Hyperboloid of Revolution

The hyperboloid of revolution is a ruled surface. It is generated by revolving a straight line about another non-parallel, non-intersecting line as its axis. Figures 18.38 and 18.39 show this arrangement. It is clear from the diagram that any section of a hyperboloid of revolution which is perpendicular to the axis will produce a circle.

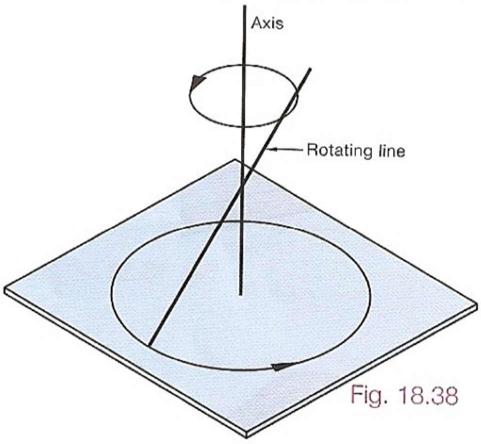


Fig. 18.38

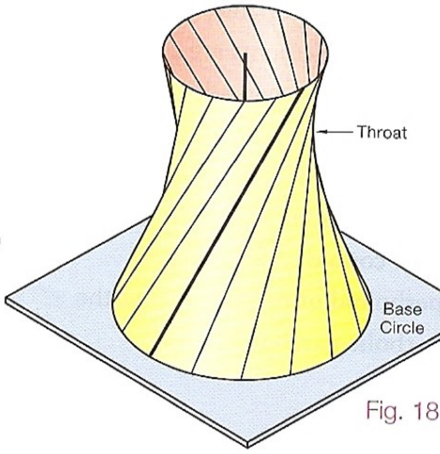


Fig. 18

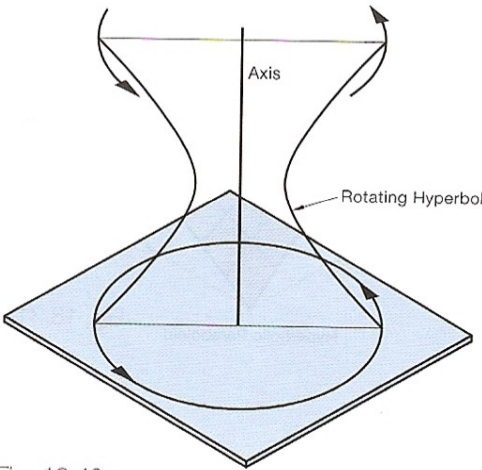


Fig. 18.40

The curves produced at the sides are hyperbolas. The narrowest part of the hyperboloid of revolution is called the throat or the throat circle.

The extreme limits of this shell structure are the cylinder and cone. As the throat circle and the base circle became closer to each other in size the hyperboloid of revolution becomes more cylindrical. As the throat circle decreases in size and nears a radius of zero the hyperboloid becomes more cone-like.

A hyperboloid of revolution can also be constructed by rotating one arm or both arms of a double hyperbola about the conjugate axis. Figures 18.40 and 18.41 show such a double hyperbola.

