).

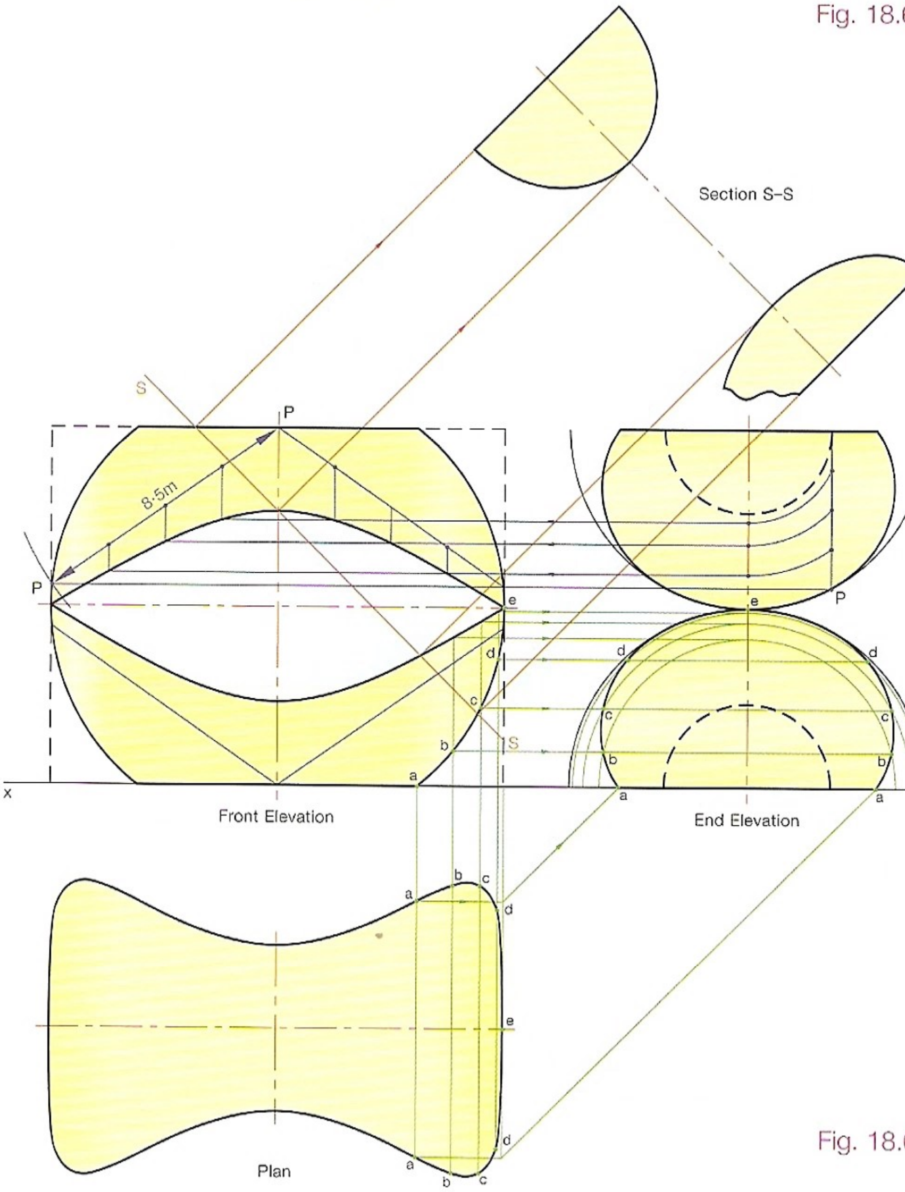


Fig. 18.

HIGHER LEVEL

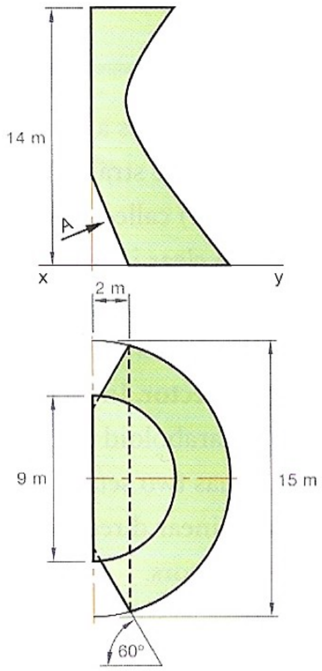


Fig. 18.65

Fig. 18.65 shows the plan and elevation of a semi-hyperboloid of revolution.

(i) Draw the plan and elevation of the structure.

(ii) Project a new elevation that will show the true shape of surface A.

Scale 1:100

(1) Draw the two semicircles in plan and project to elevation.

(2) The straight line in plan projects as a straight line in elevation and is therefore a portion of an element. As an element it will form a tangent to the throat circle in plan. Extend the line and draw the throat circle.

(3) Draw the asymptote in plan and project to elevation. Where the asymptote crosses the axis gives the position of the throat.

(4) Complete the plan and elevation.

(5) The new elevation is projected perpendicular to surface A. Surface A, when projected, will be made up of straight lines. The widths are found in the plan.

