Terms Used in Gearing

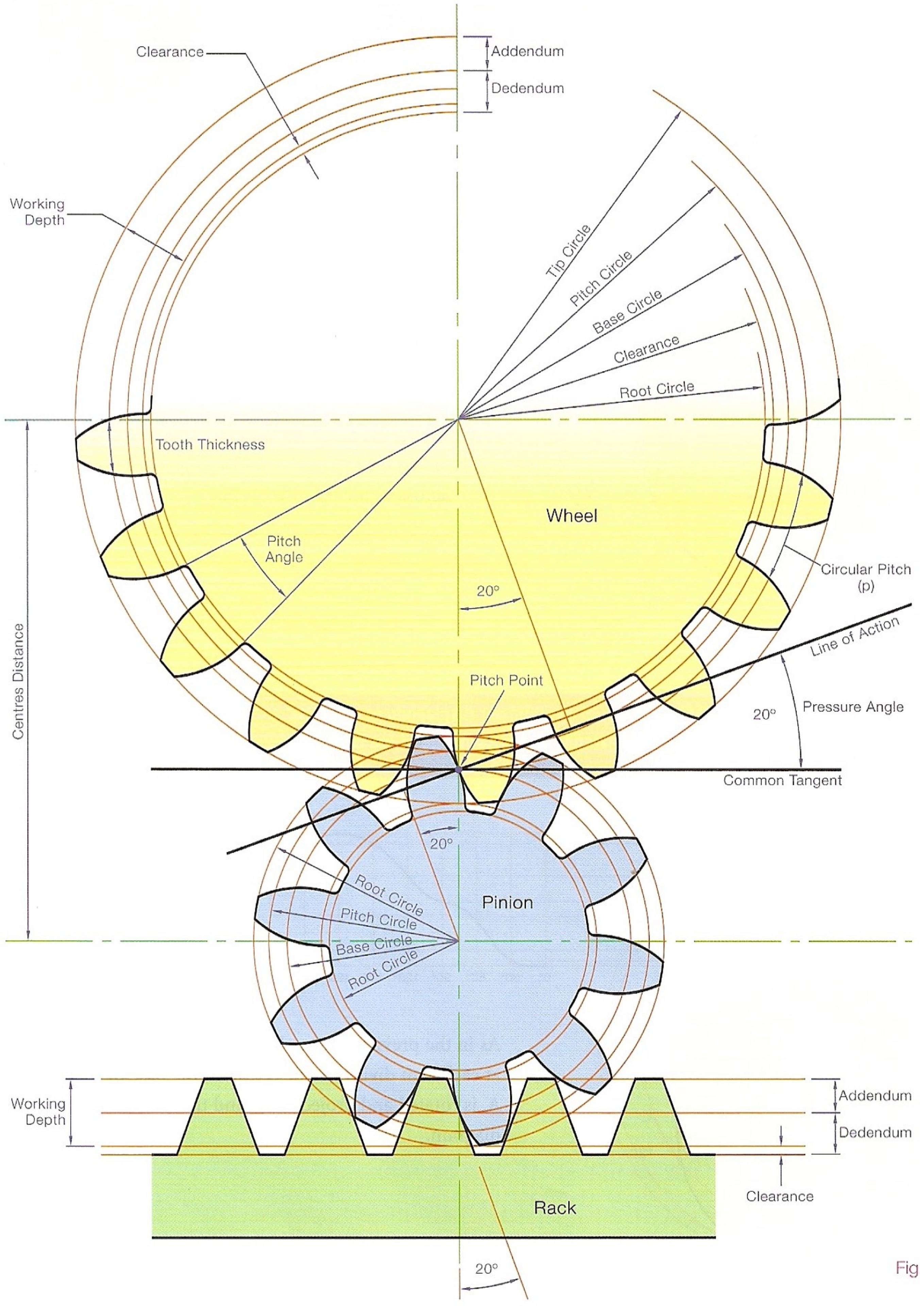

Fig. 17.134

Addendum (a)	The part of the tooth that extends outside the pitch circle or pitchline. The addendum will always equal the module. **a = m**
Base Circle	An imaginary circle from which the tooth shape is generated. The base circle diameter = the pitch circle diameter × cos (pressure angle) BCD = PCD × cos (pressure angle)
Circular Pitch (p)	Circular pitch is the distance from a point on one tooth to the corresponding point on the next tooth, measured round the pitch circle. **p = πm**
Circular Tooth Thickness	The thickness of a tooth measured along the pitch circle. **Circular tooth thickness** $= \frac{p}{2} = \frac{\pi m}{s}$
Clearance (c)	Clearance equals one quarter of the addendum. The clearance is the space underneath the tooth when it is in mesh. **c = d – a = 0.25a = 0.25 m**
Dedendum (d)	The part of the tooth which is inside the pitch circle or pitch line. The dedendum equals 1.25 × addendum. **d = 1.25 × a**
Line of Action	Contact between the teeth of meshing gears takes place along a line tangential to the two base circles. This line passes through the pitch point.
Module (m)	The module is the pitch circle diameter divided by the number of teeth. $m = \frac{PCD}{t} = \frac{\text{Pitch circle diameter}}{\text{No. of teeth}}$ For example, a gear having a PCD of 200 and 20 teeth will have a module of 10.
PCD	Pitch circle diameter.
Pinion	When two gears are in mesh the smaller gear is called the pinion.
Pitch Angle	360° divided by the number of teeth.
Pitch Circle (PC)	This is the circle representing the original cylinder which transmitted motion by friction.
Pitch Point	When two gears are in mesh their pitch circles will be tangential to each other. The pitch point is the point of contact between these two circles.
Pressure Angle	The angle between the line of action and the common tangent to the pitch circles at the pitch point. The pressure angle is normally **20°** but may be **14.5°**.
Tip Circle	A circle through the tips of the teeth.
Wheel	When two gears are in mesh the larger one is called the wheel.
Whole Depth	This is the depth of a tooth from tip to root. The whole depth equals the addendum and dedendum.
Working Depth	The whole depth minus the clearance.