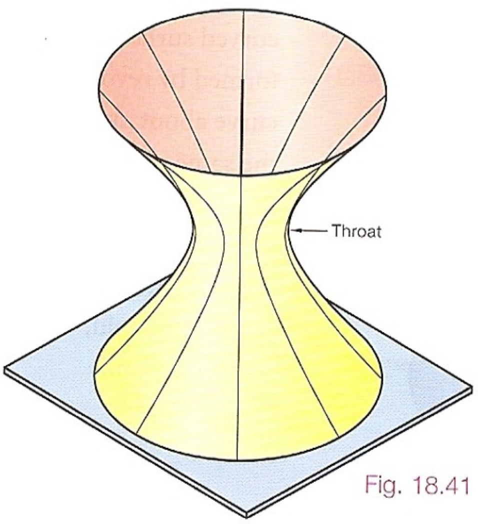
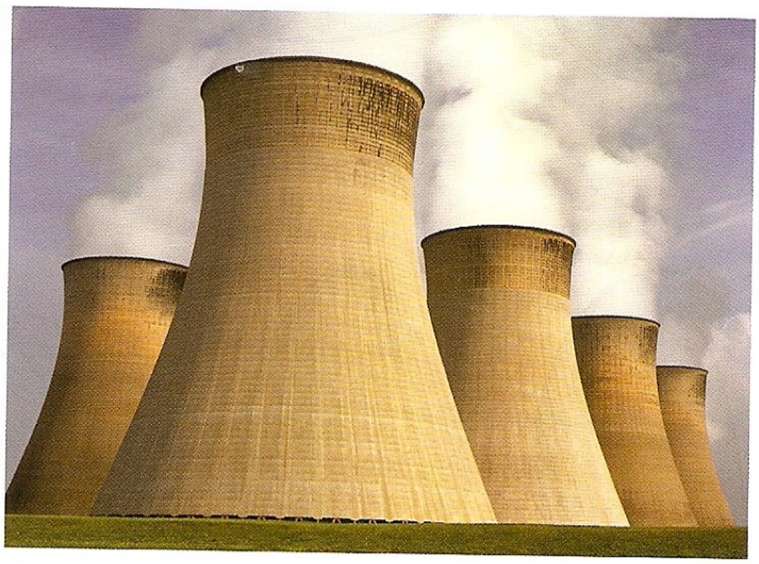
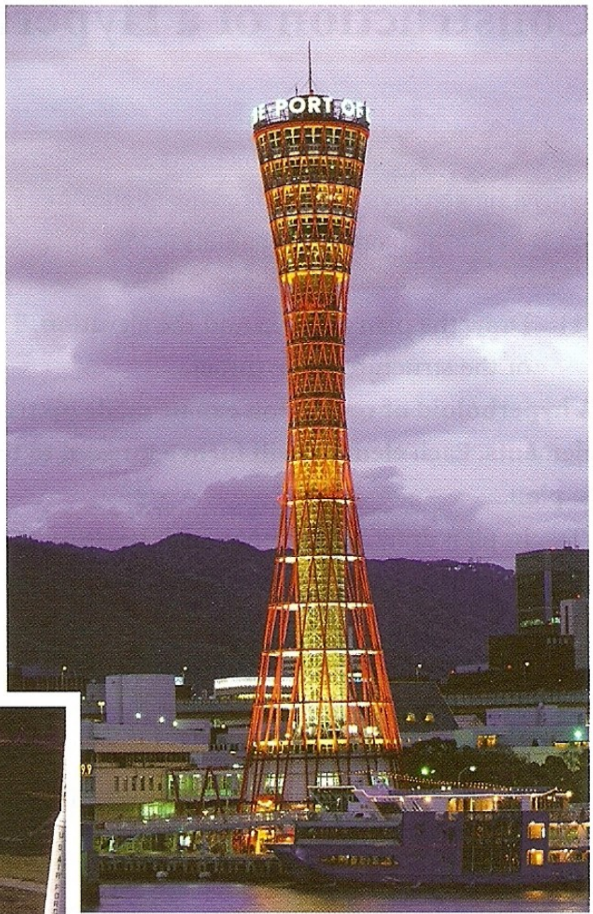
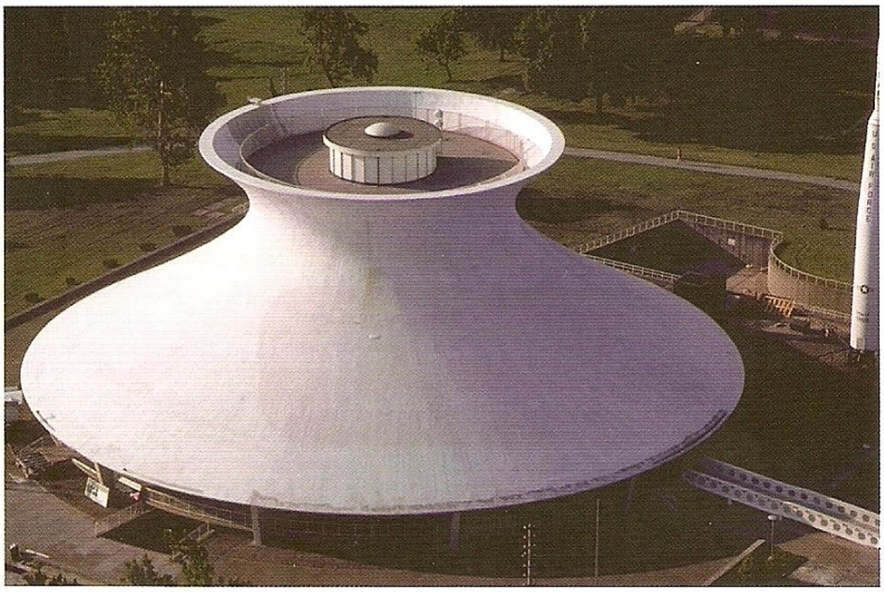