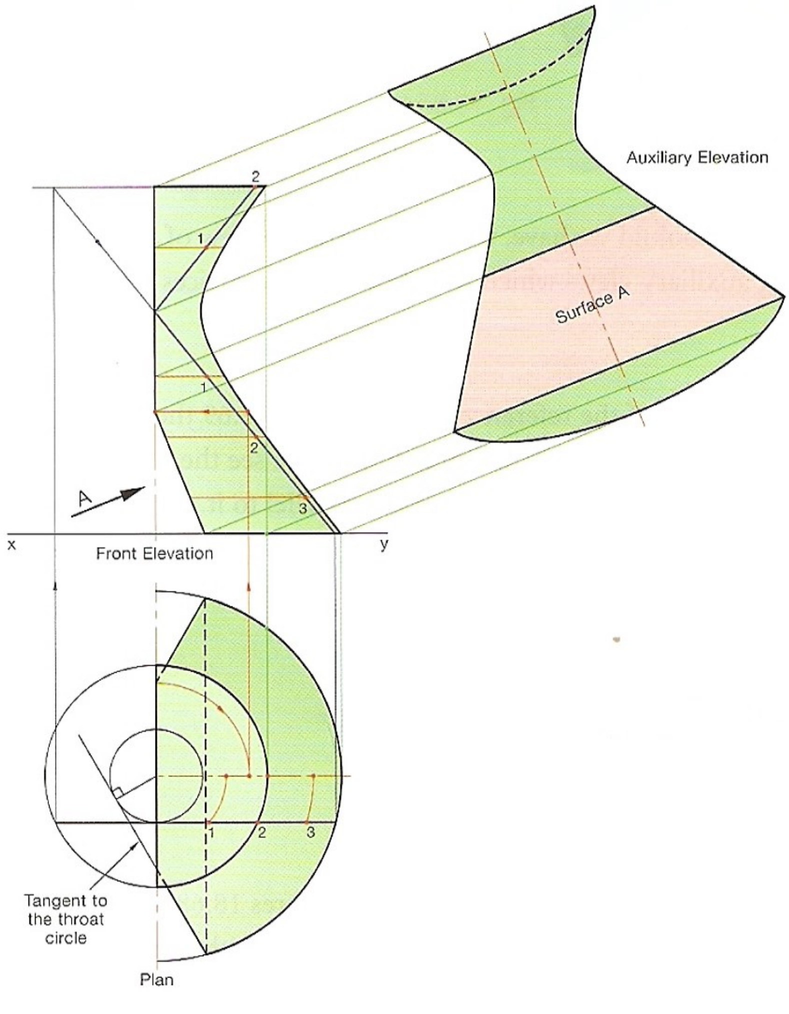


Fig. 18.66

Hyperbolic Paraboloid (contd.)

The hyperbolic paraboloid is a warped surface and therefore cannot be developed. It can also be referred to as a warped quadrilateral. We have seen earlier that it can be considered to be a surface generated by a straight line. The straight line is called a **generatrix.** This straight line moves along two non-parallel, non-intersecting lines (skew lines) called **linear directrices.** All this can be clearly seen from the previous work on this surface. What perhaps is not so clear is that the generatrix, as it slides along the linear directrices, must always stay parallel to a plane, called the **plane director.** In fact, because the hyperbolic paraboloid is a doubly ruled surface, it has two sets of generatrices, two sets of linear directrices and two sets of plane directors.

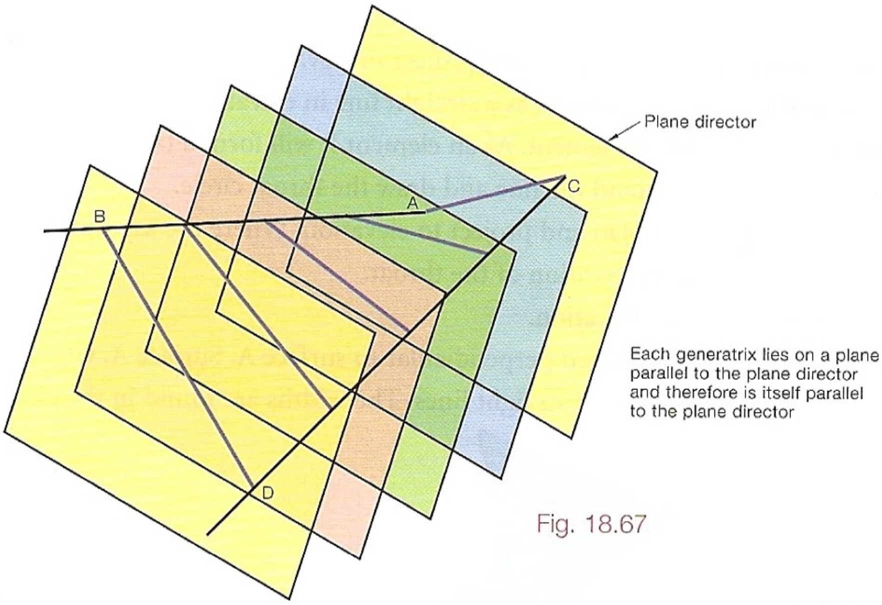


Fig. 18.67

Fig. 18.68 shows the plan and elevation of a hyperbolic paraboloid having two linear directrices ad and bc. The vertical plane is the plane director. Since the xy line in plan represents the edge view of the vertical plane, all the generatrices will be parallel to the xy line in plan.

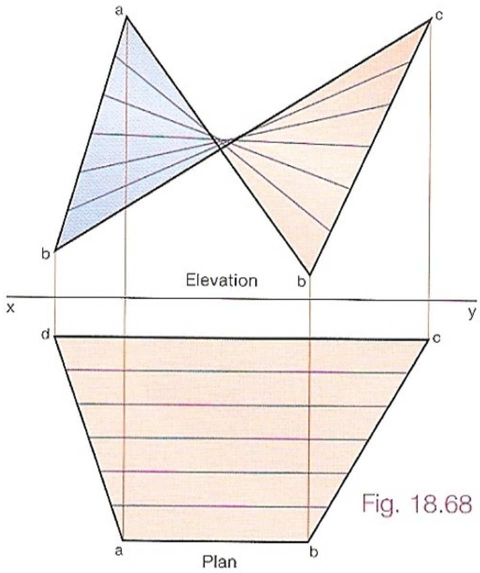


Fig. 18.68

No matter what hyperbolic paraboloid we have, we can get an edge view of its plane director by getting auxiliary views which will show its generatrices as parallel.

The plane director need not be one of the reference planes. In Fig. 18.69 the plane director is a vertical plane that is simply inclined. In plan we see the edge view of the plane director and all the elements appear parallel to it.

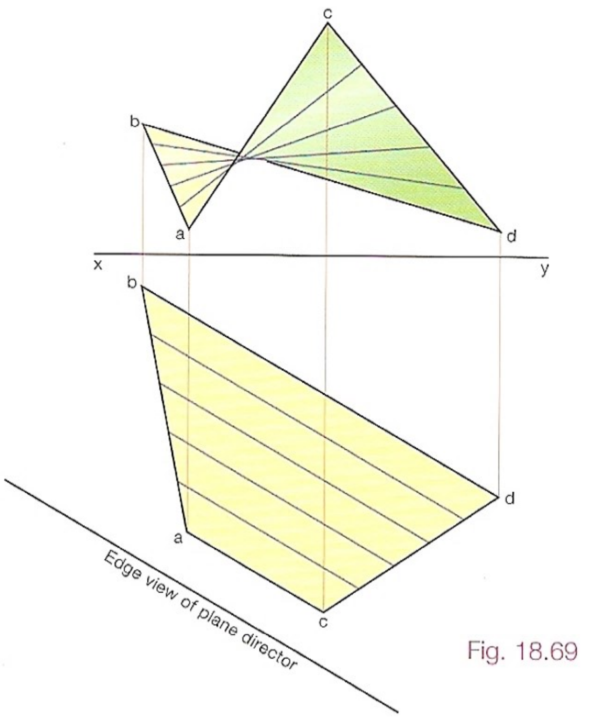


Fig. 18.69

In both Figures 18.68 and Fig. 18.69 the elements of the structure appear parallel either in plan or elevation. If the plane director is an oblique plane, the elements will not appear parallel in either of these views and the use of an auxiliary view is required.

g. 18.70 shows an example of such a hyperbolic paraboloid. order to find the plane director we must find a view of the ructure that will show the generatrices as appearing rallel. The extreme elements ab and cd are used in Fig. .69 to find this view. The construction used is exactly that ed in skew lines problems to show the two lines as parallel.