Gear Basics

We will start by considering the theoretically perfect gears – two toothless disks. These gears touch at a single point. The rotation of one gear is perfectly transmitted to the other. There is no friction between the gears and there is no friction or wear on the bearings. Unfortunately, if these gears are not held tightly together they will slip. Furthermore, when they are held tightly together we will have friction and wear on the gears themselves and on the bearings and we will have a considerable loss of power.

To overcome this difficulty we cut teeth into the disks, so that they will engage each other without slipping and without unduly increasing the friction. **The diameters of these perfect, toothless, gears is the Pitch Circle diameters of the gears.**

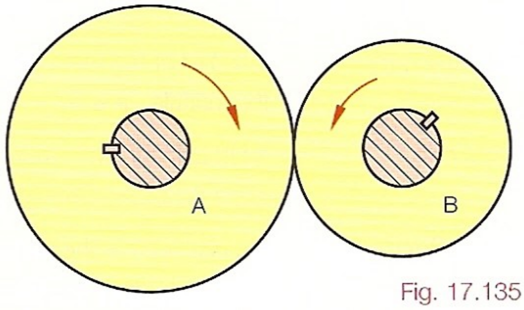

Fig. 17.135

Gear teeth

The aim when designing teeth shape is that the faces of the teeth will roll across each other, minimising the sliding friction. There are two types of curves commonly used, epicycloidal (the curve generated by tracing a point on a circle as it rolls around another circle) and involute (the curve generated by unwinding a line from a circle).

Gear ratio

The relative speed of rotation of the two disks is proportional to their radii. Since the circumferences of the circles are in contact a point on the circumference of disk A will move the same distance as a point on the circumference of disk B as they both rotate.

Circumference large disk A $2\pi R$: Circumference of smaller disk B $2\pi r$

$$\cancel{2\pi}R : \cancel{2\pi}r$$

$$R : r$$

The ratio of the relational speeds is called the **gear ratio**. If the wheel has a PCD of 100 mm and the pinion has a PCD of 50 mm then the gear ratio will be 2:1. The pinion rotates twice as fast as the wheel. This, of course, will only apply if both **gears have the same pressure angle and module**.

Gear tooth design

As mentioned previously the ideal for all gearing is to have only rolling contact between the tooth surfaces of mating gear teeth, thus reducing both wear and friction. When two teeth interact as a pair of gears rotate, the mating curves must satisfy certain conditions to obtain this rolling action. Given almost any reasonable curve for one tooth, a mating tooth can be derived that will give this rolling action. Such a pair of curves are said to be **conjugate**. It is especially neat if the two conjugate curves are based on the same construction. Involute gears satisfy this requirement.

Involute Gears

A piece of string is wrapped tightly around a disk. Unwind the free end, keeping the string tight at all times. The free end as it unravels will trace out an involute curve, Fig. 17.136. By using the same construction for another disk, another involute is formed which is conjugate to the first. Use these two curves, or parts of them, as the sides of gear teeth and the teeth will roll together as they mesh. Modern machinery uses involute gearing predominantly.

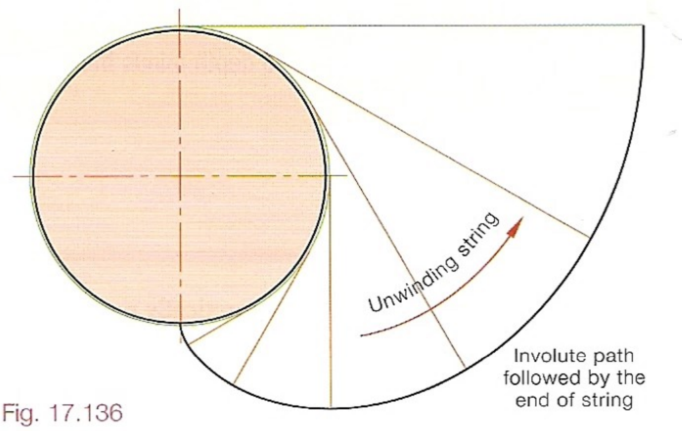

Fig. 17.136

Epicycloidal Gears

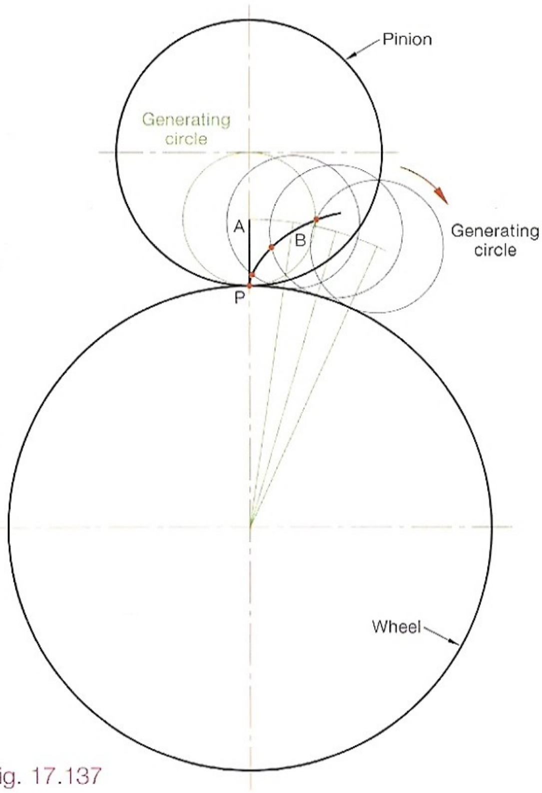

Fig. 17.137

Cycloidal gearing requires two different curves to obtain conjugate action. Fig. 17.137 shows two disks, the wheel and the pinion. A third disk is introduced which is used to generate the tooth profile. If we plot the path of point P on the generating circle as it rolls around the outside of the wheel circle it generates curve B, which is an epicycloid. Using the same generating circle to roll on the inside of the pinion circle a conjugate curve is formed which is a hypocycloid. When the generating circle has a diameter equal to the radius of the pinion circle, the hypocycloid formed is a straight line, A, as shown. The addendum of the wheel is an epicycloid curve B and the dedendum of the pinion is a hypocycloid, straight line A in this example.

