

18

Structural Forms

SYLLABUS OUTLINE

Areas to be studied (in an applied context):

- Structural forms, natural and manufactured.
- Singly and doubly ruled surfaces.
- The hyperbolic paraboloid as a ruled surface.
- *The hyperbolic paraboloid as a surface of translation.*
- *Plane directors.*
- The hyperboloid of revolution, projections and sections.
- Sections through singly and doubly ruled surfaces.
- *The geodesic dome of not more than four points of frequency.*

Learning outcomes

Students should be able to:

Higher and Ordinary levels

- Investigate the development of structural forms in a historical context.
- Identify the key structural forms including arches, domes, vaults, frames and surface structures.
- Produce line drawings of the basic structural forms.
- Produce two-dimensional drawings of arches, domes, vaults and surface structures.
- Construct a hyperbolic paraboloid as a ruled surface.
- Determine the true shape of sections through curved surfaces.
- Project views and sections of a hyperboloid of revolution.

Higher level only

- *Relate the key properties of structural forms to their design and construction.*
- *Produce three-dimensional drawings of arches, domes, vaults and surface structures.*
- *Determine plane directors for ruled surfaces, and construct ruled surfaces given plane directors and directrices.*
- *Project views of a hyperbolic paraboloid defined as a surface of translation.*
- *Construct geodesic domes of not more than four points of frequency.*
- *Investigate and represent structural forms as they occur in the environment.*

In this chapter we will be looking at the historical development of some common structural forms including the arch, the dome and the vault. We will then move on to look at some structural forms of special interest, the hyperbolic paraboloid and the hyperboloid of revolution.

Column and Beam

Using columns and beams is the simplest way to make an opening in a wall. The column or post is the vertical member and the beam is the horizontal member. The beam supports the weight (load) above it and its own weight. This weight is then transferred to the columns and from these to the lower structure. This type of construction was used in prehistoric times and is still used in modern day structures.

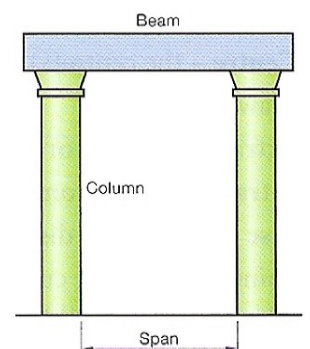


Fig. 18.1

The range of materials available to ancient Romans, Egyptians and Greeks was quite limited (timber and stone) and therefore the spans that could be crossed with beams were relatively short. Stone, in particular, is weak in bending. The longer the beam is, the more likely it is to bend. Long stone beams would be susceptible to failure. Furthermore, since a beam is a single member, it was difficult to obtain in long lengths. Modern day materials such as steel and concrete allow longer spans, particularly when used together.

A bending beam is under compression at the top and under tension at the bottom. Concrete is very strong in compression and is good at the top of the beam. Steel is strong in tension and is good at the bottom of the beam. Steel and concrete together make a strong beam.

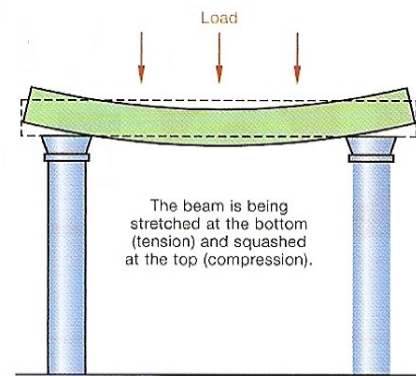
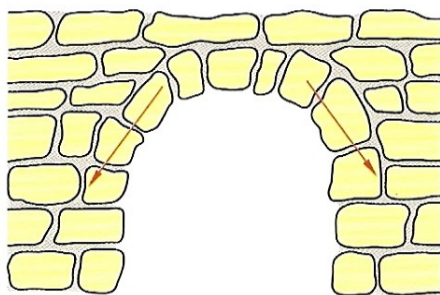


Fig. 18.2

The construction of columns presented less problems in ancient times for two reasons. Firstly, columns are fundamentally members under compression and the main building material, stone, is excellent under compression. Secondly, columns could be made up from small segments stacked on top of each other. The load when applied actually compresses the joints making it stronger.

The Arch



Thrust downwards and outwards

Fig. 18.3

The arch was developed as a means of crossing larger spans than was practical/possible with monolithic stone beams. The arch spans an opening without using any beams at all, just a lot of small stones or bricks. These smaller building elements support each other.

The invention of the arch is credited to the Etruscans, before the Roman Empire was established. The Etruscans used the arch for gates, bridges and drains. When the Roman Empire conquered the Etruscans, they adopted the arch into their architecture and used it widely when building bridges, aqueducts, gates, entrances etc.

The forces exerted by an arch, because of its bend upwards, tend to be both downwards and outwards. The walls to the side of an arch must be of sufficient mass to counteract this diagonal thrust. A single arch therefore will not be stable on two columns unless the columns are heavy enough to buttress against these forces. A series of arches will buttress each other and may be supported on light columns.