Fig. 18.41



As mentioned already, the hyperboloid is a ruled surface. A ruled surface is a surface that for every point on it, there is a straight line passing through it, which lies on that surface for its entire length. When a point on the surface has two such lines passing through it, it is called a **doubly ruled surface**. The hyperboloid of revolution is such a surface as is evident from Fig. 18.43 on the next page. There are only three doubly ruled surfaces: the hyperboloid of revolution, the hyperbolic paraboloid and the plane.





The hyperbolic paraboloid is a shell structure – one of many types. Shells are as old as nature and derive their strength not from the thickness of the shell but from the shape. A good example of this is an egg. The shell of an egg is relatively thin and is composed of a brittle material. In spite of this, an egg will withstand a huge load as long as it is evenly distributed. The strength is derived from the doubly curved surface.

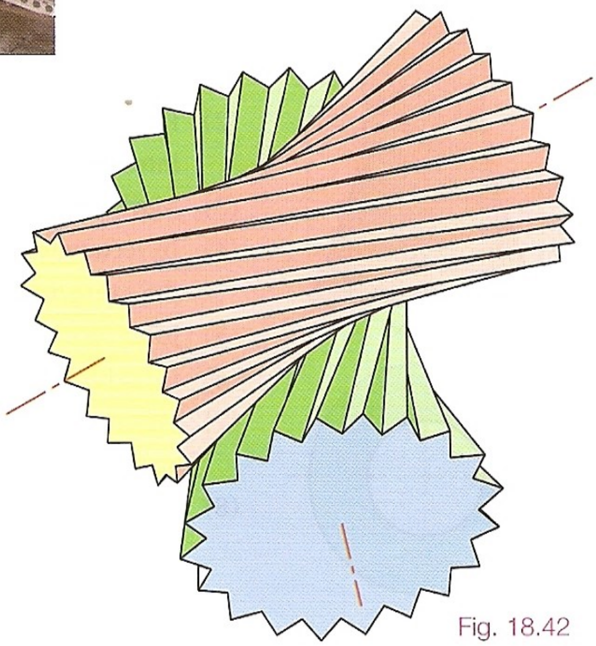


Fig. 18.42

Hyperboloidal gears transmit motion to a skew shaft

Construction of a Hyperboloid of Revolution

Method 1

Given the base circle, throat circle and height of a hyperboloid of revolution. Construct the shape using elements.

(1) Draw the plan as given and the elevation. The narrowest part of the structure is the throat.

A hyperboloid of revolution can be made up straight-line elements. Each element will form a tangent to the throat circle in plan.

(2) In plan draw an element which starts on the base circle, is tangential to the throat circle and continues to hit the base circle again, e.g. element 1.