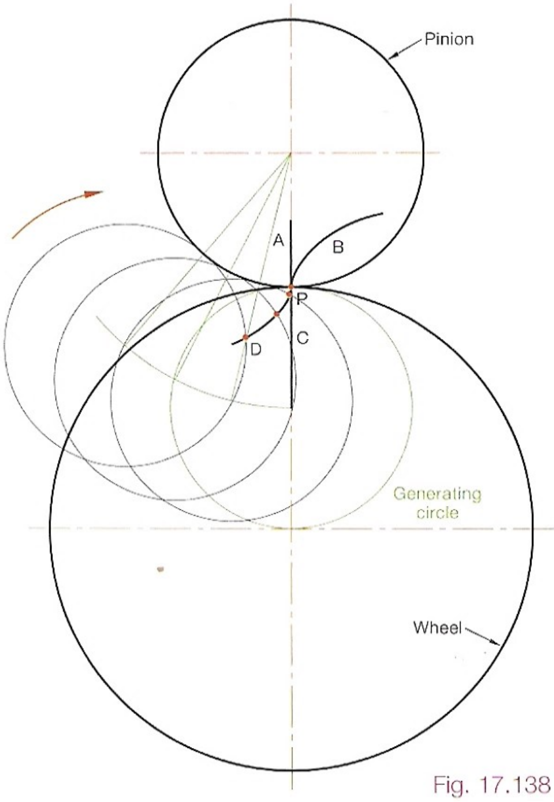

Fig. 17.138

In the same way a generating circle in the wheel generates the flank C of the tooth on the lower gear and the addendum D of the pinion, Fig. 17.138. Between them the two generating circles have generated the tooth shape for each of the two gears. Of particular interest here is that if the sizes of the generating circles are chosen carefully the dedendum of the teeth will be radial straight lines. Fig. 17.139 shows the proof that a hypocycloid generated by a circle rolling inside a circle of twice its radius will be a straight line.

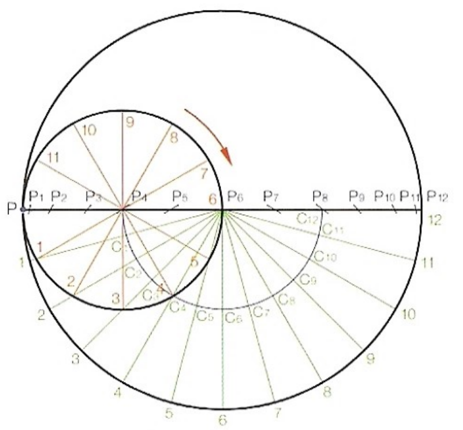

Fig. 17.139

Law of Gearing

Gears are arranged to have sliding contact between pairs of surfaces formed by the teeth of the gears, the rotation of the gears bringing successive pairs into contact. The teeth surfaces in sliding contact have a common tangent and the pressure being exerted between these two teeth will be normal to this tangent. This normal forms the line of action for the gears shown in Fig. 17.140. The line of action and the common tangent between the pitch circles form an angle, the pressure angle. For involute gears this pressure angle is a constant and is usually 20° or 14.5°. For cycloidal gear teeth the pressure angle is variable, becoming zero for contact at the pitch point.

In order that gear motion is smooth, quiet and free from vibration, the Law of Gearing must be satisfied.

Law of Gearing
The normal to the common tangent between two gear teeth surfaces must pass through the pitch point of the gears.

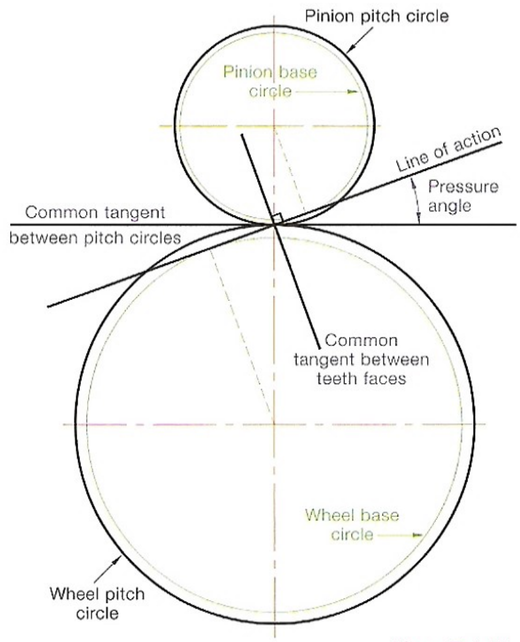

Fig. 17.140

HIGHER LEVEL

Some Things to Note about the Size of the Generating Circle

We have already noted that by using a pair of generating circles having radii equal to half that of the pitch circles, it produces the neat result of generating **radial dedenda** for each gear tooth. If the generating circle is **smaller** than half the radius of the pitch circle, the roots of the teeth are wider and stronger but not radial. A generating circle **larger** than half the radius of the pitch circle produces undercut, necked teeth which may be weak.

The teeth of a cycloidal rack are cycloids generated by the rolling generating circle. They are not straight, like they are in an involute rack, and their shape depends on the radius of the generating circle.