Arch Terminology

Centre – the centre(s) from which the curve(s) of the arch is drawn.

Extrado – the top or outer surface of a voussoir.

Intrado – the bottom or inner surface of a voussoir.

Rise – the vertical distance from the spring line to the highest point on the inner curve of the arch

Span – the inner width of the arch.

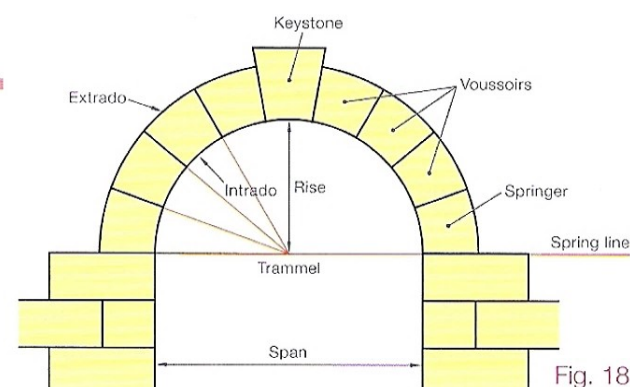


Fig. 18.

Spring Line – the line from which the curve of the arch starts.

Springer – the first voussoir on the left and the right of the arch.

Voussoirs – the individual elements that make up the arch. Usually tapered.

Arch Shapes

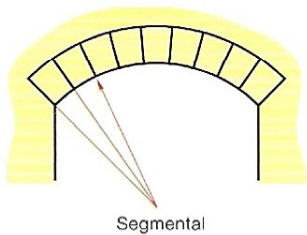


Fig. 18.5

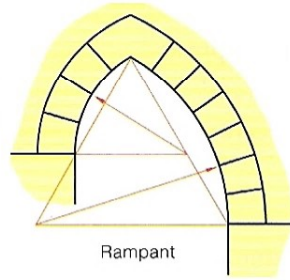


Fig. 18.6

A segmental arch forms a curve which is a segment of a larger arch.

A rampant arch is one that starts at one level and finishes at another.

There are a large variety of arch shapes. Some of these are shown below. For each arch shape the joint lines of the voussoirs are generally normals to the inside curve.