(3) In this example the larger circle in plan represents both the base circle and the top circle of the structure. Project point 1 to the base of the elevation and point 'a' to the top of the elevation. Join these points.

(4) At suitable spacing around the circle draw more elements in plan and project to elevation.

(5) The shape forms as the elements are plotted.

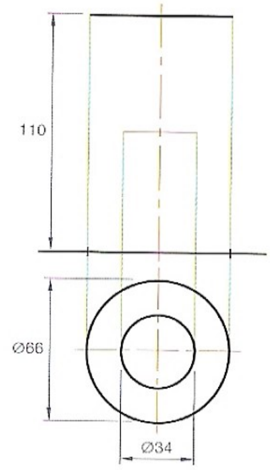


Fig. 18.43

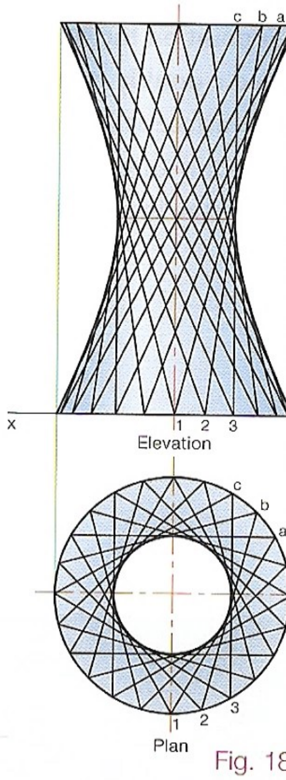


Fig. 18

Method 2

Given the same information as in the previous example, construct a hyperboloid of revolution using the rectangle method.

This method is based on the construction of a double hyperbola using the rectangle method.

(1) Draw the given plan and mark off the heights in elevation.

(2) Project up the outer extremities of the circles from the plan thus creating the rectar into which the curves will fit.

(3) Mark V_1 and V_2 the vertices of the curves.

(4) In rectangle $ABCV_1$ we divide edge AB into a number of equal parts and edge AC ir the same number of equal parts.

(5) Join the divisions on AB to vertex V_1.

(6) Join the divisions on AC to vertex V_2.

(7) Where line $V_1$1 and line $V_2$1 cross gives a point on the hyperbola. Similarly for lines and $V_2$2 etc.

(8) Repeat construction for other sections of the curves.

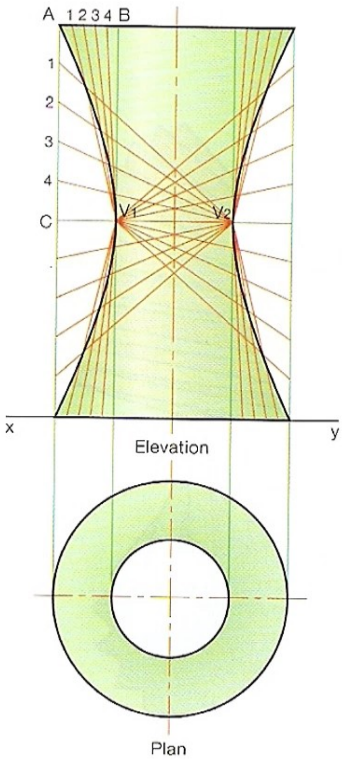


Fig. 18.45

Method 3
Given the same information as in Method 1 construct a hyperboloid of revolution using the asymptote method.

The asymptotes to the curves are elements which are seen as true lengths in elevation.
The hyperbola will get closer to the asymptote as it extends but will never touch it.
The asymptotes will always cross at throat level.