HIGHER LEVEL

Worked Examples

Given a pitch circle diameter of 160 mm and a module of 10 construct the spur gear. Show teeth on half the gear and use conventions for the other half. Teeth to be constructed by the involute method. Pressure angle 20°.

(1) Draw the pitch circle, tip circle and root circle. We are given the pitch circle diameter of 160 mm or radius 80 mm. The addendum equals the module. Therefore the addendum equals 10 mm. The tip circle radius will be equal to the radius of the pitch circle plus the addendum, 80 mm + 10 mm = 90 mm. The root circle radius will be equal to the radius of the pitch circle minus the dedendum. The dedendum will be 12.5 mm, therefore the root circle radius will be 67.5 mm (dedendum = 1.25 module).

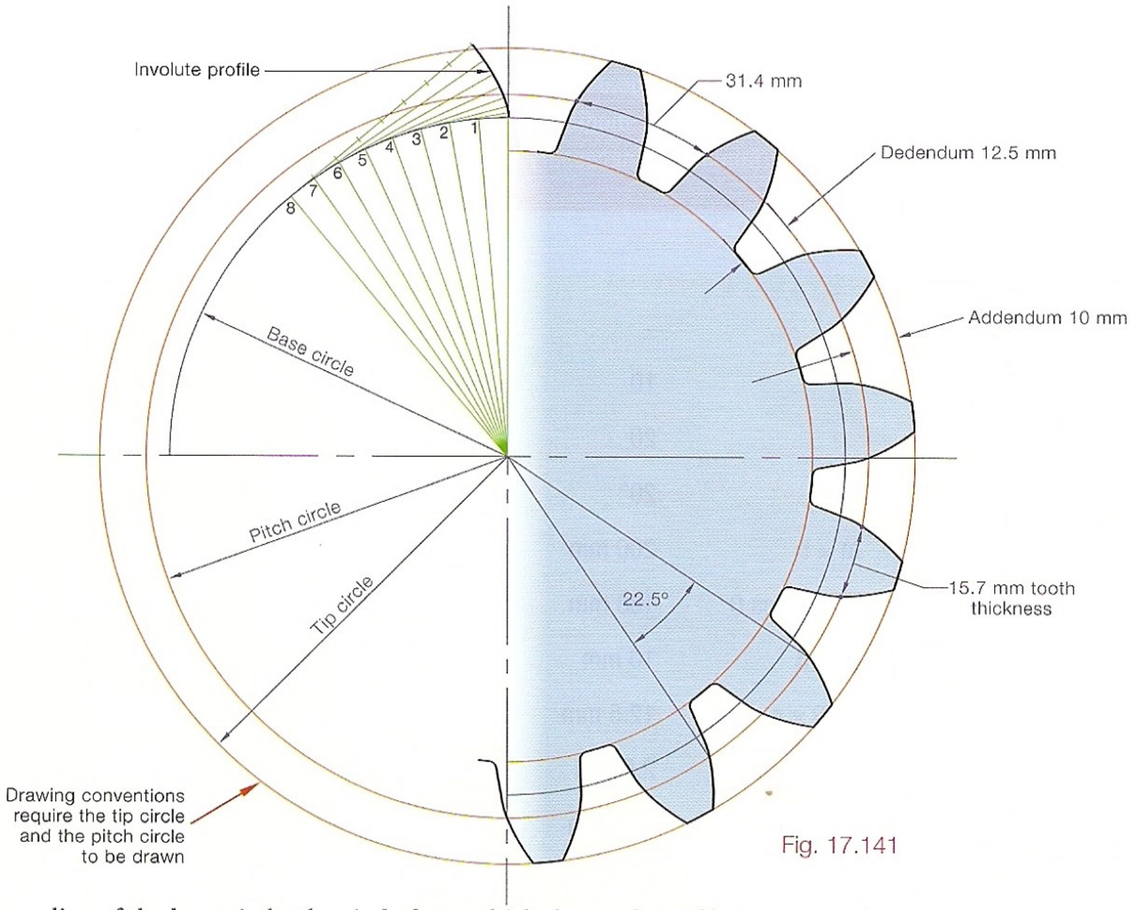

Fig. 17.141

(2) Calculate the radius of the base circle, the circle from which the tooth profile is generated.

Base circle diameter = Pitch circle diameter × cos (pressure angle)

BCD = 160 mm × cos 20°

BCD = 150.4 mm

Draw the base circle.

(3) Calculate the number of teeth.

PCD = module (m) × number of teeth (t)

$$\Rightarrow \frac{PCD}{m} = t$$

$$\frac{160}{10} = 16 \text{ teeth}$$

(4) Calculate the circular pitch, the distance from one point on a tooth to a similar point on the next.

$$p = \pi m$$

$$p = 31.4 \text{ mm}$$

The tooth thickness measured on the pitch circle equals half of this, 15.7 mm.

The angular pitch equals $\frac{360^\circ}{t} = \frac{360^\circ}{16} = 22.5^\circ$

(5) Set out the teeth spacing on the pitch circle either by angular measurement or by measurement along the circumference.

(6) Generate one involute from the **base circle** which is used to draw all the teeth profiles. The construction of an involute has been covered earlier and is shown on Fig. 17.141. The involute is unwound until the tip circle is rea

(7) The portion of the dedendum inside the base circle usually radiates toward the centre or can curve slightly.

(8) For the other half only the pitch circle and the tip circle are drawn.

Draw two involute spur gears showing the gears in mesh. Show five teeth on each gear. The gear ratio is 5:4.
Driver gear details: module 10, 20 teeth, pressure angle 20°.
Tabulate all necessary data for the two gears.

