Construct one convolution of an Archimedian Spiral given the shortest radius vector of 15 mm and an increase in vector length of 3 mm every 20°.

(1) Draw a 15 mm circle.
(2) Choose a starting point on the circumference.
(3) Divide the circle up into 20° sections from this point.
(4) For each radial measure the required distance outside the circumference of the 15 mm circle.
    9 mm for radial 3 (3 × 3 mm)
    24 mm for radial 8 (8 × 3 mm)
    45 mm for radial 15 (15 × 3 mm)
(5) Draw the curve to pass through these points.

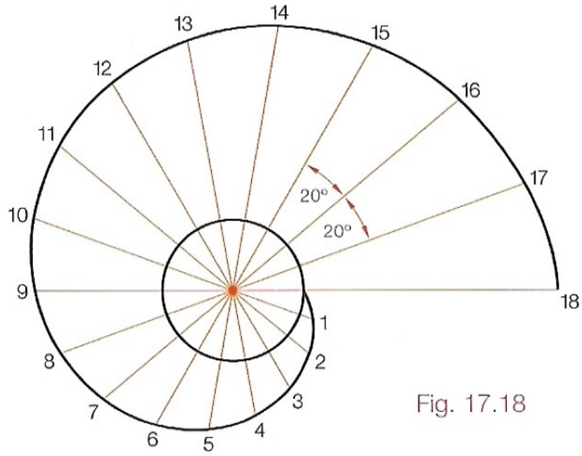


Fig. 17.18

Cycloid

A cycloid is the path traced out by a point on the circumference of a circle as it rolls along a fixed straight line without slipping.

To draw a cycloid given the circle, the base line and the point on the circumference.

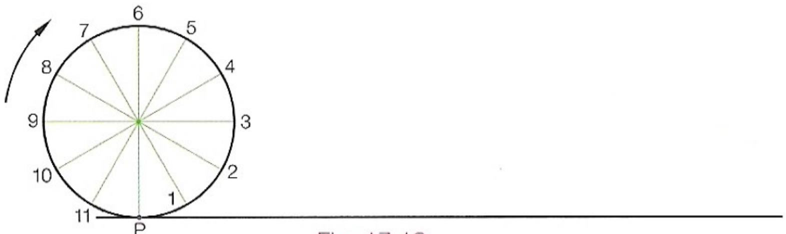


Fig. 17.19

(1) Draw the circle, the base line and point P.
(2) Divide the circle into a number of equal parts, e.g. 12 parts.
(3) The circle is to make one revolution so it will travel a distance equal to the circumference of the circle. Take one-twelfth of the circumference and step it off, from P, twelve times.

(4) Index the points as shown.
(5) As the circle rolls, the centre of the circle will travel parallel to the base line. We are going to look at the circle at twelve stages during its travel. Locate the centre point of the circle for each of these stages by erecting perpendiculars from 1,2,3 etc. on the base line.

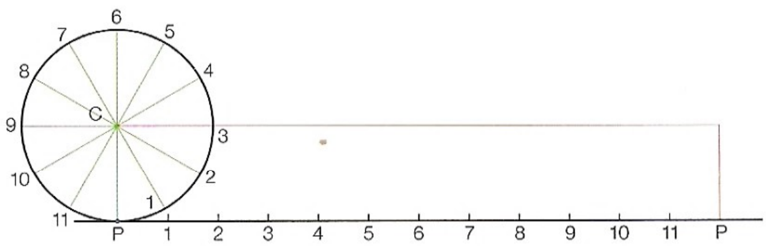


Fig. 17.20

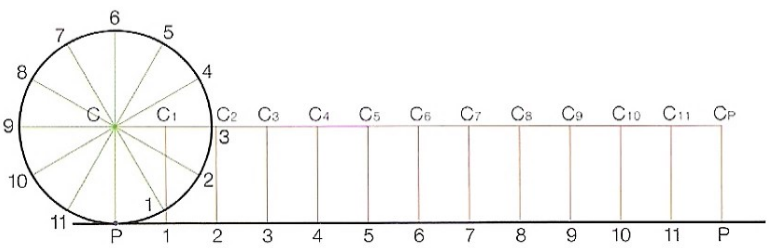


Fig. 17.21

(6) As the circle rolls clockwise, point 1 on the circumference moves onto point 1 on the base line. At the same time the centre will have moved to C_1. Point P will have moved also and can be located by triangulation. Take the distance from point 1 on the circumference to point P. Place the compass on the new point 1 on the base line and scribe an arc. Take a second

distance from the circle centre to point P (the radius) and scribe an arc from C_1 to intersect the first arc, giving the location of point P_1 on the locus.

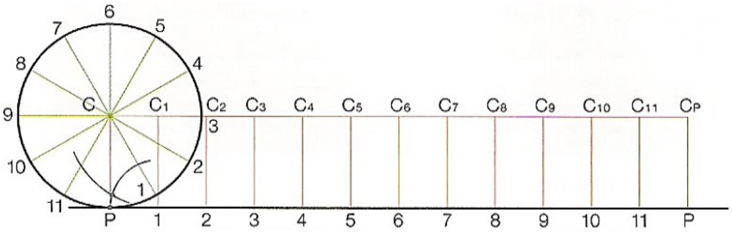


Fig. 17.22

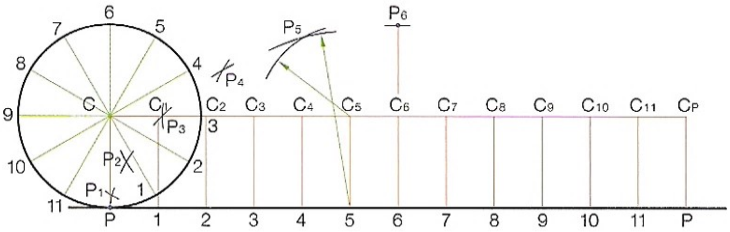


Fig. 17.23

(7) Take a similar approach for the rest of the points. For example P_5 is found by taking the distance from 5 on the circumference. Place the compass on 5 on the line and scribe an arc. Take a second distance from the centre of the circle to point P (the radius) and scribe an arc from C_5. The intersection of the arcs gives P_5, the fifth point on the cycloid.