(1) Draw the given plan and set off the heights in elevation.
(2) Since the asymptotes are true lengths in elevation they must be horizontal in plan. Draw the horizontal line AB in plan tangential to the throat circle.
(3) Projecting A and B from plan to the top and bottom of the elevation will find the asymptotes AB.
(4) Pick any number of points on the asymptote in plan, e.g. points 1 to 4.
(5) Project these points onto the asymptote in elevation.
(6) Rotate point 3 for instance, in plan, onto the central axis.
(7) Project the rotated point to elevation and across from point 3 on the asymptote in elevation. This locates a point on the curve.
(8) Repeat for the other points.
(9) The shape is completed by using symmetry.

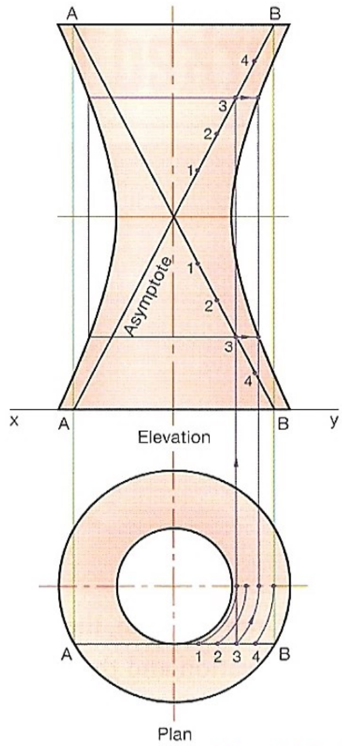


Fig. 18.46

Important Things to Remember about the Hyperboloid of Revolution

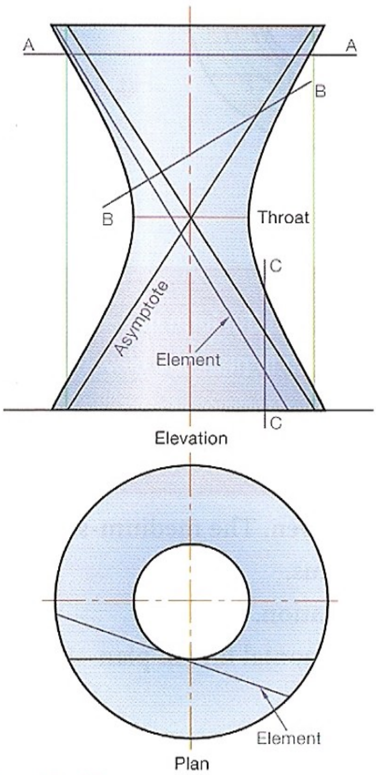


Fig. 18.47

(1) Sections cut perpendicular to the axis will be circles.
(2) A straight line on the surface is an element and will be a tangent to the throat circle in the plan.
(3) The asymptotes are elements which are seen as true lengths in elevation.
(4) The asymptotes cross each other where the axis and the throat meet.

Hyperbolas of revolutions are used in cooling towers and in gear profiles.

Section A–A is a circle
Section B–B is an ellipse
Section C–C is a hyperbola

Worked Examples

Fig. 18.48 shows the outline plan and elevation of a cooling tower. It is in the form of a hyperboloid of revolution.

(i) Draw the given plan and elevation.

(ii) Determine the true shape of the section S–S.

Scale 1:500

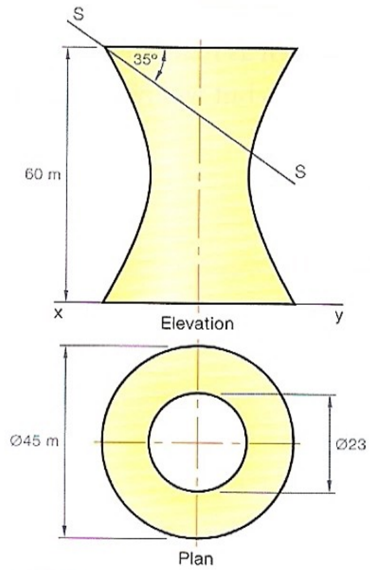


Fig. 18.48

(1) Draw the plan, the axis in elevation and the height in elevation.

(2) Construct the hyperboloid of revolution as described earlier.