Driver Gear			**Driven Gear**		
Module (m)		10	Module (m)		10
No. of teeth (t)		20	No. of teeth (t)	5:4 = 20:16	16
Pressure angle (θ)		20°	Pressure angle (θ)		20°
Pitch circle diameter	m × t	200 mm	Pitch circle diameter	m × t	160 mm
Base circle diameter	PCD × cos θ	188 mm	Base circle diameter	PCD × cos θ	150.4 mm
Addendum (a)	a = m	10 mm	Addendum (a)	a = m	10 mm
Dedendum (d)	1.25 × a	12.5 mm	Dedendum (d)	1.25 × a	12.5 mm
Clearance	0.25 × m	2.5 mm	Clearance	0.25 × m	2.5 mm
Tip circle diameter	PCD + a + a	220 mm	Tip circle diameter	PCD + a + a	180 mm
Root circle diameter	PCD – d – d	175 mm	Root circle diameter	PCD – d – d	135 mm
Circular pitch (p)	π × m	31.4 mm	Circular pitch (p)	π × m	31.4 mm
Tooth thickness	$\frac{p}{2}$	15.7 mm	Tooth thickness	$\frac{p}{2}$	15.7 mm
Pitch angle	$\frac{360^\circ}{t}$	18°	Pitch angle	$\frac{360^\circ}{t}$	22.5°

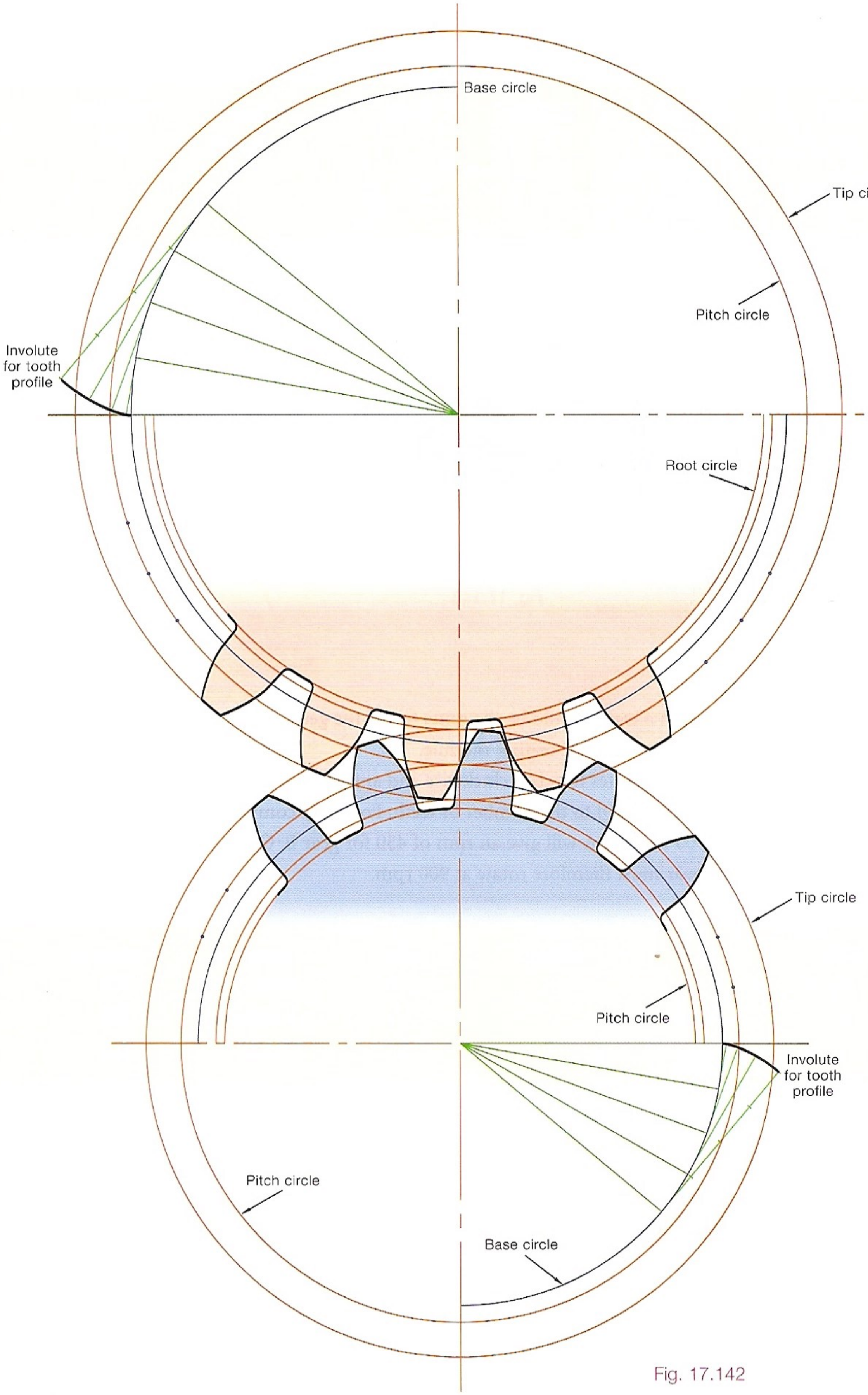
Base circle
Tip circle
Pitch circle
Involute for tooth profile
Root circle
Tip circle
Pitch circle
Involute for tooth profile
Pitch circle
Base circle

Fig. 17.142

HIGHER LEVEL

Fig. 17.143 shows a spur gear train. Draw the following table. Complete it by inserting the missing gear train information.