(8) The right half is completed in the same way with the arcs swung to the right. Join the points up freehand to form a smooth curve, see Fig. 17.24.

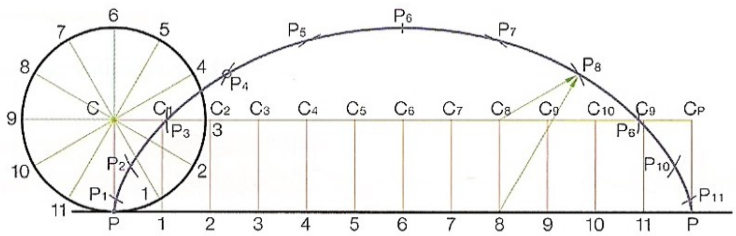


Fig. 17.24

To draw a cycloid given the base line, circle and the point P on the circle circumference. Point P does not fall on one of the twelve divisions.

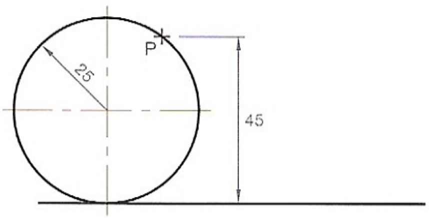


Fig. 17.25

(1) Divide the circle into twelve equal parts and index.

(2) Take one-twelfth of the circumference and step along the base line twelve times.

(3) Find the twelve centres. Index both sets of points.

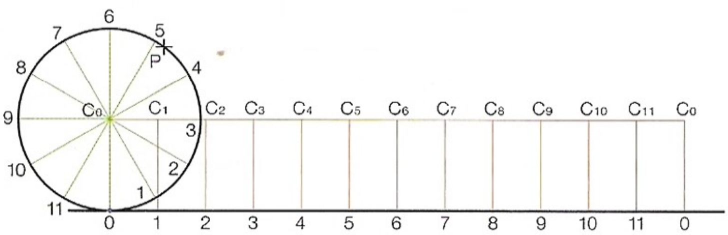
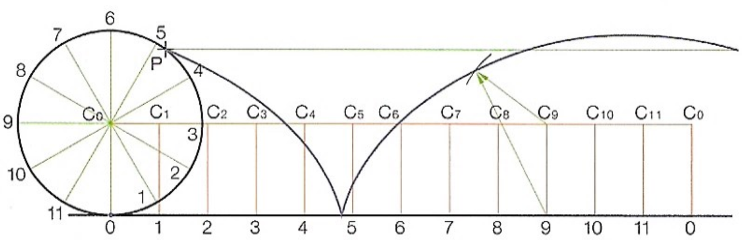


Fig. 17.26

(4) Point P falls between 4 and 5 on the circumference and will therefore hit the base line between 4 and 5. It is also to the right of the centre line. The cycloid will therefore be dropping at the start.

(5) Plot the points as before.



(6) The starting point and the end point must be at the same level.

Fig. 17.27

Epicycloid

If a circle rolls without slipping round the outside of a fixed circle then a point P on the circumference of the rolling circle will produce an epicycloids.

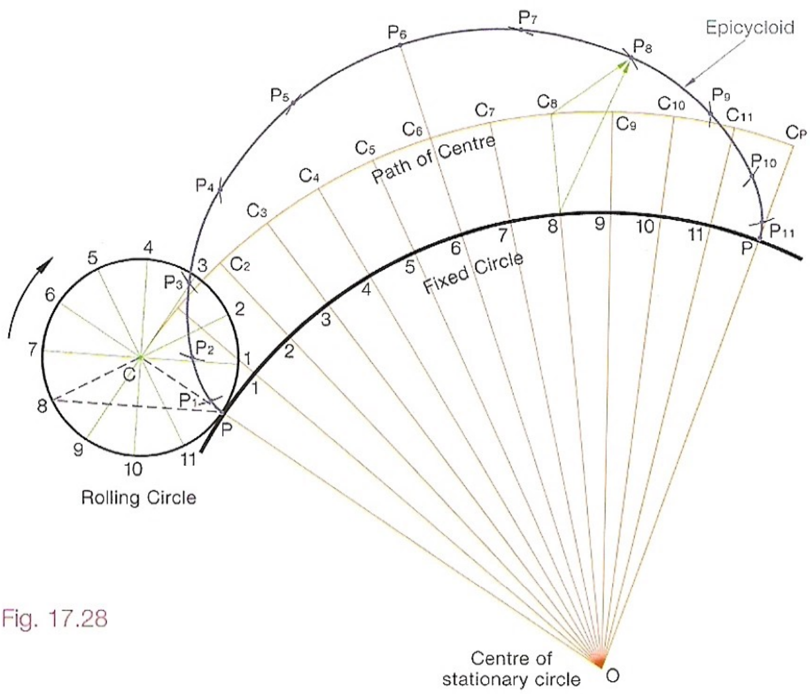


Fig. 17.28

(1) Join the centre of the rolling circle and the centre of the fixed circle.
(2) Divide the circle into twelve using this line as one of the division lines.
(3) Step-off the twelve steps along the outside of the fixed circle and index.
(4) With O as centre, swing an arc from C. This will be the path of the centre as the circle rolls.
(5) Using radians from O through points 1, 2, 3 etc. On the circumference of the fixed circle locate the centres C_1, C_2, C_3 etc.
(6) Plot the locus as before.

Hypocycloid

If a circle rolls without slipping round the inside of a fixed circle, then a point P on the circumference of the rolling circle will produce a hypocycloid.