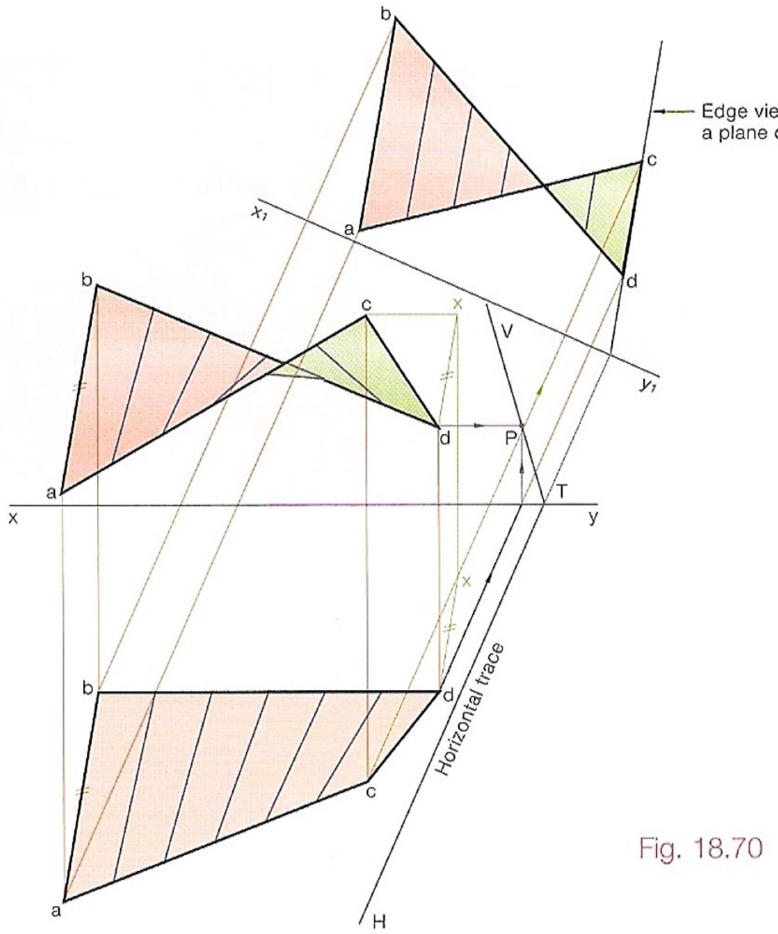


Fig. 18.70

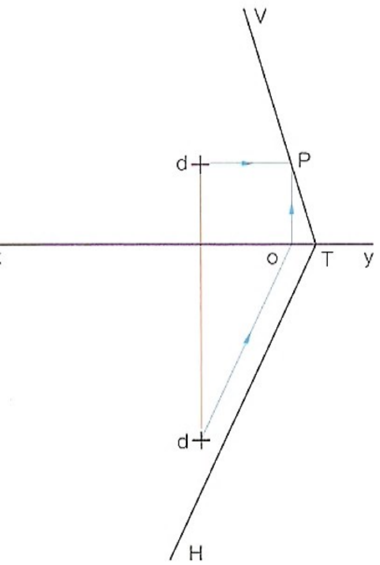
Generatrices ab and cd

) Draw a level line from c and from d draw a line parallel to ab to intersect at x.

) Project x down to plan. In the plan draw a line from d parallel to ab in plan to intersect at x in plan.

) Join c to x and view in this direction to project a view of the structure showing all elements as parallel.

) The plane director can be drawn in the auxiliary. It is seen as an edge view and will be parallel to the generatrices.



g. 18.71

(5) In Fig. 18.70 the plane director is containing element cd. Extend the edge view o the plane director to hit the x_1y_1. This point is a point view of the horizontal trace. Project the horizontal trace back to plan.

(6) The vertical trace is found by using a point on the plane director, e.g. point c or point d. We have point d in plan and elevation, we have the horizontal trace and we require the vertical trace.

(7) Project d in plan, to the xy line, parallel to the horizontal trace to give point o.

(8) Project vertically and from d in the elevation project horizontally to give point p Point p is a point on the vertical trace.

There is an infinite number of plane directors for any hyperbolic paraboloid surface. They will all be parallel to each other but their position can vary.

Given two skew line directrices ab and cd and the traces of the plane director of a hyperbolic paraboloid. To determine the elements of the surface.

a = 30, 10, 68
b = 75, 21, 40
c = 18, 45, 10
d = 82, 9, 27

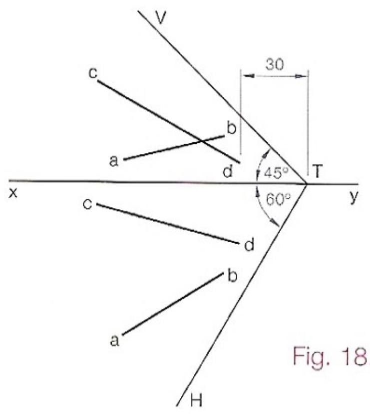


Fig. 18.72

(1) Find the edge view of the plane director by projecting an auxiliary view in the direction of the horizontal trace.
(2) Draw the directrices ab and cd in this auxiliary.
(3) The elements can now be drawn in the auxiliary, parallel to the edge view of the plane director. The most extreme elements are found first and further elements spaced out evenly between these.
(4) Project the elements back to plan and project to elevation.