Gear	Teeth	Module	PCD	Rotation	Speed rpm
A	36	6		Clockwise	300
B	24				
C	12				

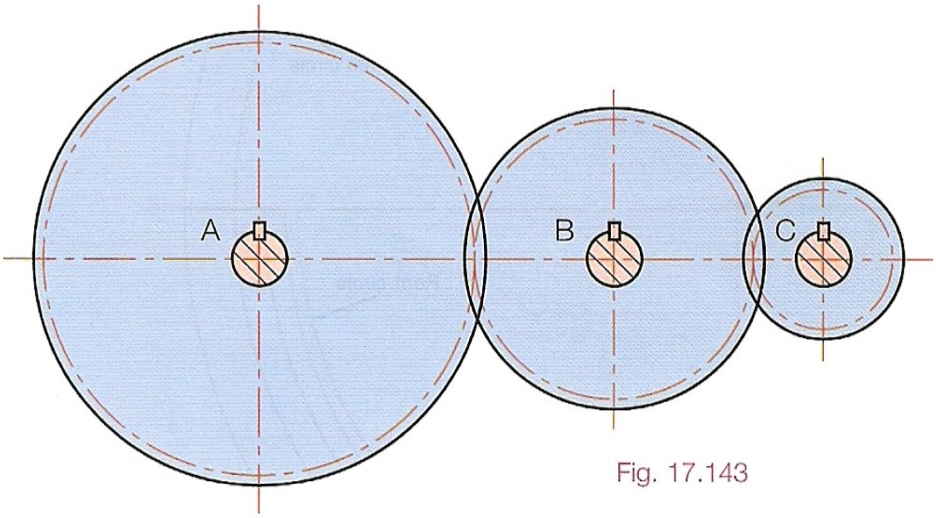

Fig. 17.143

Solution

(1) The module for meshing gears in a train should be the same for all the gears or they will vibrate and wear badly.

(2) The PCD, pitch circle diameter = number of teeth × module.

(3) The rotation of gears in a train alternates between clockwise and anti-clockwise.

(4) The speed of the rotating gears is related to the number of teeth. For every complete turn of gear A, gear B must rotate 1.5 times. An rpm of 300 for gear A will give an rpm of 450 for gear B. Gear C rotates twice for every compl turn of gear B. The smallest gear must therefore rotate at 900 rpm.

Gear	Teeth	Module	PCD	Rotation	Speed rpm
A	36	6	216	Clockwise	300
B	24	6	144	Anti-clockwise	450
C	12	6	72	Clockwise	900

An involute gear wheel with 24 teeth, 20° pressure angle and module 10 is in mesh with a rack. Draw full-size, the gear and rack in mesh, showing five teeth on the gear and five teeth on the rack. Tabulate all relevant information and calculations.

Gear Wheel			**Rack**	
Module (m)		10	Module	10
No. of teeth (t)		24	Pressure angle	20°
Pressure angle (θ)		20°	Addendum	10 mm
Pitch circle diameter	m × t	240 mm	Dedendum	12.5 mm
Base circle diameter	PCD × cos θ	225.5 mm	Clearance	2.5 mm
Addendum (a)	a = m	10 mm	Pitch	31.4 mm
Dedendum (d)	1.25 × a	12.5 mm	Tooth thickness	15.7 mm
Clearance	d × a	2.5 mm		
Tip circle diameter	PCD + 2a	260 mm		
Root circle diameter	PCD – 2d	215 mm		
Circular pitch (p)	π × m	31.4 mm		
Tooth thickness	$\frac{p}{2}$	15.7 mm		
Pitch angle	$\frac{360°}{t}$	22.5°		

(1) Draw the pitch circle, tip circle and root circle.
(2) The base circle can be found by calculation or by finding the circle that is tangential to the line of action.
(3) Draw the pitch line, tip line and root line for the rack. The pitch line and pitch circle are tangential at point P.
(4) Construct the involute tooth profile from the base circle and using tracing paper reproduce this curve to pass through point P.