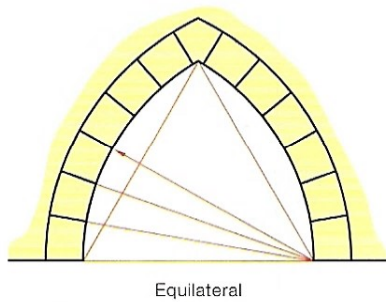


Fig. 18.7a

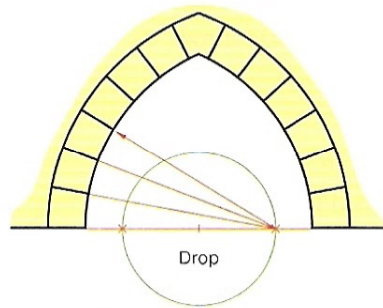


Fig. 18.7b

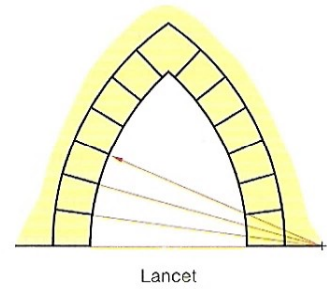


Fig. 18.7c

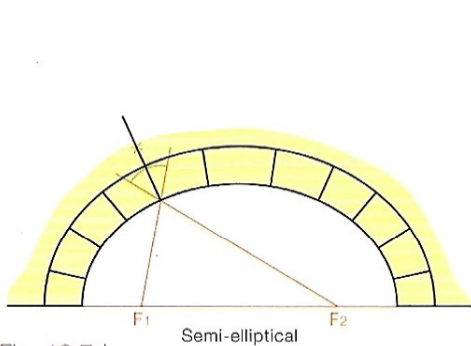


Fig. 18.7d

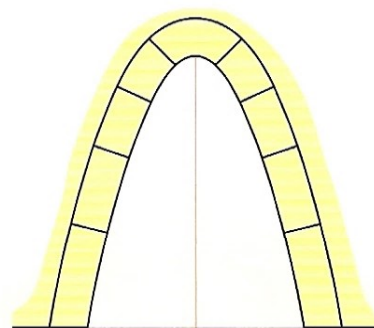


Fig. 18.7e

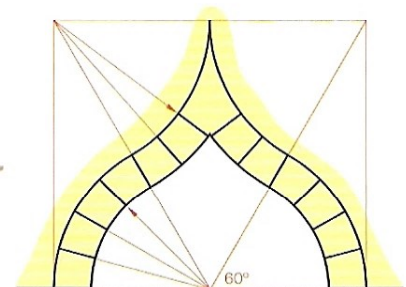


Fig. 18.7f

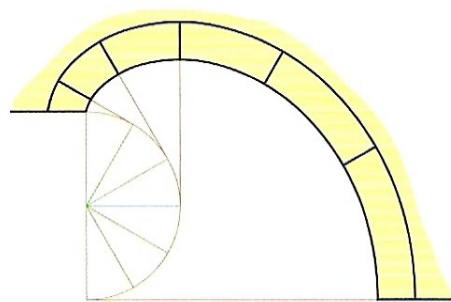


Fig. 18.7g

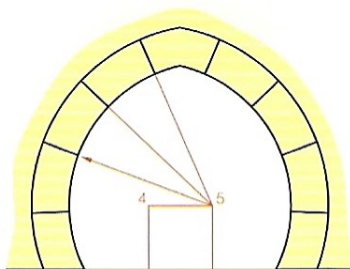


Fig. 18.7h

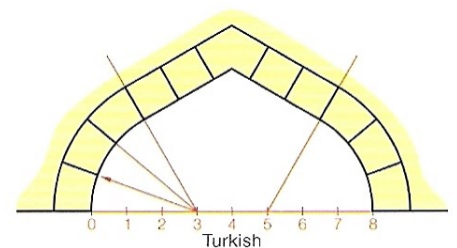


Fig. 18.7i

The Vault

The logical progression from the arch is the vault. When the arch is deepened enough to form a part of a cylinder or barrel we have a vault.

The Egyptians were one of the first civilisations to have a widespread use of the vault. They used the vault in tombs, storage rooms and drains. The Romans later adopted the vault into their architecture and developed it further to form the groined vault.

A groined vault is made up from two barrel vaults of the same size and height joining each other at right angles. It will cover a square area. The lines of intersection are called the groins. For a groined vault the whole roof area is supported on four corner piers. This opened up the floor area and was used to great advantage in the construction of large buildings. Many medieval cathedrals show great examples of both barrel and groined vaults.

The construction of a vault consisted of building an arch at the ends. Between the arches a long tunnel was formed from concrete. Centerings or temporary supports were introduced to support the concrete while the vault was being constructed. Buttresses were often used to give the heavy vaults support.