(1) Join the centres O and C and extend.
(2) Divide the rolling circle into twelve equal parts relative to this line.
(3) Step-off the twelve steps along the circumference of the fixed circle and index.
(4) With O as centre swing an arc from C giving the path of the centre as the circle rolls.
(5) Locate the centres C_1, C_2, C_3 etc. by using radians from O to points 1, 2, 3 etc. on the circumference of the fixed circle.
(6) Plot the locus as before.

Note: Point P need not be on one of the twelve divisions.

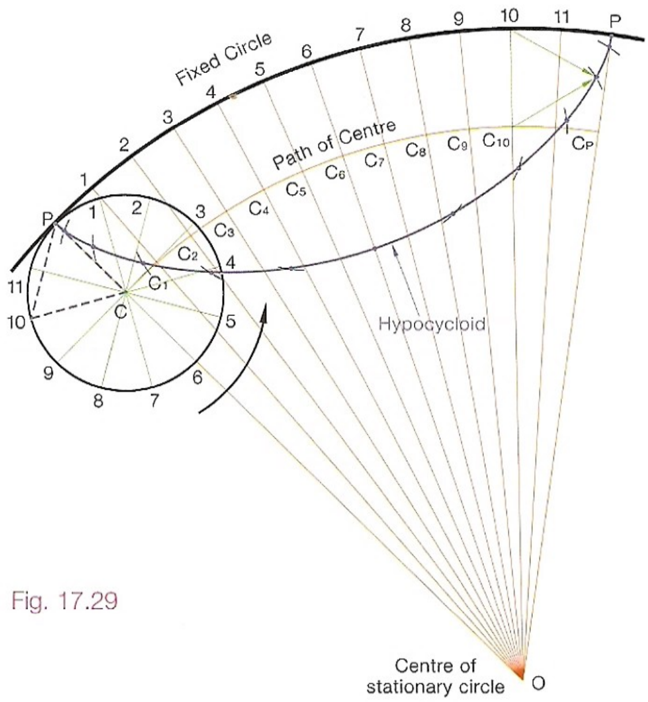


Fig. 17.29

Inferior Trochoid

When a circle rolls, without slipping, along a straight line, then a point P inside the circle will follow the path of an inferior trochoid.

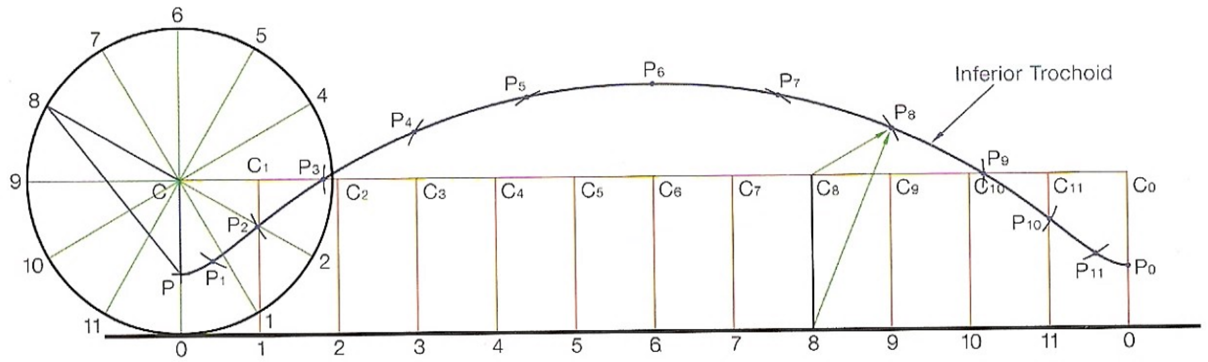


Fig. 17.30

The construction is similar to the cycloid.

(1) Divide the circle into twelve equal parts and index.

(2) Step-off the twelve steps along the straight line to find the length of the circumference.

(3) Find the path of the centre line and locate C_1, C_2, C_3 etc., as shown in Fig. 17.30.

(4) The points along the locus are located as previously described for the cycloid, measuring to point P inside the circle each time.

(5) Join the points to give a smooth curve.

Superior Trochoid

When a circle rolls, without slipping, along a straight line, then a point P outside the circle will follow the path of a superior trochoid.

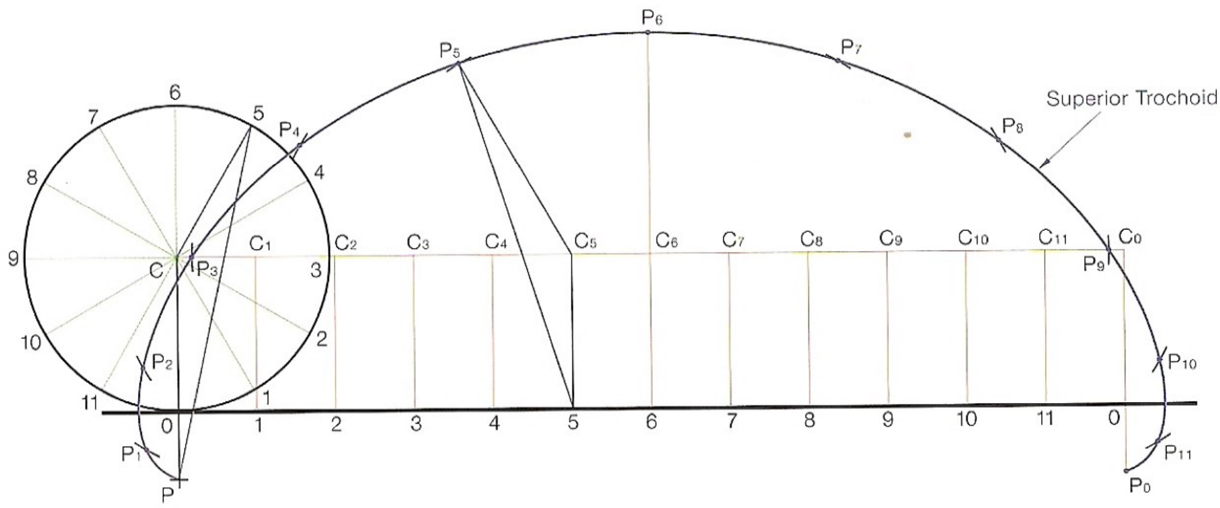


Fig. 17.31

(1) Divide the circle into twelve equal parts and index.