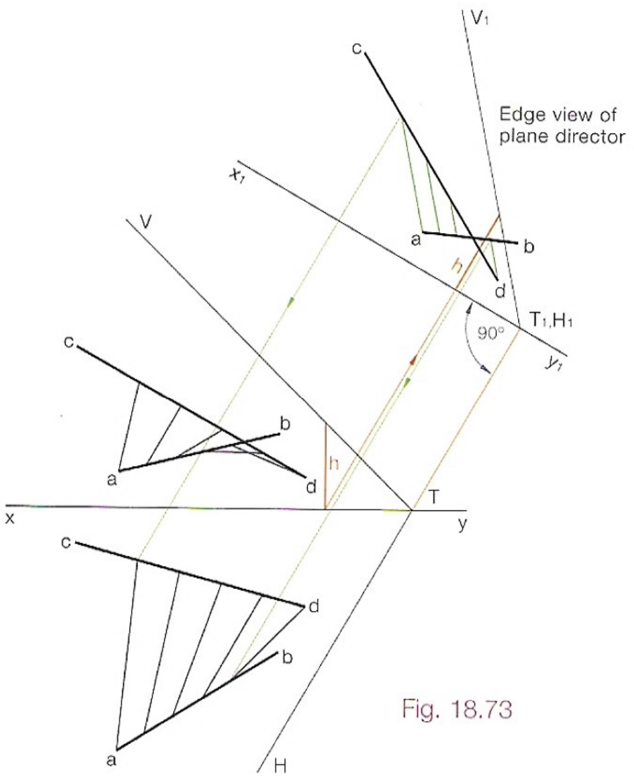
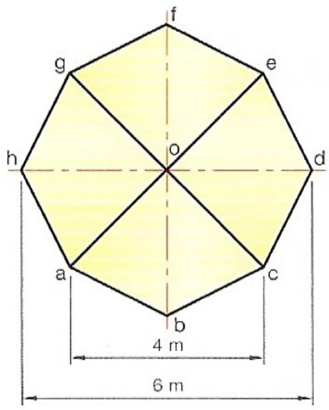


Fig. 18.73

HIGHER LEVEL



g. 18.74

Fig. 18.74 shows the outline plan of a rain shelter. The structure is in the form of four hyperbolic paraboloid surfaces, abco, cdeo, efgo and ghao. Points b, d, f and h are at ground level. Points a, c, e and g are 5 m above ground level and point o is 6 m above ground level.

(i) Draw the plan and elevation of the surfaces ghao, abco and cdeo.

(ii) Find the curvature of the roof along the line joining b to e.

(iii) Find the traces of the plane director for the edges ab and co and having its horizontal trace containing point b.

Scale 1:50

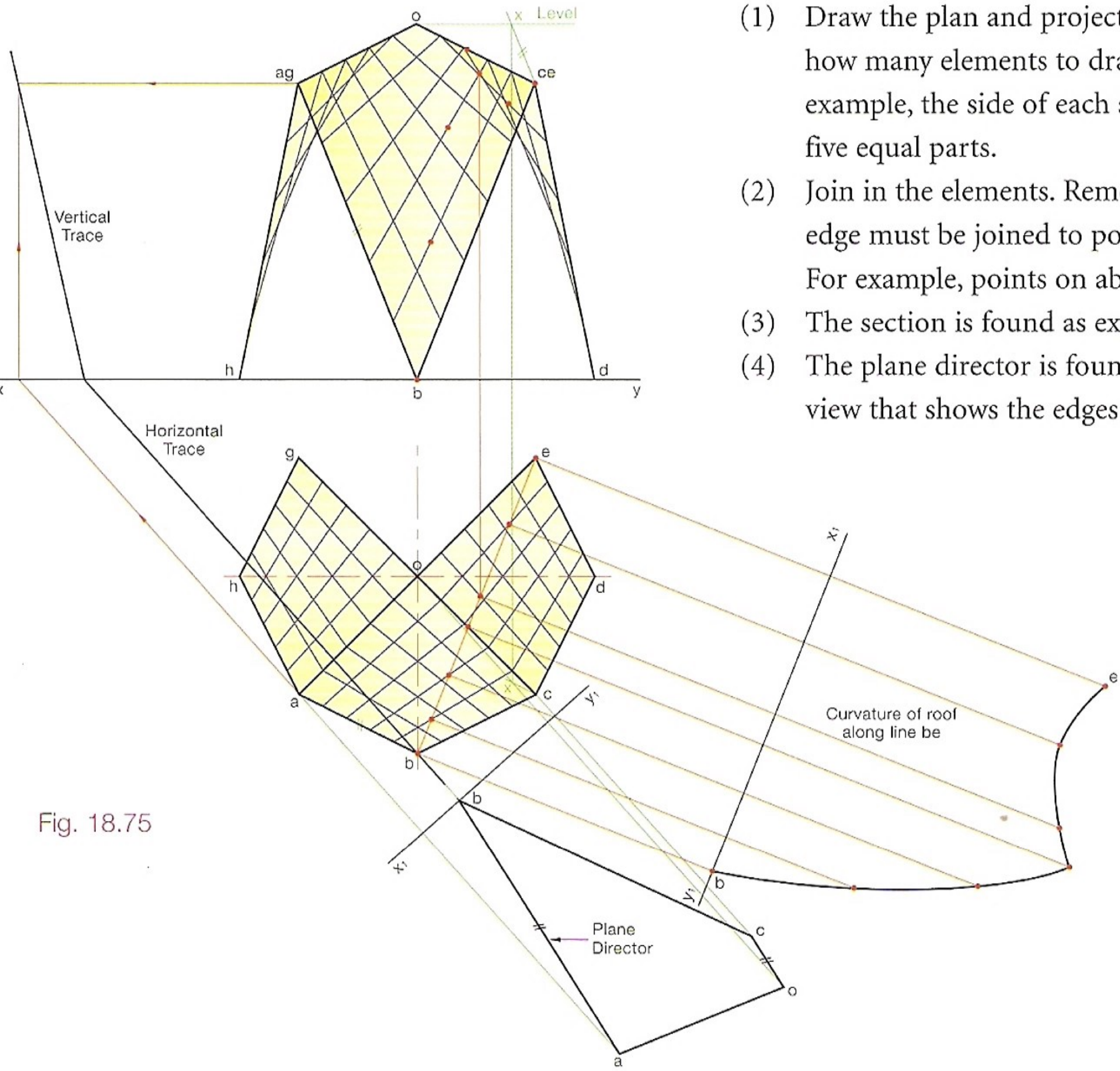


Fig. 18.75

(1) Draw the plan and project the elevation. Decide on how many elements to draw on each surface. In this example, the side of each structure was divided into five equal parts.

(2) Join in the elements. Remember that points on one edge must be joined to points on its opposite side. For example, points on ab join to points on oc.

(3) The section is found as explained before.

(4) The plane director is found by finding an auxiliary view that shows the edges ab and co as parallel. This is a skew lines problem and the construction is as explained earlier.

(5) Where the plane director meets the x_1y_1 gives the horizontal trace which is projected back to plan.

(6) Point 'a' which is also on the plane director is used to find the vertical trace.