Section S–S

(1) Select a number of points on the section line, e.g. a, b and c.

(2) Take horizontal sections through each of these points. These horizontal sections will produce circles in plan. The radii of these sections are r_1, r_2 and r_3.

(3) Project points a, b and c from elevation onto the appropriate circles in plan.

(4) The section can now be drawn by projecting perpendicular to the section line. Widths W_1, W_2 and W_3 are taken from plan. The section is an ellipse.

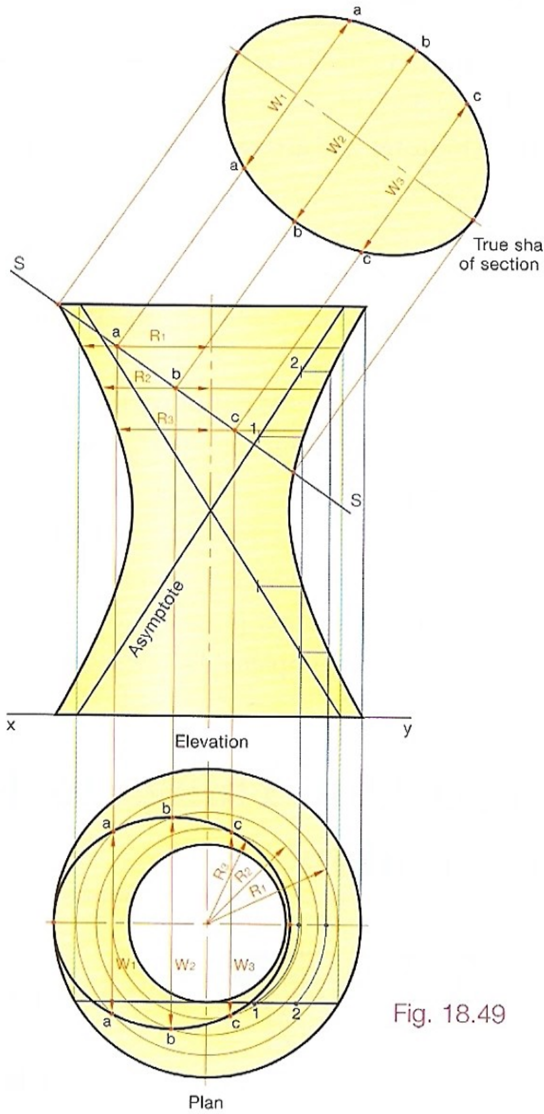


Fig. 18.49

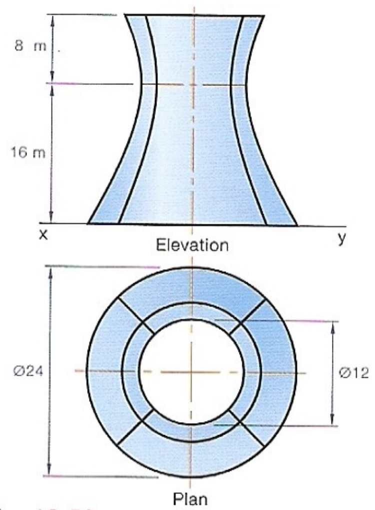


Fig. 18.50

Fig. 18.50 shows the outline plan and elevation of a building. It is in the form of a hyperboloid of revolution. The joint lines on the surface are shown in plan and elevation. Draw the given views.

Scale 1:200

(1) In plan, draw the base circle and the throat circle as given. The medium-sized circle cannot be drawn yet as we do not know its radius.

(2) Draw the xy line, throat line, top line and axis in elevation.

(3) Draw the asymptotes in plan. These are seen as a horizontal line in plan tangential to the throat circle.

(4) Where the asymptote line in plan hits the base circle at P will give the starting points of the asymptotes in elevation on the xy line at P_1.

(5) The asymptotes always cross at the centre of the throat line in elevation. Draw the asymptotes.