(2) Step-off the twelve steps along the straight line to find the length of the circumference.

(3) Locate C_1, C_2, C_3 etc.

(4) Locate P_1 to P_0 as shown previously.

(5) Draw the locus.

Inferior Epitrochoid

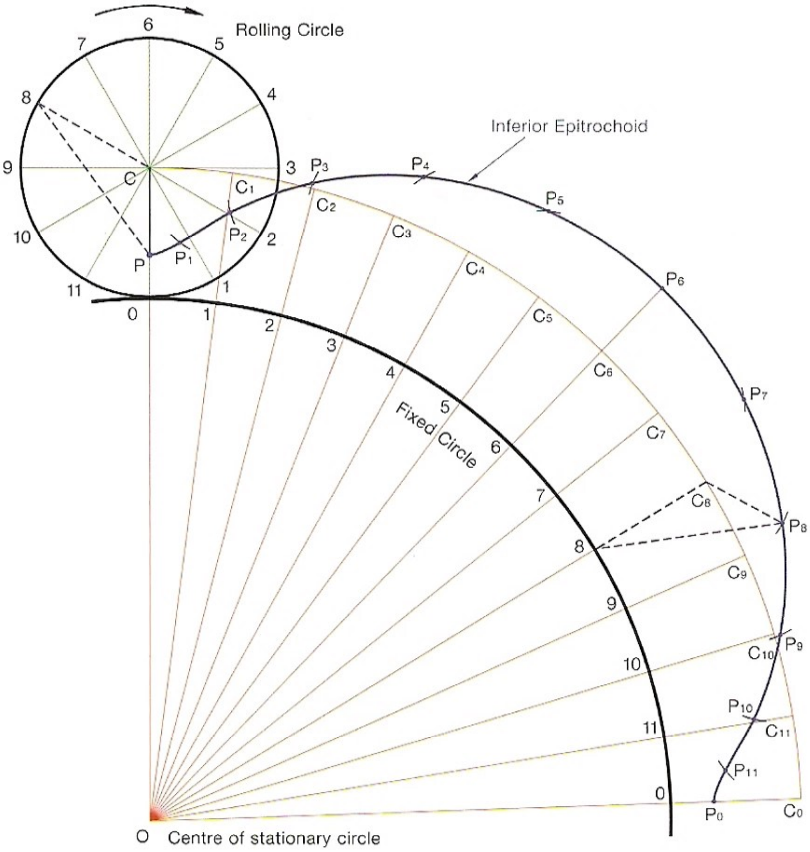


Fig. 17.32

When a circle rolls, without slipping, around the outside of a fixed circle, then a point P inside the circle will follow the path of an inferior epitrochoid.

(1) Divide the circle into twelve equal parts.
(2) Set circumference off on the fixed circle.
(3) Rotate C about O and locate the twelve centres.
(4) Locate the points on the locus as before.

Superior Epitrochoid

When a circle rolls, without slipping, around the outside of a fixed circle, then a point P outside the circle will follow the path of a superior epitrochoid.

The construction of the locus follows the same method as before.