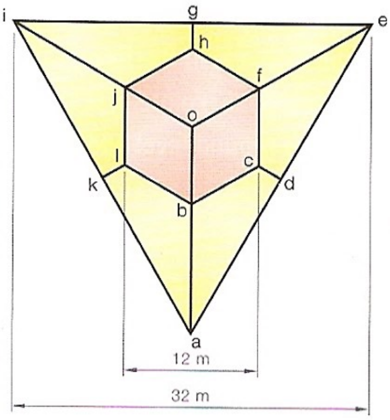


Fig. 18.76

Fig. 18.76 shows the outline plan of a roof made up of nine adjoining hyperbolic paraboloid surfaces. The outline is an equilateral triangle and the three internal surfaces make a regular hexagon in plan.

(i) Draw the plan and elevation of the three surfaces abcd, cdef and bcfo.

(ii) Determine the curvature of the roof along a line joining A to C.

(iii) Determine the plane director for the elements cf and de. Find the traces of the plane director having its horizontal trace passing through e.

Scale 1:200

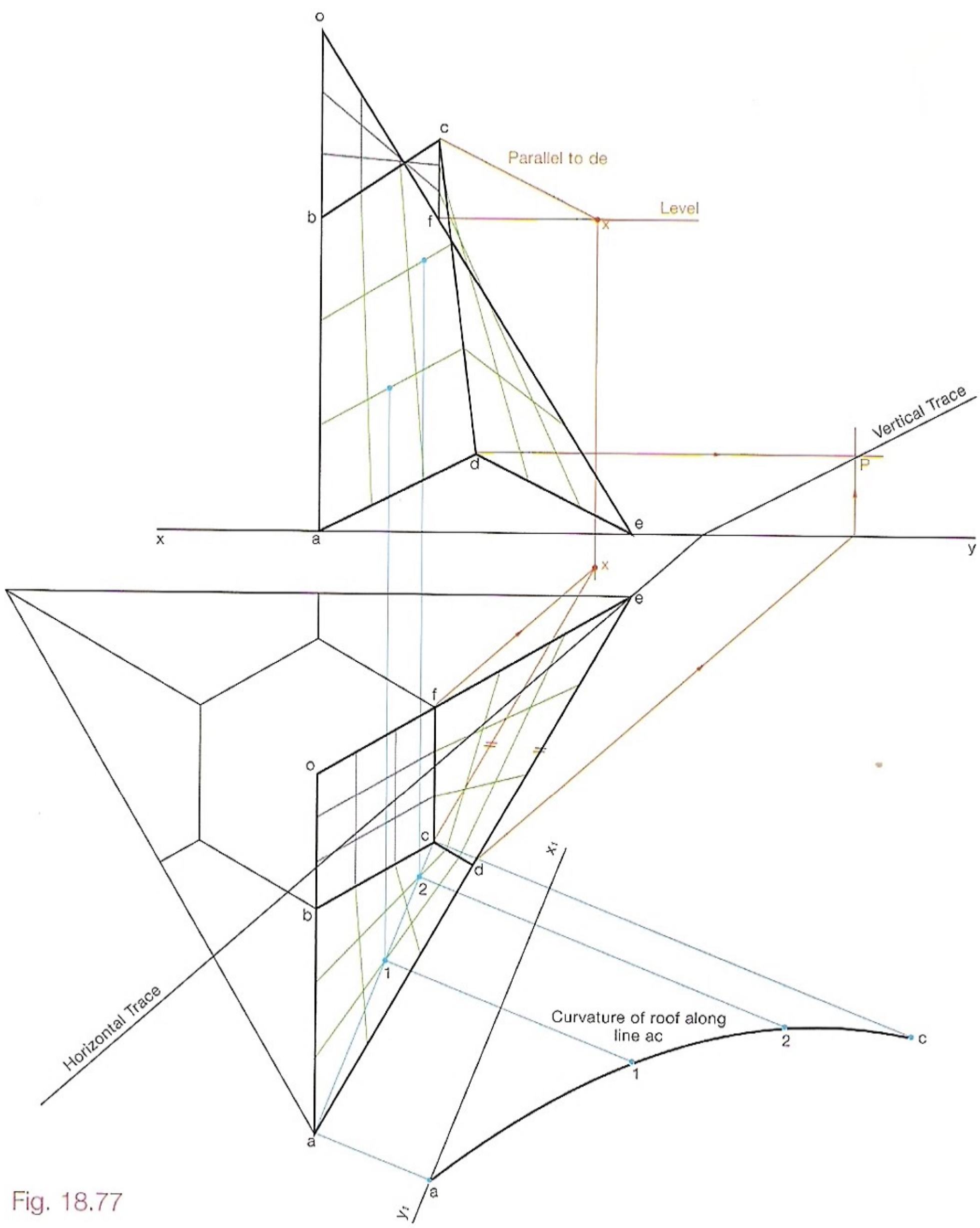


Fig. 18.77

(1) Because of the complexity o the drawing we have only shown a limited number of elements.

(2) The plane director is to be found for cf and de. Draw a level line from f in elevatio and draw from c parallel to de giving x. Drop x to plan.

(3) In plan, draw from c paralle to de in plan to locate the exact position of x.

(4) Join x back to f. This is the direction of the horizontal trace. Draw the trace through e.

(5) Draw from d parallel to HT to xy. Project vertically.

(6) Project from d in elevation, horizontally to find P. Point is a point on the vertical trace. Draw the trace.

Fig. 18.78 shows the plan of a shell structure in the form of a hyperbolic paraboloid. Curve DBA is semi-elliptical in plan. Curves DE and BF are found by extending the surface ABCD until it meets ground level.

(i) Draw the plan and elevation of the structure.

(ii) Project the end view.

Scale 1:200

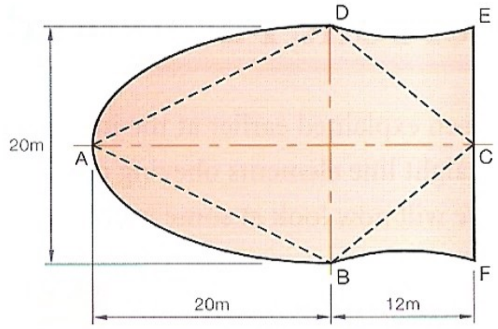


Fig. 18.78

(1) Set up ABCD in plan and elevation.

(2) Draw the semi-ellipse in plan using the trammel or similar method.

(3) Divide the sides of the hyperbolic paraboloid into a suitable number of equal spaces, both in plan and elevation.