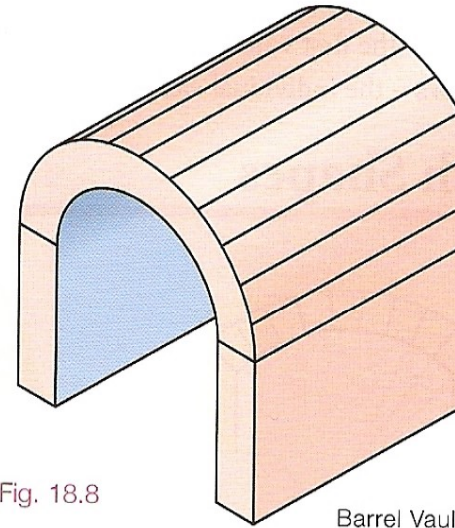


Fig. 18.8

Barrel Vault

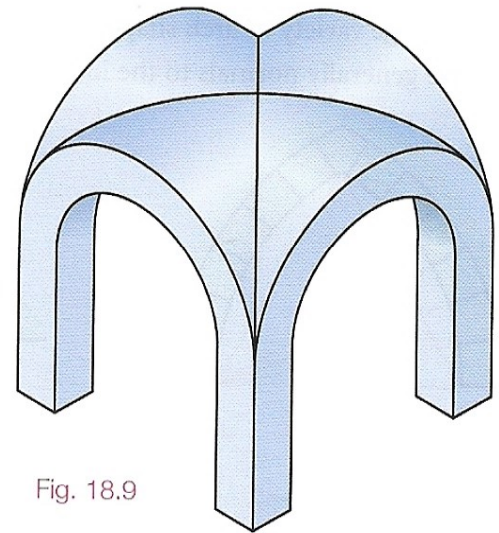


Fig. 18.9

Groined Vault

Domes

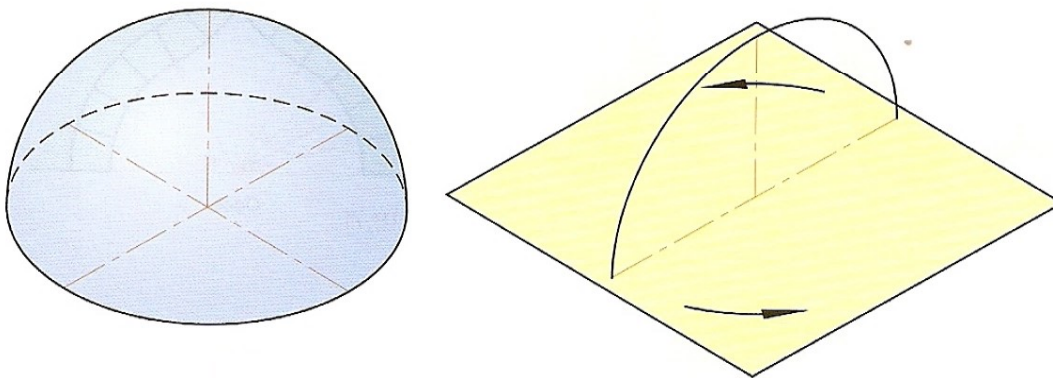


Fig. 18.10

Domes, like the vault, evolved from the arch. A dome is an arch which has been rotated about its centre line. The masonry dome was first constructed around 100 AD by the Romans. The forces exerted by a dome are equal around its perimeter. Because of the limited variety of building materials available at these times (stone and cement), the domes that were constructed were heavy, requiring extensive buttressing.

The availability of steel to architects and builders in the nineteenth century made dome construction less difficult. Steel has a high strength-to-weight ratio compared to stone and it can also be made into continuous forms. For awkward shapes like domes, this is a great advantage.

The geodesic dome, which is a skeletal frame of a spherical dome, was developed by Buckminster Fuller in the 1940s. It is a lightweight construction and generally sits at ground level.

Most domes are created by rotating a semicircle or similar curve about a centre line.

A parabola, hyperbola, semi-ellipse or similar but non-regular curve will produce a dome. However, the curve must be convex as opposed to concave. In other words, it must bulge outwards.

The geodesic dome is a dome created with triangles. It is structurally very stable and can be built to a very large scale. Its construction is not based on rotation of a curve about an axis but rather on the platonic solids. More on the geodesic dome later.

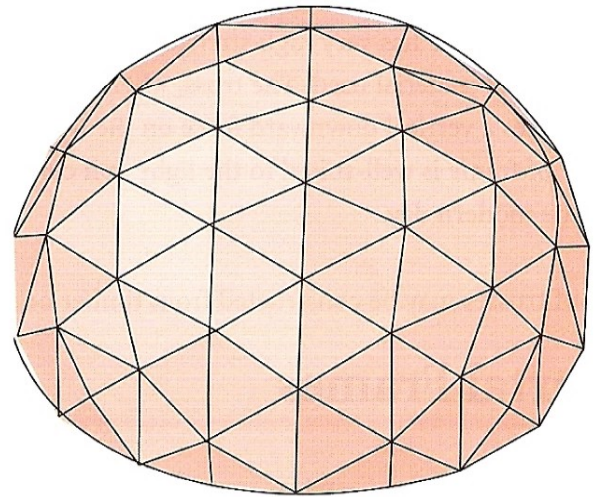


Fig. 18.11

Frames

There are many examples in modern architecture where frames of one type or another are used to speed up and expand the limits of the construction process. The prefabricated frame can be made in ideal conditions, with exact accuracy and to a uniform standard.

Lattice Girder

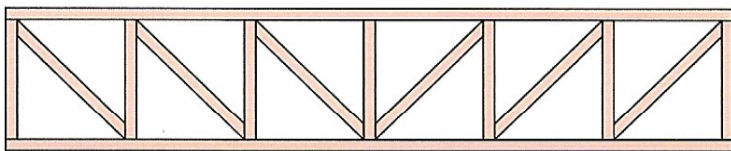


Fig. 18.12

It is only common sense that the wider the span that a beam has to cross, the larger that beam must be in cross-section. As the span widens it will eventually become uneconomical to span across it with solid section beams because of the amount of material used. The lattice girder (Fig. 18.12) provides a strong, light alternative. The lattice girder will be deeper and wider than the solid beam, yet uses less actual material and is stronger if constructed properly.

The lattice girder is broken into triangles. The triangle is the most stable geometric shape and will not be distorted in shape unless one of its sides is lengthened or shortened. Pin-jointing at the ends of each member produces a well-braced, stable frame.

Truss

The triangular truss also bases its strength on triangulation. The larger triangle is often broken into smaller triangles using struts (members under compression) and ties (members under tension). The resulting frame has very high strength in relation to the amount of material used. The truss, when in place, produces a vertical downward force on the walls. This type of frame is well-suited to the light wall construction of the modern day.

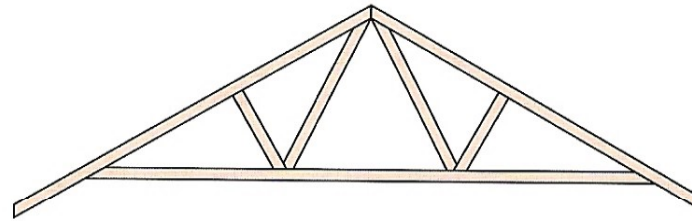
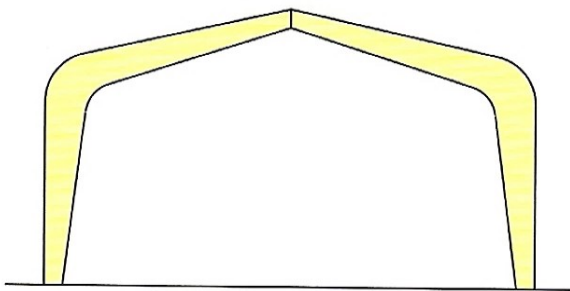


Fig. 18.13

Roof trusses may be constructed from timber or steel and vary hugely in shape and size.