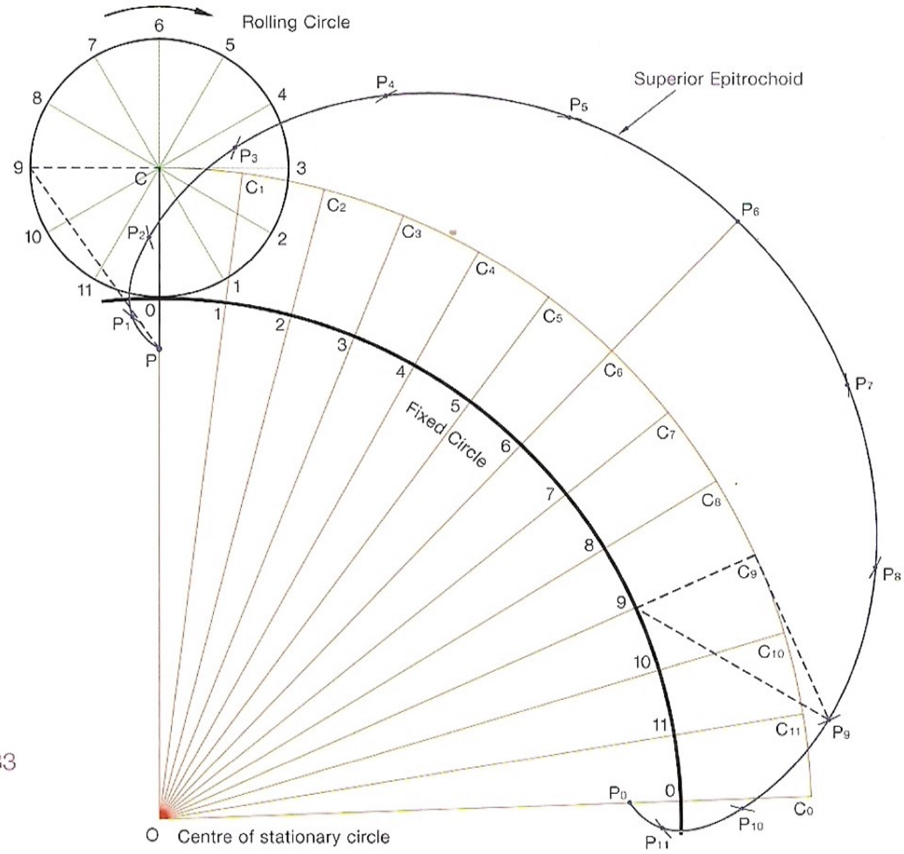


Fig. 17.33

Inferior Hypotrochoid

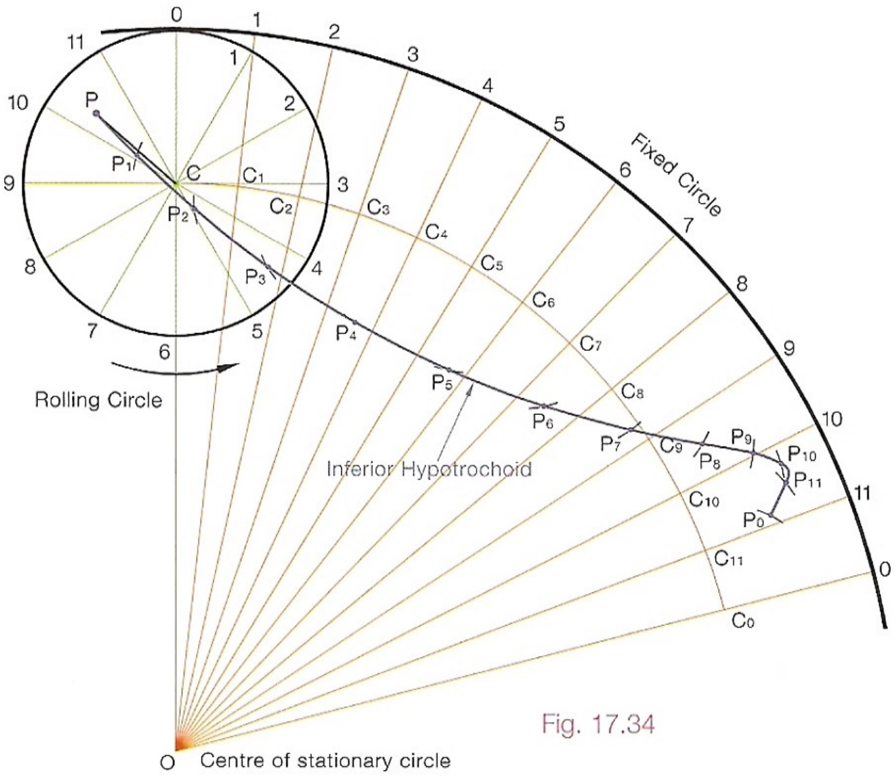


Fig. 17.34

When a circle rolls, without slipping, around the inside of a fixed circle, then a point P inside the circle will follow the path of an inferior hypotrochoid.

Construction as before. Note the position of point P. The position of point P does not effect the method of construction.

Superior Hypotrochoid

When a circle rolls, without slipping, around the inside of a fixed circle, then a point P outside the circle will follow the path of a superior hypotrochoid.

Construction as before.

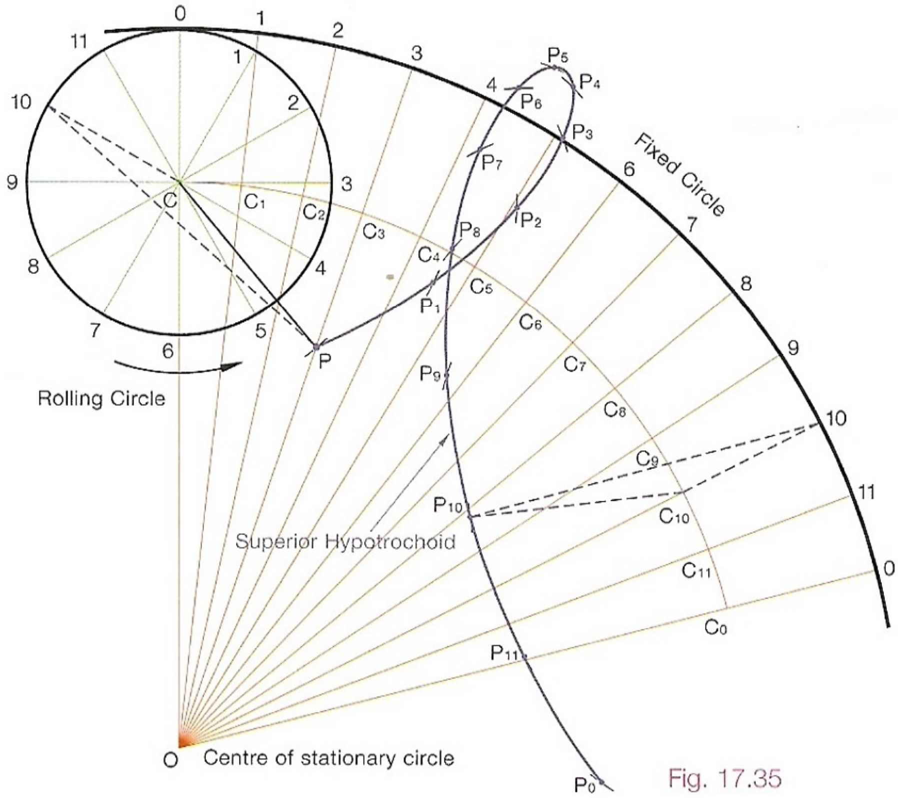


Fig. 17.35

Use of Templates to Solve Problems on Cycloids etc.