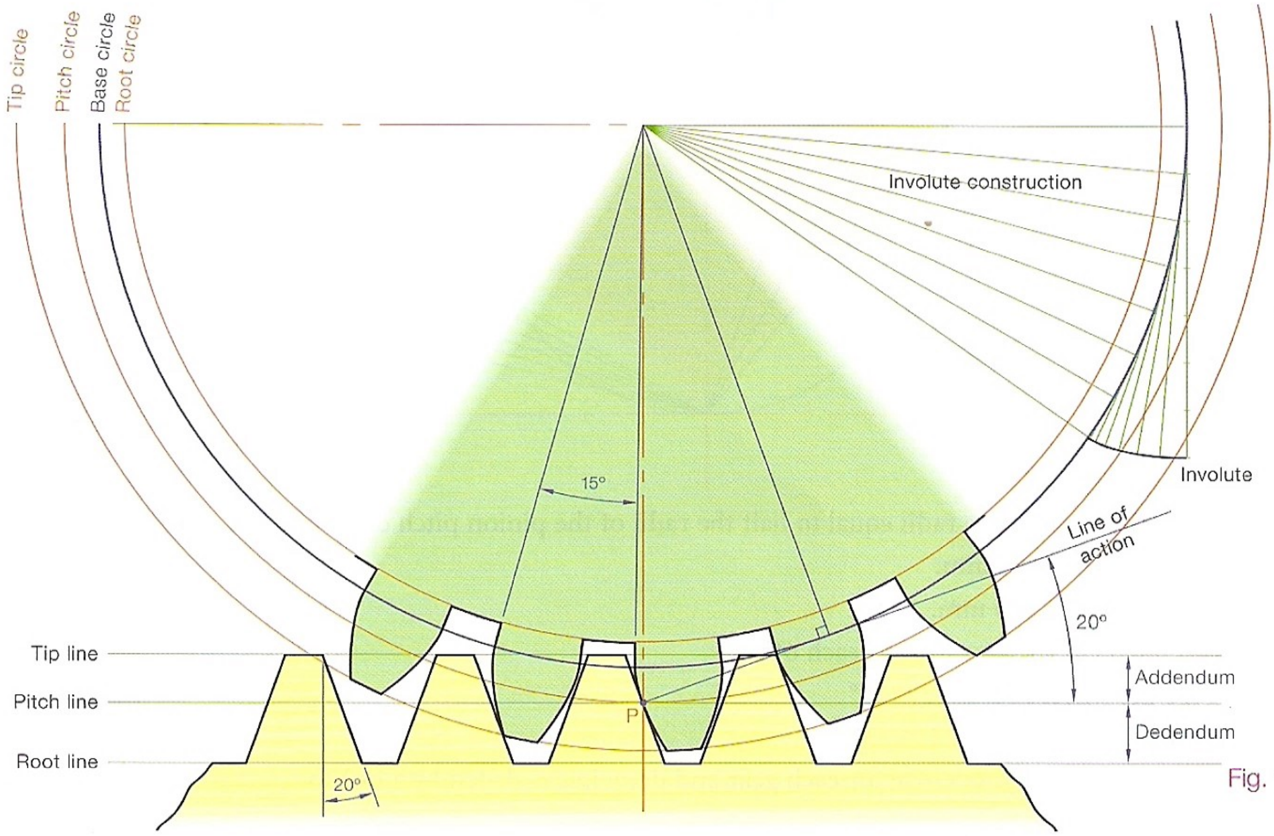

Fig. 17.144

HIGHER LEVEL

(5) The circular pitch, measured along the pitch circle, is 31.4 mm or 15°. Construct five teeth using these spacings.

(6) The rack should be considered to be a gear of infinite radius. The tooth thickness is measured along the pitch line and the tooth angle is 20° as shown in Fig. 17.144.

A cycloidal gear wheel with 18 teeth and a module of 10 is in mesh with a cycloidal pinion gear with 12 teeth and module of 10. Draw full-size, the gears in mesh showing five teeth on each gear. The generating circles used are to produce radial dedenda on each gear.

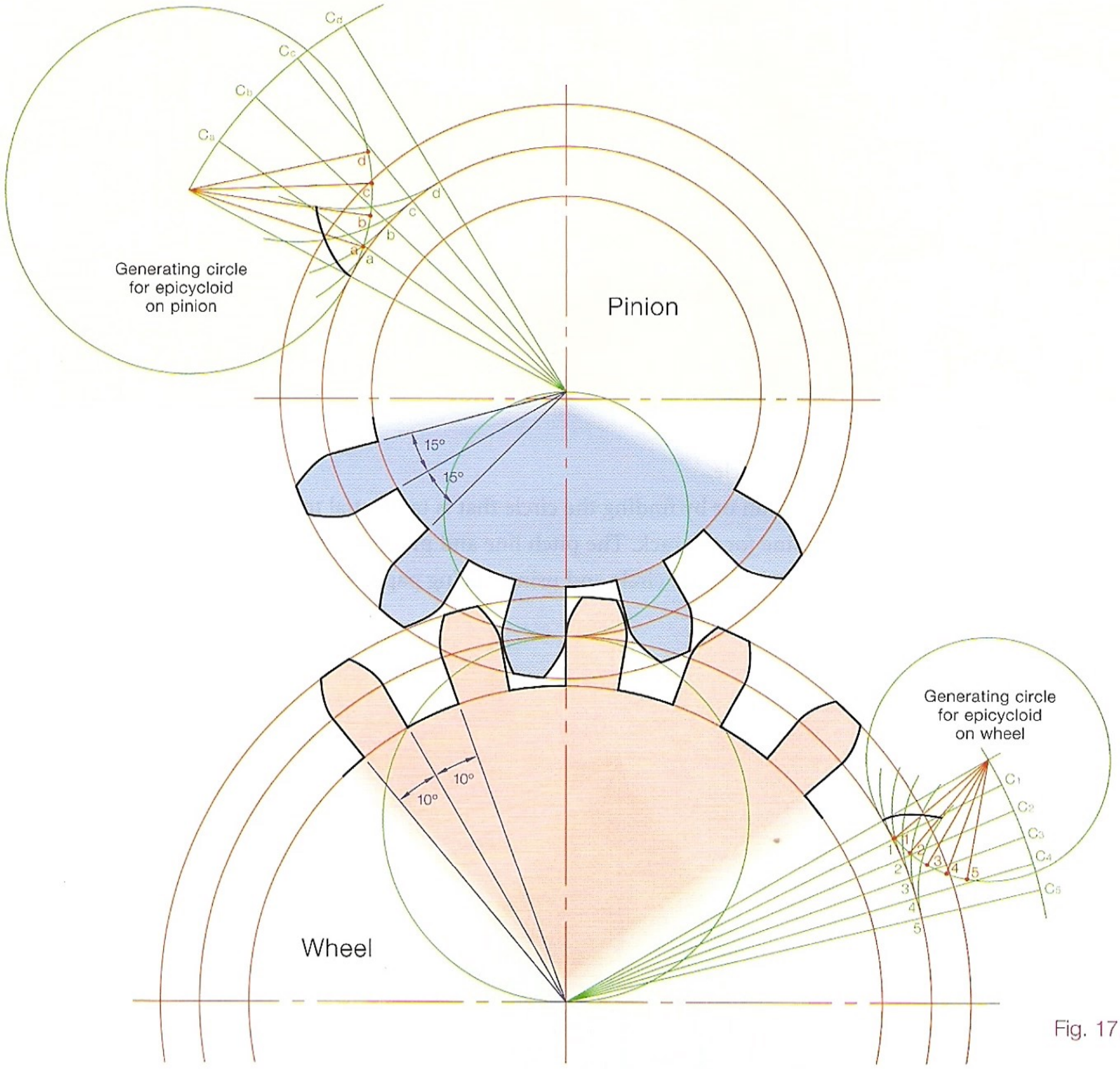

Fig. 17.145

(1) The generating circles will have radii equal to half the radii of the pinion pitch circle and the wheel pitch circle.