(6) Construct the outer curves of the elevation as explained before.

(7) Where the asymptote meets the top surface at 2 is projected down to point 2 on the asymptote in plan. This is a point on the medium-sized circle. Draw the circle.

JOINT LINES

(1) Rotate points 1, 2 and 3 on the asymptote in plan onto the joint line, giving 5, 6 and 7. These points on the joint line, because they are on the same horizontal section, can be projected to elevation as shown.

(2) Points 4 and 8 are on the throat circle and base circle respectively and can be projected to elevation. The right joint line is a symmetrical image of the left.

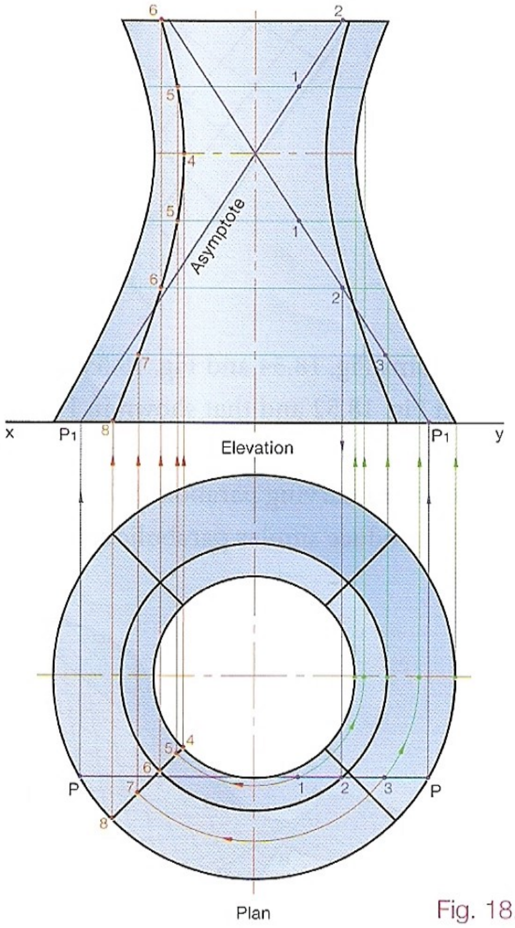


Fig. 18.51

Hyperbolic Paraboloid

A hyperbolic paraboloid surface is obtained by translating a parabola with a downward curvature (ABC) along a parabola with an upward curvature (RST). The vertex of parabola ABC stays in contact with the parabola RST and the parabola hangs vertically at all times.

Horizontal sections produce a double hyperbola while vertical sections produce a portion of the parabola ABC.

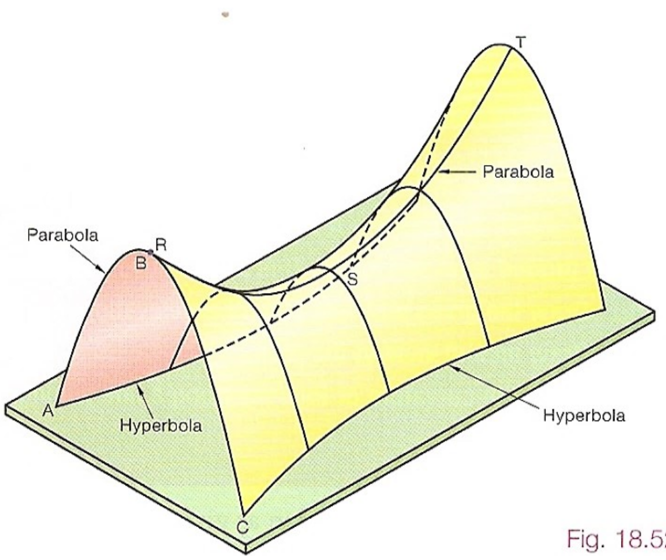


Fig. 18.52

The hyperbolic paraboloid surface can also be generated by straight lines as shown in Fig. 18.53. It is called a **doubly ruled**