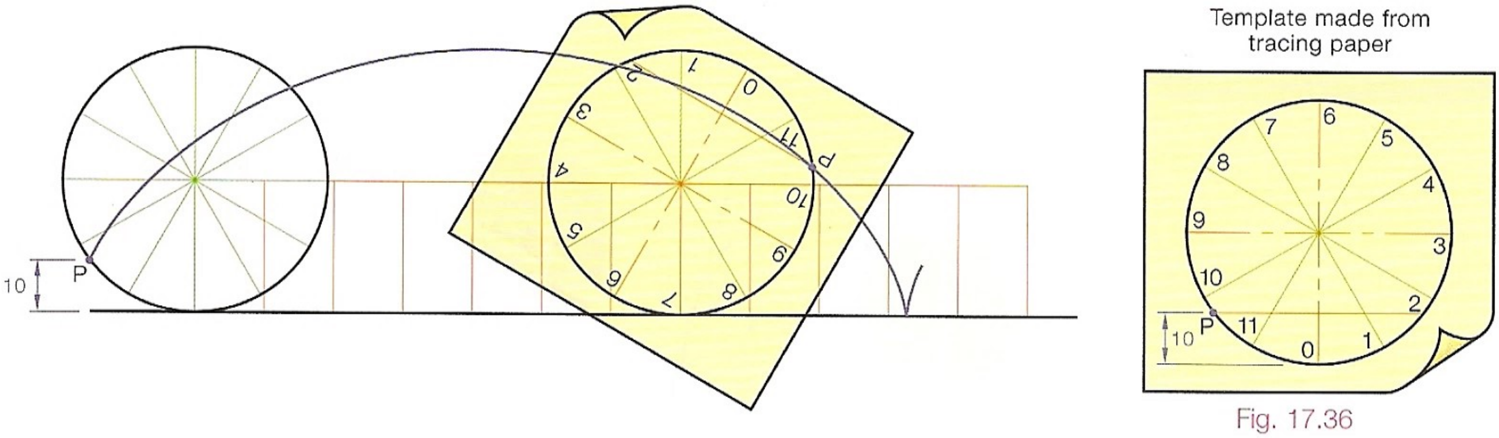


Fig. 17.36

Most of these loci problems can be solved using a template. The template is moved into position and point P is plotted with a pencil or compass point, see Figures 17.36 and 17.37.