Portal Frames



Portal Frame

Fig. 18.14

Portal frames may be constructed of reinforced concrete, steel and often laminated wood. The frame is thickened at the corners to help transfer the load from the top section to the vertical section. This type of frame is widely used in factory and warehouse construction because it forms wall and roof frame in the same unit. The portal frame is a relatively modern form of construction.

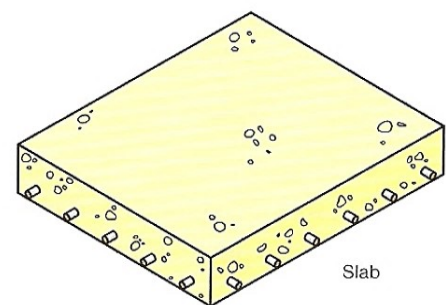
Surface Structures

A surface structure is one whose surface both encloses a space and provides support. The material used is usually reinforced concrete because of its strength and because of the versatility it offers in shape.

Slab

The simplest form of surface structure is the horizontal slab or the vertical panel. A slab combined with columns or vertical panels will quickly create a structure which is self-supporting.

By folding or corrugating thin materials their stiffness can be increased enormously. This property has been applied to the simple slab to produce many varied types of surface structures called shell structures.



Slab

Fig. 18.15

Shell Structures

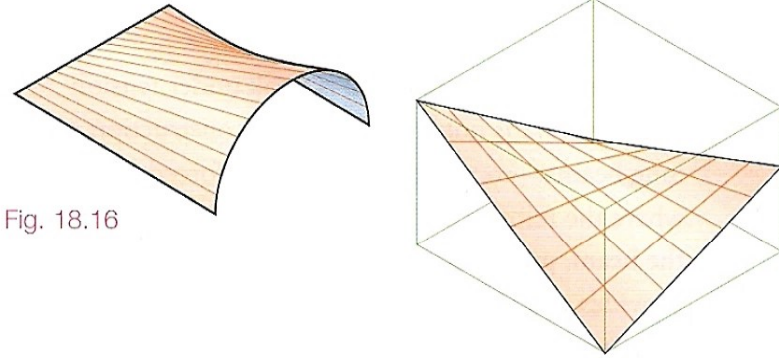


Fig. 18.16

These generate their strength from their shape rather than the thickness of material used. They are generally made from thin concrete with a mesh of steel reinforcement. Shell structures, as a building form, were first used in the twentieth century and were a result of improvements in cement and concrete production. The shapes produced by single shells or combinations of shells can be aesthetically pleasing and produce free-flowing designs. It is these shell structures which form our main topic of interest in this course and they will be examined in detail later on in this chapter.

Structures: Properties, Design and Construction

The Column

A column is a member under compression. The load is vertical downwards and is resisted by an equal upwards force. Columns do not, however, resist lateral thrust because of their height and slenderness. Concrete is an ideal material for building columns because of its high resistance to compression. Steel reinforcing would be used in the column to help prevent it from bulging under heavy load conditions. A thickening of the column at the head and base helps the transfer of loads from the beam above to the floor/foundation below. The column itself is often tapered inwards slightly for the top two-thirds of its height. This idea was developed by the ancient Greek architects and produced a column that appeared straight with parallel sides. If there is an increase in height of a column there must be a corresponding increase in thickness. This ratio, height to thickness, is called the Slenderness Ratio.

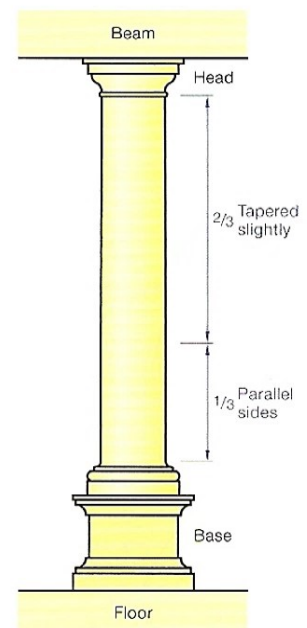


Fig. 18.17

The Beam

A beam is a member under both tension and compression. The loads applied are usually perpendicular to the longitudinal axis. The beam develops internal stresses to resist these applied loads. Fig. 18.18 shows a loaded beam which is deflecting. The bottom of the beam is stretching and is therefore under tension. The top of the beam is shortening and is under compression. There is a neutral axis in the centre and both these forces increase as the top and bottom of the

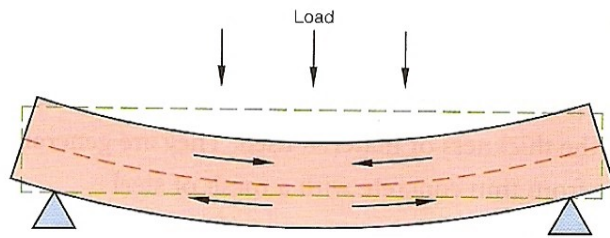


Fig. 18.18

beam are approached. A properly sized beam will bend very little but the load is supported by the beam's resistance to these forces.