(4) The curve DAB is found in elevation by extending the elements in plan to reach the semi-ellipse at 1, 2, 3 and 4. These points are projected to elevation to meet the elements extended.

(5) The curves DE and BF are found by extending the elements in elevation to meet the xy line at p and q.

(6) p and q are projected to plan to intersect the elements extended.

(7) The end view is found by projection from the other two views.

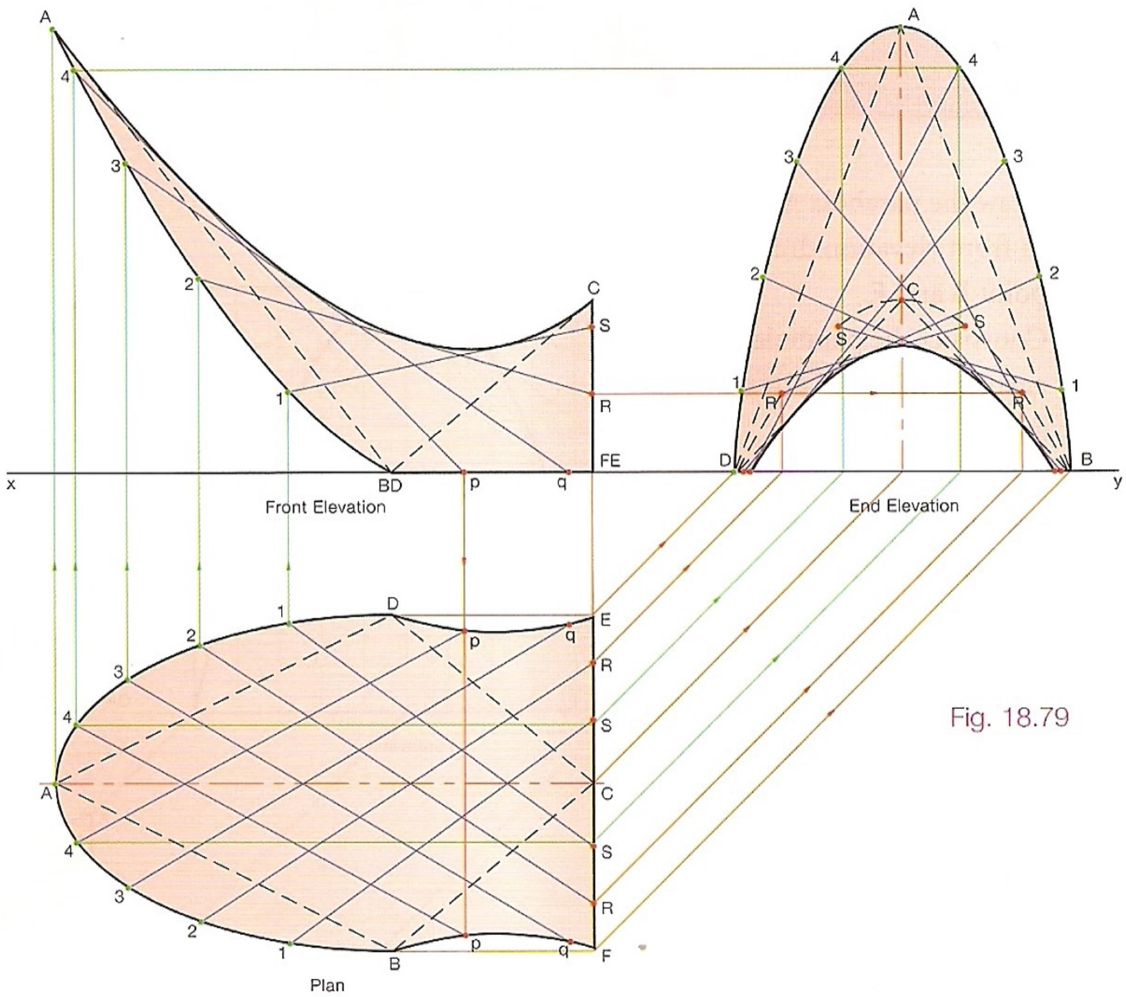


Fig. 18.79

Hyperbolic Paraboloid as a Surface of Translation

As has been explained earlier at the introduction to this topic, the hyperbolic paraboloid can be seen as a structure mac up of straight line elements obeying certain rules or as a parabolic curve sliding on its vertex along an inverted parabol curve. We will now look at some problems based on this type of model.

Fig. 18.80 shows a pictorial view of a shell structure. The surface of the structure is generated by translating the parabola ABC in a vertical position along the parabola BE whose vertex is at E.
Draw the plan and elevation of the structure.

Scale 1:200

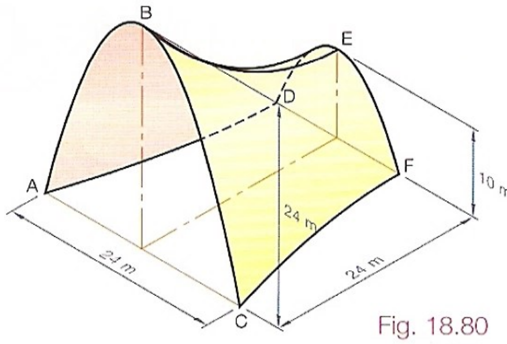


Fig. 18.80

(1) Draw the parabola ABC.
(2) In front elevation draw point B and E.
(3) Construct the parabola BE ensuring that the vertex is at E.
(4) All vertical sections will produce parabolas which are part of the ABC parabola. The end curve DEF is part of the ABC parabola. The width w is found in plan by stepping height h down from the top of parabola ABC giving width w.
(5) Curves AD and CF are hyperbolas and are constructed as explained earlier.