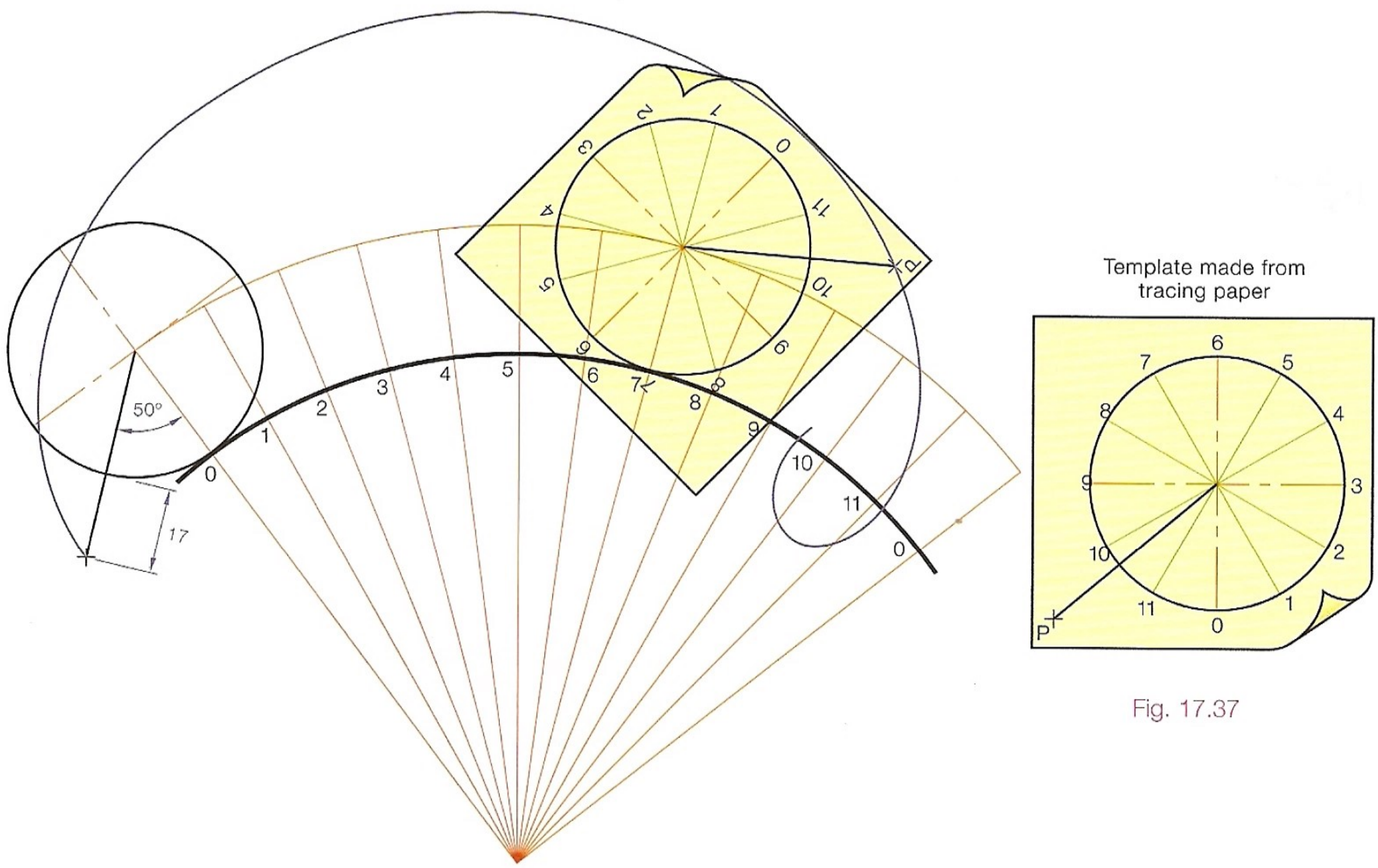


Fig. 17.37

Tangent to an Involute, Archimedian Spiral, Cycloid, Epicycloid and Hypocycloid

Tangent to an Involute

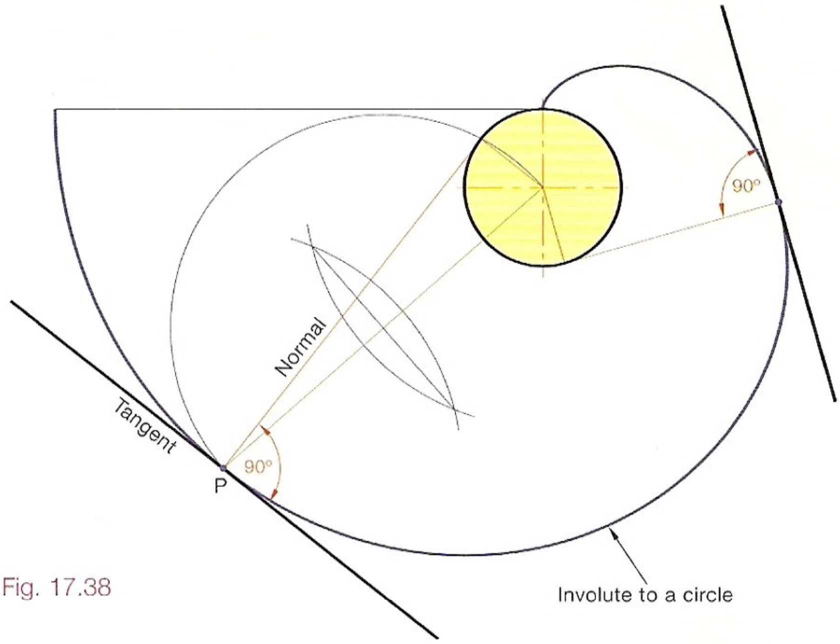


Fig. 17.38

For an involute to a circle it can be seen that tangents to the circle will form normals to the involute.

(1) Select any point P on the involute.
(2) From P draw a tangent to the circle.
(3) The tangent to the circle is the normal for the tangent to the involute. Draw the tangent at 90° to the normal.

For an involute of a square the curve is made up of circular portions.

(1) Select any point P on the involute.
(2) Join P back to the centre point of the arc that formed that section of the involute. This is the normal.
(3) Draw the tangent perpendicular to the normal.

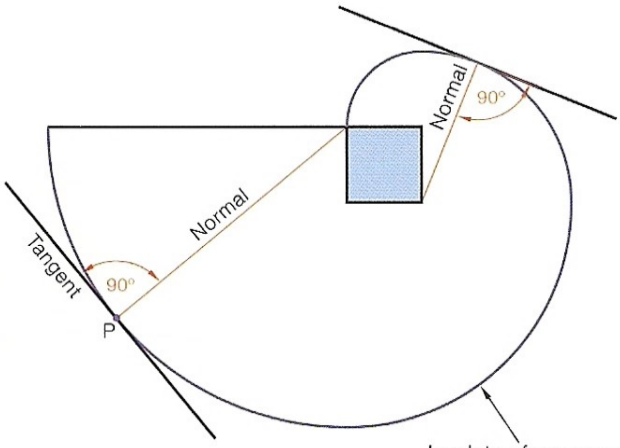


Fig. 17.39

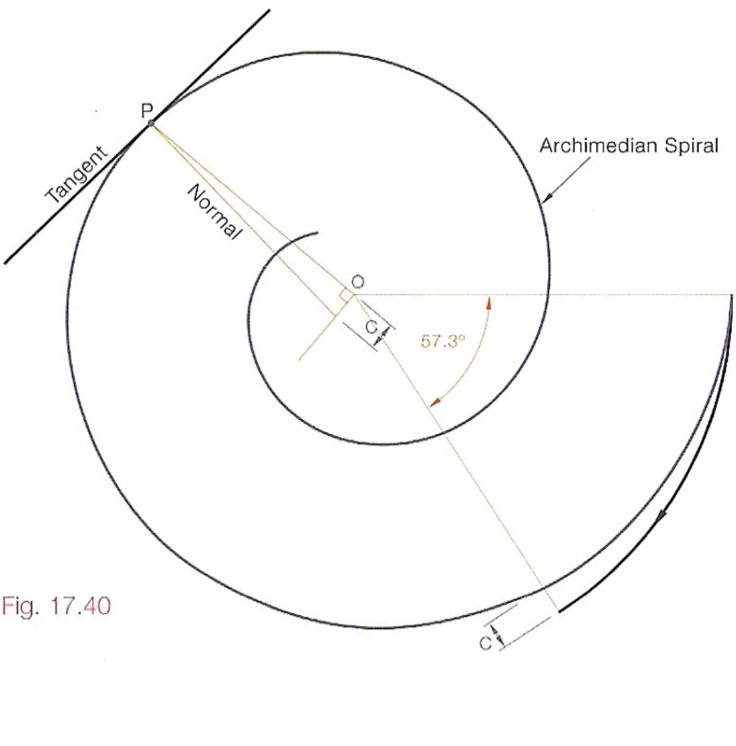


Fig. 17.40

Tangent to an Archimedian Spiral.

(1) Select any point P on the spiral.
(2) Join P to the pole, point O.
(3) Draw a perpendicular to PO at O.
(4) Measure out the constant C and draw the normal.
(5) The tangent is perpendicular to the normal. The constant c is the distance the spiral has moved closer to the pole over an angular distance of one radian. A radian equals approximately 57.3°, see Fig. 17.40.