Stone as a building material is weak under tension and therefore is not suitable for the construction of long beams – concrete is similar. Reinforced concrete where the reinforcing is placed toward the bottom of the beam, produces a good beam because the steel provides good tensile strength where the concrete is at its weakest.

The Arch

The shape of the arch means that its upper edge (extrado) is longer than its inner edge (intrado). Each of the individual blocks or elements of the arch must therefore be wedge-shaped. They press against their neighbours on either side and are under compression. The load supported by the arch is passed from element to element until the sides of the arch are reached. The resulting downward and outward thrust must be resisted by the walls at the side, buttressed columns or the next arch in a row of arches.

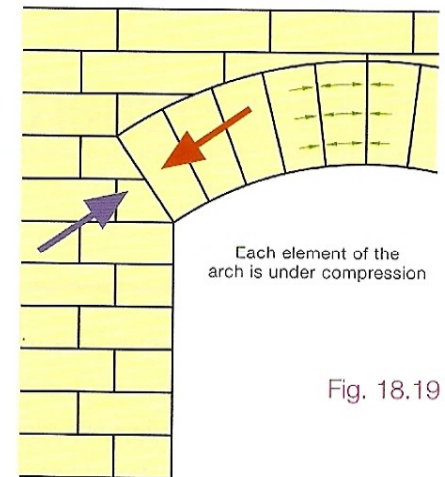
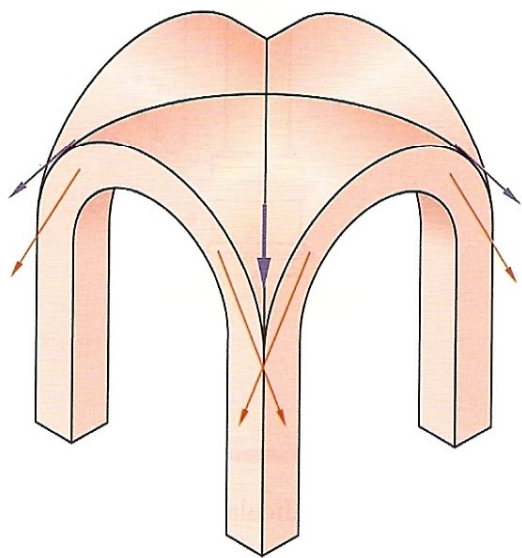


Fig. 18.19

The Vault



Oblique thrust from three different directions at each corner

Fig. 18.20

The barrel vault exerts the same forces on its supports as an arch does and must have heavy buttressing walls at the sides. The groined vault focuses these forces into the corners where heavy buttressed columns are used. A series of these groined vaults together will help support each other and in this case the columns will only need to support vertical forces and can be lightened considerably. Fig. 18.20 shows how the diagonal arch and two side arches direct the load in three different directions at each corner. Groined vaults are usually built to a square, grid, plan, layout and support each other.

The Dome

The traditional masonry dome exerts thrust evenly all the way around its circumference. The supporting walls must be buttressed or be of very large mass to counteract these forces. The geodesic dome is usually of extremely light construction and because of all the triangulation is very stable. The geodesic dome will exert a downward force on its supports.

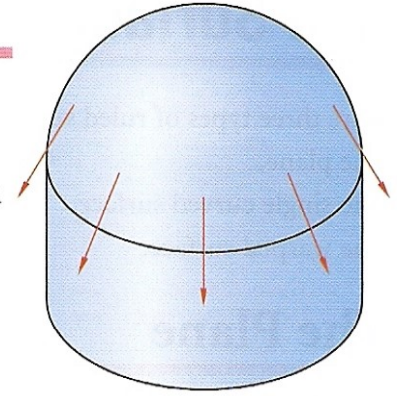


Fig. 18.21

Frames

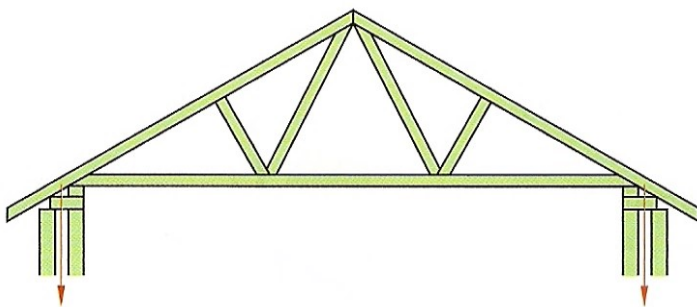


Fig. 18.22

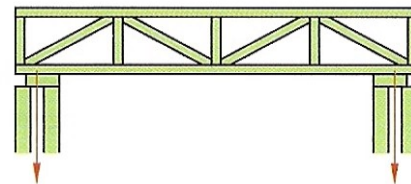


Fig. 18.23

Lattice girders and trussed rafters base their strength on a series of triangles. The individual members may be struts (members under compression) or ties (members under tension). Both the lattice girder and the triangular truss are self-contained units and exert only a vertical load on their supports.