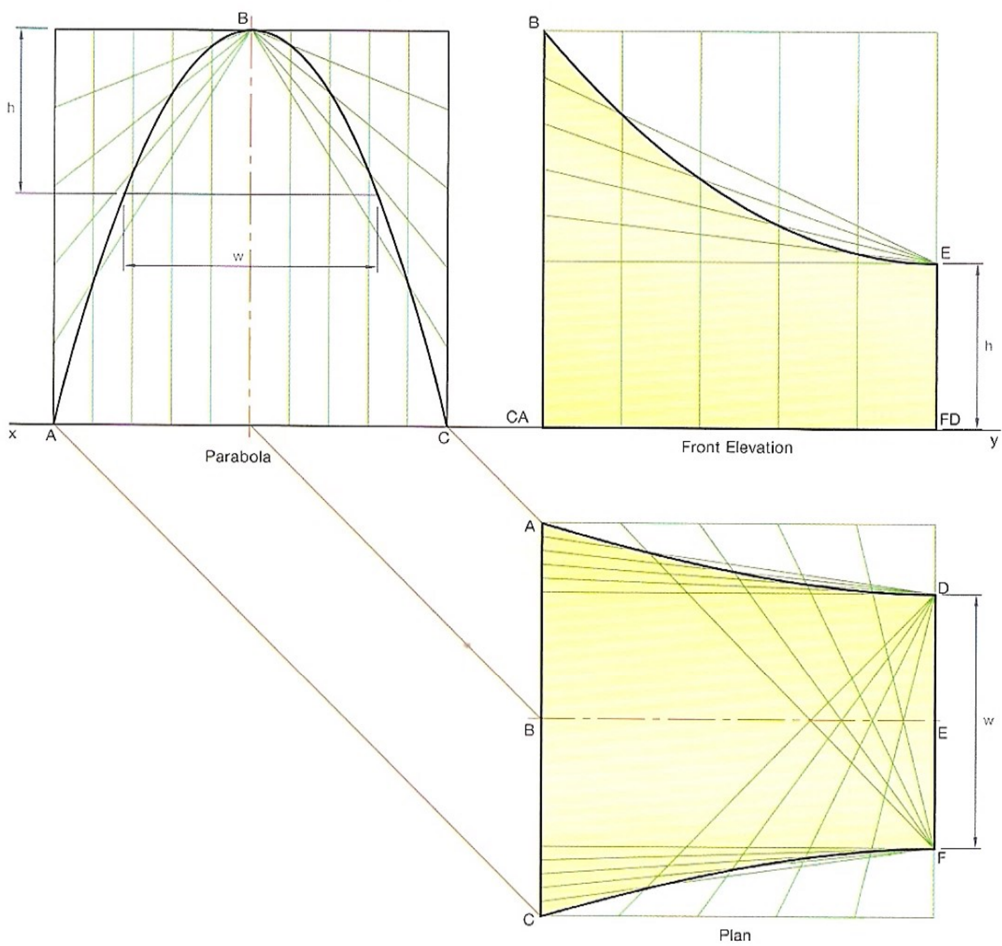


Fig. 18.81

HIGHER LEVEL

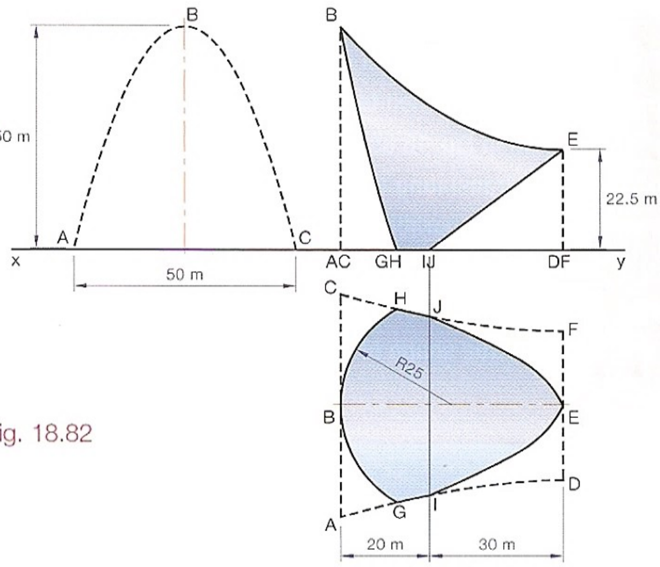


Fig. 18.82

Fig. 18.82 shows the plan and elevation of a shell structure which is in the form of a hyperbolic paraboloid. It is formed by sliding parabola ABC in a vertical position along the parabola BE whose vertex is at E. The shell has been cut as shown.

(i) Draw the plan and elevation of the unit.

(ii) Project an end view of the unit.

Scale 1:500

(1) Draw the parabola ABC.

(2) Construct the parabola BE in elevation having its vertex at E.

(3) In the plan, the width of the end DEF is not given and must be found. Take the height of E in elevation and step it **down** from the top of parabola ABC. This gives the **width** of DF in plan.

(4) The left side of the plan can be completed and the right side of the elevation.

(5) w_1 and w_2 are taken from plan, stepped out from the axis of parabola ABC to find h_1 and h_2 which are stepped down from BE.

(6) h_3 and h_4 are taken from elevation, stepped down from the vertex of parabola ABC to find w_3 and w_4 which find points in the plan.

(7) The end view is projected from front elevation and plan.