(2) Wheel PCD = 18 × 10 = 180 mm.
Pinion PCD = 12 × 10 = 120 mm.

(3) Draw the two pitch circles tangential to each other.

(4) Pitch angle for the wheel 360° ÷ 18 = 20°
Pitch angle for the pinion 360° ÷ 12 = 30°

(5) Draw the tip circle and the root circle for each gear and draw in the dedendum line for each tooth based on the pitch angle and the fact that they will be radial lines.

HIGHER LEVEL

(6) Use the generating circle rolling along the pitch circle to produce the epicycloidal curve for the gear tooth.
(7) Use tracing paper to duplicate the curves.

A cycloidal pinion with 20 teeth and a module of 10 is in mesh with a rack. Draw full-size, the rack and pinion in mesh showing five teeth on the pinion and six teeth on the rack. The pinion is to have radial dedenda.

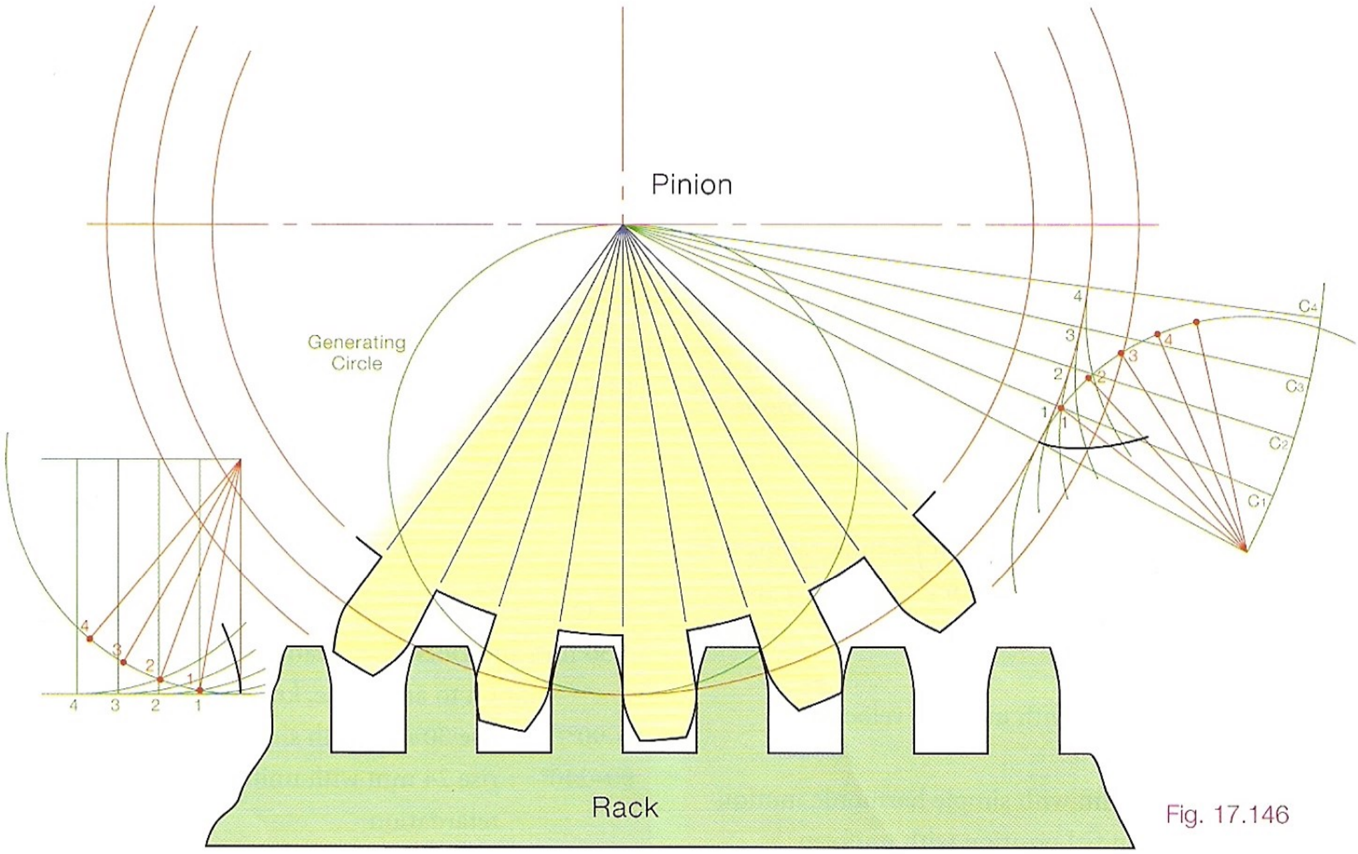

Fig. 17.146

Pinion Details

Module (m)		10
No. of teeth (t)		20
Pressure angle (θ)		20°
Pitch circle diameter	m × t	200 mm
Base circle diameter	PCD × cos θ	225.5 mm
Addendum (a)	a = m	10 mm
Dedendum (d)	1.25 × a	12.5 mm
Clearance	d × a	2.5 mm
Tip circle diameter	PCD + 2a	220 mm
Root circle diameter	PCD – 2d	175 mm
Circular pitch (p)	π × m	31.4 mm
Tooth thickness	$\frac{p}{2}$	15.7 mm
Pitch angle	$\frac{360°}{t}$	18°

Rack Details

Module	10
Addendum	10 mm
Dedendum	12.5 mm
Clearance	2.5 mm
Tooth thickness	15.7 mm

The construction of the solution is similar to the previous example. Note that the same sized generating circle is used for the pinion and rack. The teeth of the rack have parallel-sided dedenda and cycloidal addenda.