Shell Structures

When you think of shell structures, of whatever type, you think of thin, curved shells of concrete. They get their strength from their shape, not from their thickness. The shell of a bird's egg is both thin and brittle yet can withstand very large, evenly distributed loads. The curved shape helps distribute the load. Many shell structures used in modern architecture have a curved shape and yet can be made up from straight-line elements. This is helpful when constructing a reinforced concrete shell as the reinforcing bars do not need to be bent. It can also be helpful in the making of the framework.

Surface Structures

A surface may be considered to be generated by the motion of a line, the **generatrix**. Surfaces are divided into two groups:

- (1) Those that are generated by a moving straight line are called **Ruled Surfaces**.
- (2) These that are generated by a moving curved line are called **Double Curved Surfaces**.

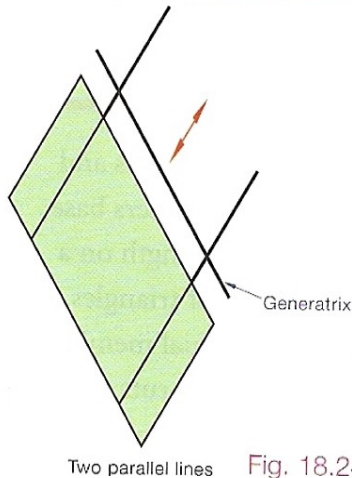
Any position of the generatrix, be it a straight line or a curve, is called an **element** of the curve.

Ruled Surface

There are three types of ruled surface:

- (1) the plane,
- (2) the single curved surface,
- (3) the warped surface.

1. The Plane

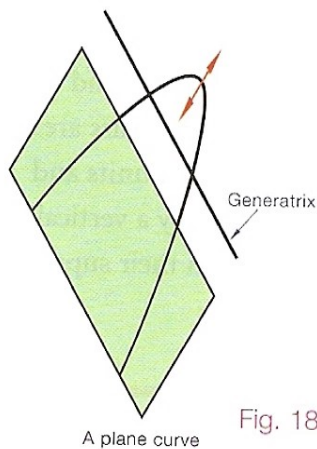


Two parallel lines Fig. 18.24

The plane is generated by a straight line moving so as to touch two other parallel, straight lines, Fig. 18.24.

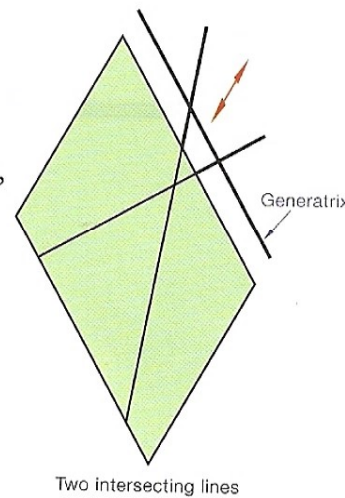
It can also be seen as a straight line moving so as to touch two intersecting straight lines, Fig. 18.25.

Alternatively, it can be seen as a straight line moving so as to touch a plane figure at two places at all times, Fig. 18.26.



A plane curve Fig. 18.26

By definition a plane is a surface such that when any two points are taken on it, the straight line joining them will lie completely on the surface.



Two intersecting lines Fig. 18.25

2. The Single Curved Surface

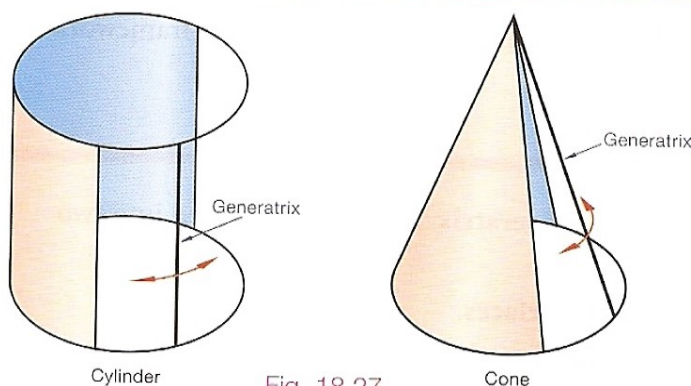


Fig. 18.27

This is a developable ruled surface. The surface unrolled to lie on a plane. The straight line elements are parallel or intersecting. Examples of single curved surfaces are the cylinder and the cone. This category would also include oblique cylinders and cones and surfaces generated by moving the generatrix around elliptical curves as well as circles.