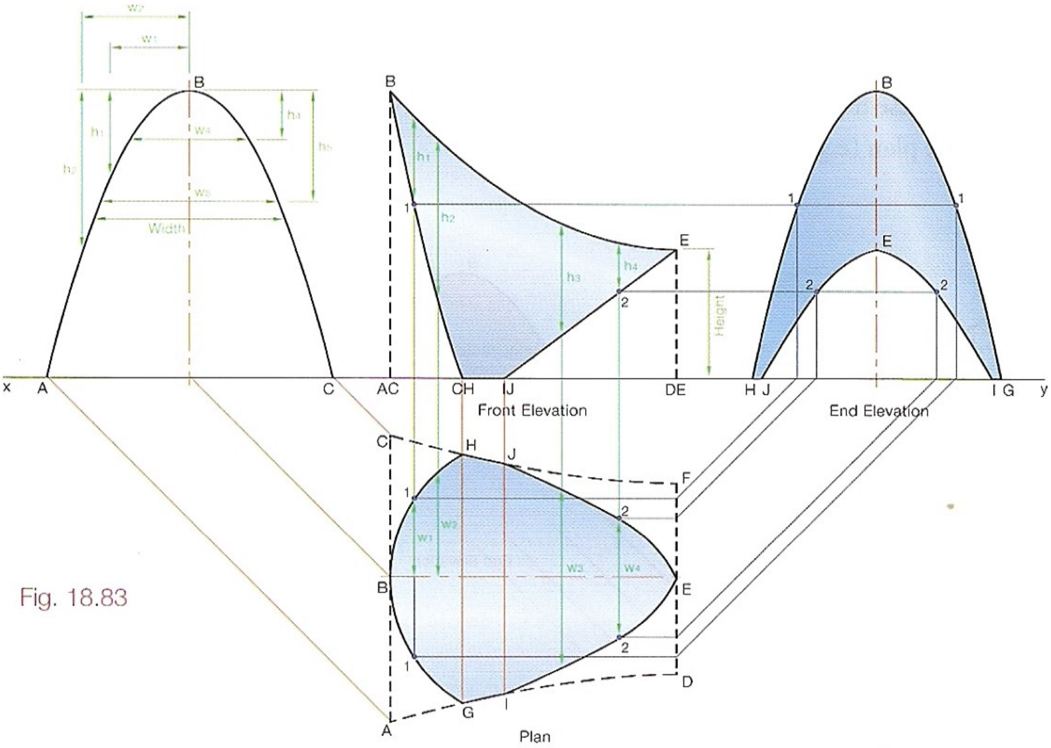


Fig. 18.83

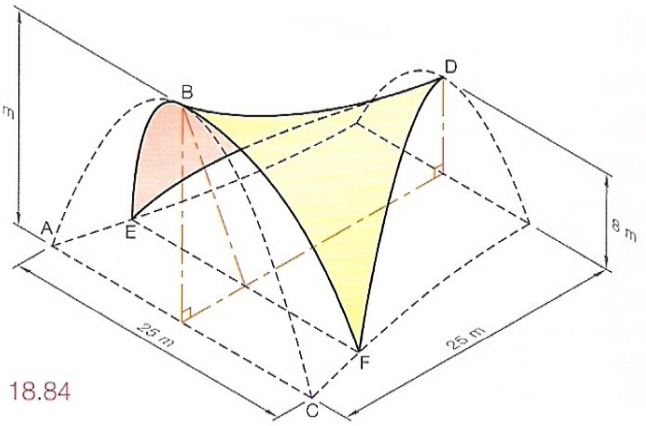


Fig. 18.84

Fig. 18.84 shows a pictorial view of a shell structure. Six of these units are combined to form a total roof surface as shown in plan in Fig. 18.85. The surface of the unit is generated by translating the parabola ABC in a vertical position along the parabola BC whose vertex is at D.

(i) Draw the plan and elevation of the unit.

(ii) Project an end view of the unit.

(iii) Find the true shape of curve DF.

Scale 1:200

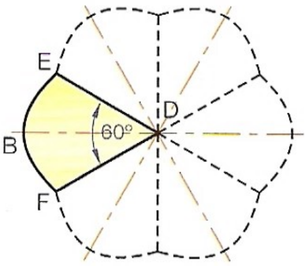


Fig. 18.85

(1) Draw parabola ABC.

(2) Draw the rectangle that contains the front elevation and construct the half parabola BD.

(3) Draw the plan of the uncut shell structure by taking heights from parabola BD to th xy line (e.g. H_1) and step these heights **down** from the top of parabola ABC to give widths (e.g. W_1) which are used in the plan.

(4) Cut the shell structure in plan to form an inclusive angle of 60° thus finding E and F

(5) Project E and F to the xy line and join to B. The left of the front elevation is complet and the right of the plan is completed.

(6) Take heights in elevation from parabola BD to straight line BFE (e.g. H_2).

(7) Step these heights **down** from the top of parabola ABC to find widths which are usec in the plan (e.g. W_2).

HIGHER LEVEL

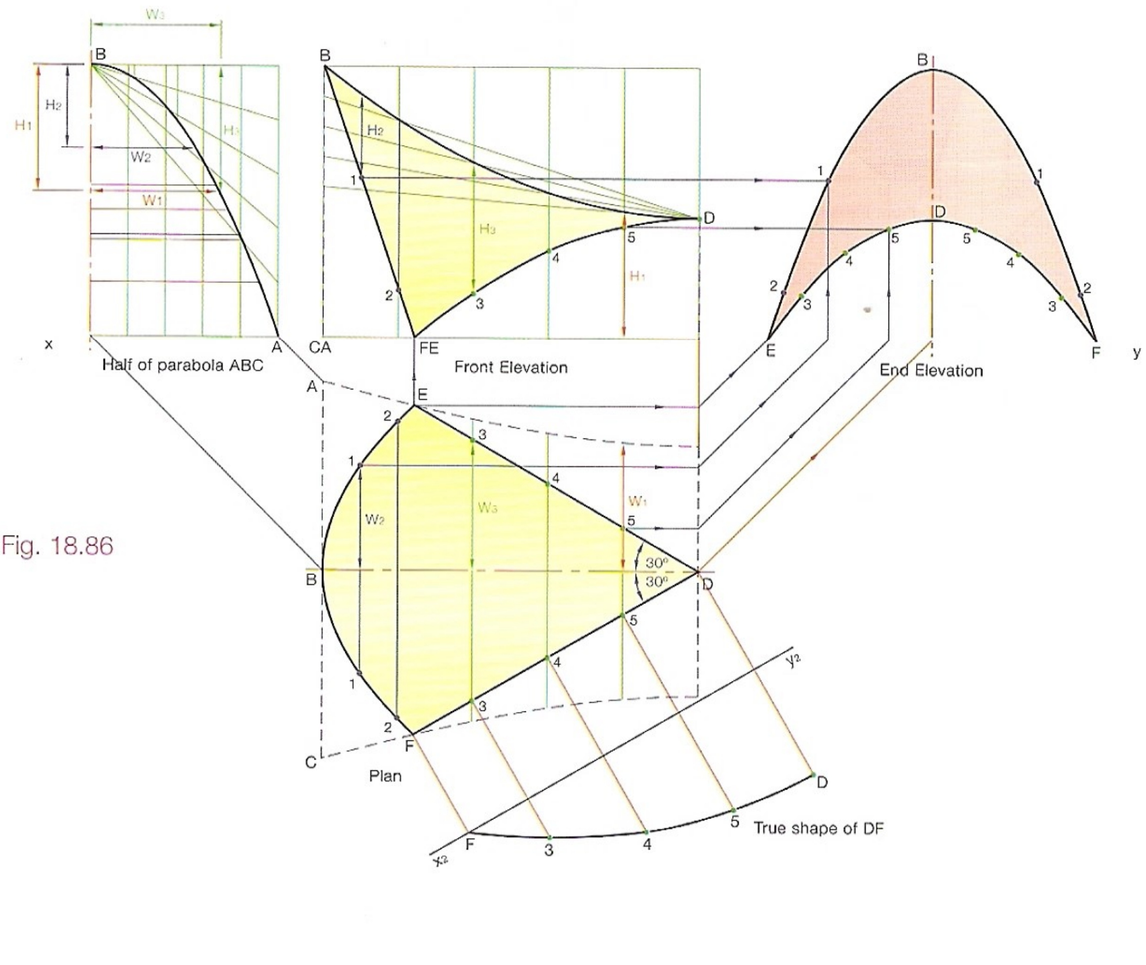


Fig. 18.86