Tangent to a Cycloid

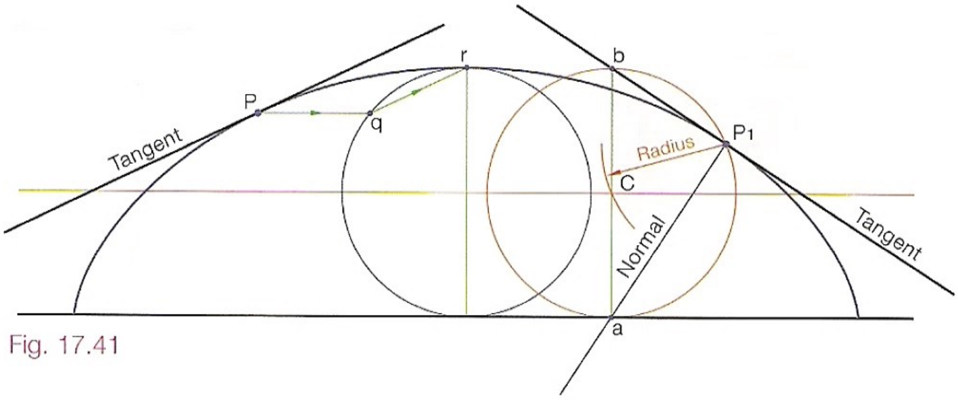


Fig. 17.41

Method 1

(1) Choose any point P on the cycloid.

(2) Draw the circle at the cycloid's highest point.

(3) Project P horizontally to q.

(4) Join q to r.

(5) The tangent will be parallel to qr.

Method 2

(1) Choose any point P_1 on the cycloid.

(2) Using the radius of the circle, strike an arc from P_1 to locate c on the centre line.

(3) Draw the circle.

(4) The circle touches the base line at a. A line drawn from a through c will locate point b.

(5) P_1b is the tangent and P_1a is the normal.

Tangent to an Epicycloid

The construction is the same as for a cycloid.

(1) Choose point P anywhere.

(2) Locate and draw the circle.

(3) From O (the centre of the fixed circle) through c will locate points a and b.

(4) Pb is the tangent and Pa is the normal.

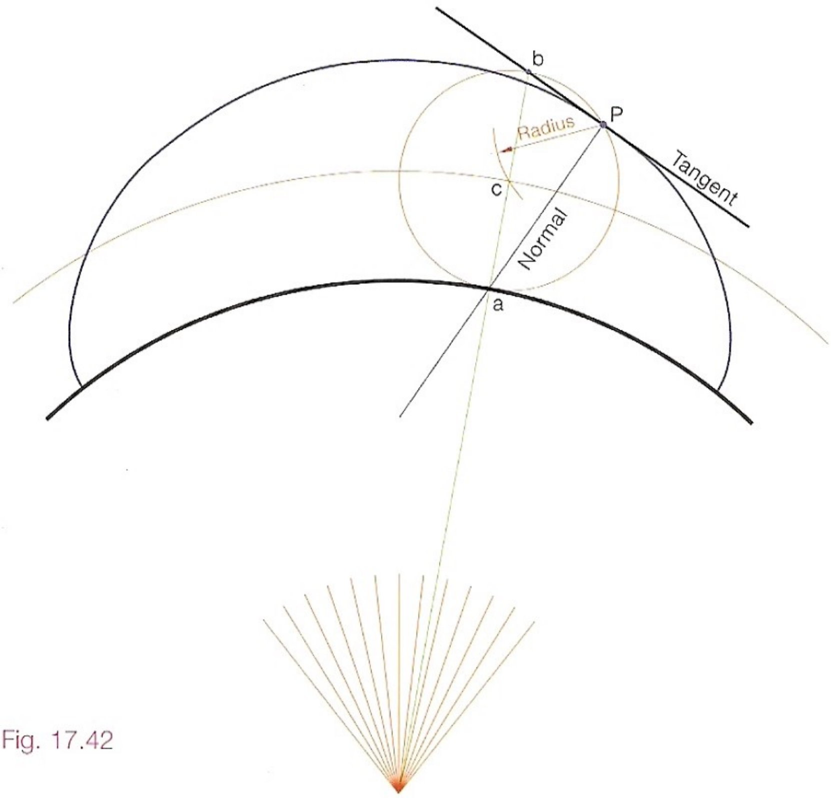


Fig. 17.42

Tangent to a Hypocycloid

The construction is the same as for a cycloid and an epicycloid.

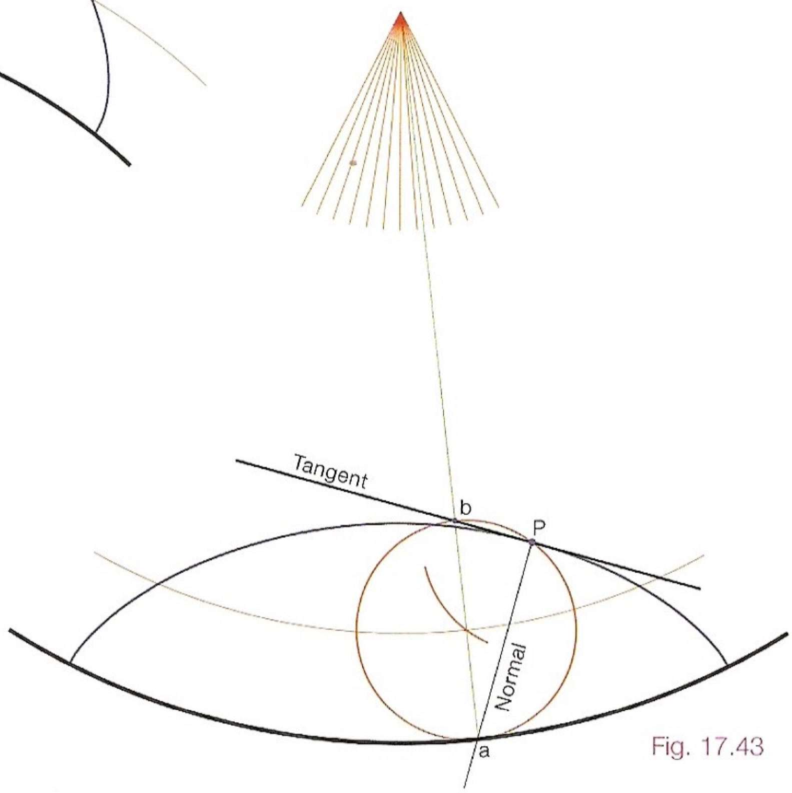


Fig. 17.43