3. Warped Surface

A warped surface is a ruled surface that is not developable. No two consecutive elements are parallel or intersecting. No two adjacent positions of the generatrix lie in the same plane. There is a huge variety of warped surfaces. Common examples are the conoid, cylindroid, hyperboloid of revolution and the hyperbolic paraboloid.

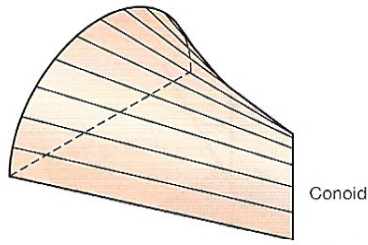


Fig. 18.28

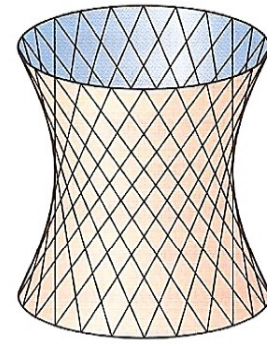


Fig. 18.30

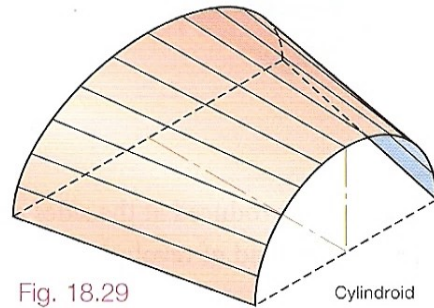
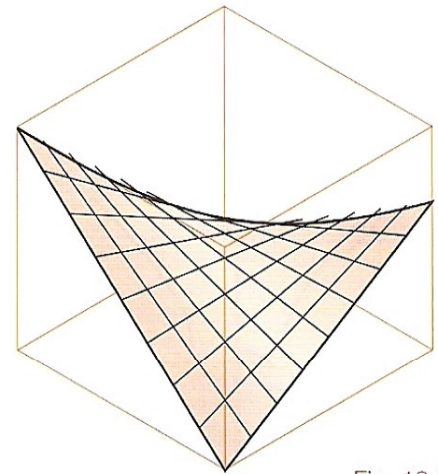


Fig. 18.29

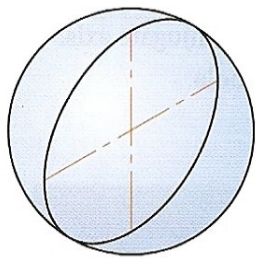
Cylindroid



Hyperbolic Paraboloid

Fig. 18.31

Double-curved Surfaces



Sphere

Fig. 18.32

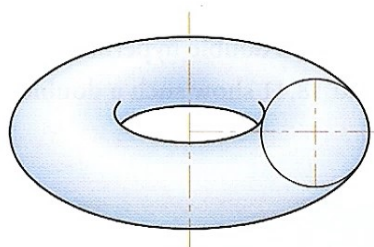
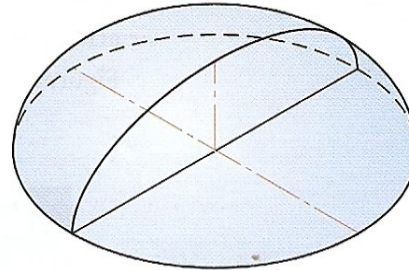


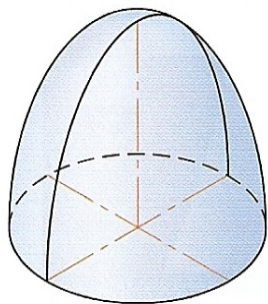
Fig. 18.33

Torus



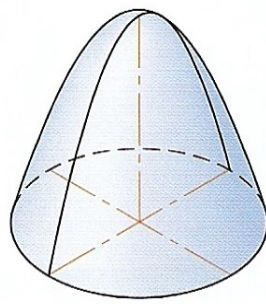
Oblate Ellipsoid
(Semi-ellipse)

Fig. 18.34

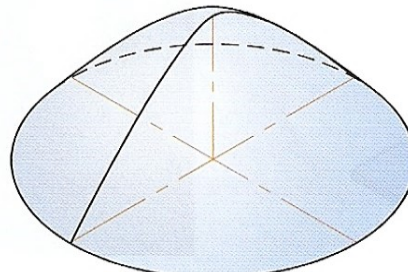


Prolate Ellipsoid
(Semi-ellipse)

Fig. 18.35



Paraboloid
Fig. 18.36



Hyperboloid

Fig. 18.37

Double-curved surfaces are generated by a curved line moving according to a certain law. The most common double-curved surfaces are formed by revolving a curve about an axis in the same plane. Examples of these would be the sphere, torus, oblate ellipsoid, prolate ellipsoid, paraboloid and hyperboloid.