Q21.

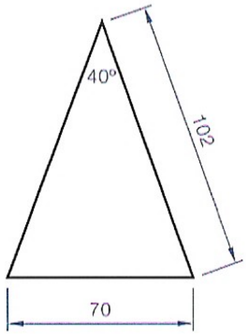

Fig. 1.39

Q22.

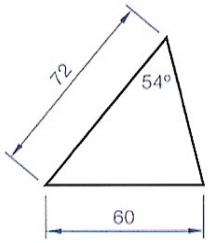

Fig. 1.40

Q23.

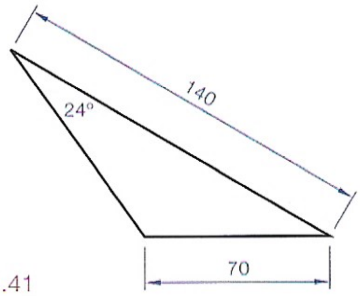

Fig. 1.41

Q24. Construct a triangle given the perimeter of 180 mm and the ratio of the sides of 4:3:2.

Q25. Construct a triangle given the perimeter of 220 mm and the ratio of the sides of 4:3:5.

Q26. Construct a triangle given the perimeter of 200 mm and the ratio of the sides 7:6:2.

Polygons

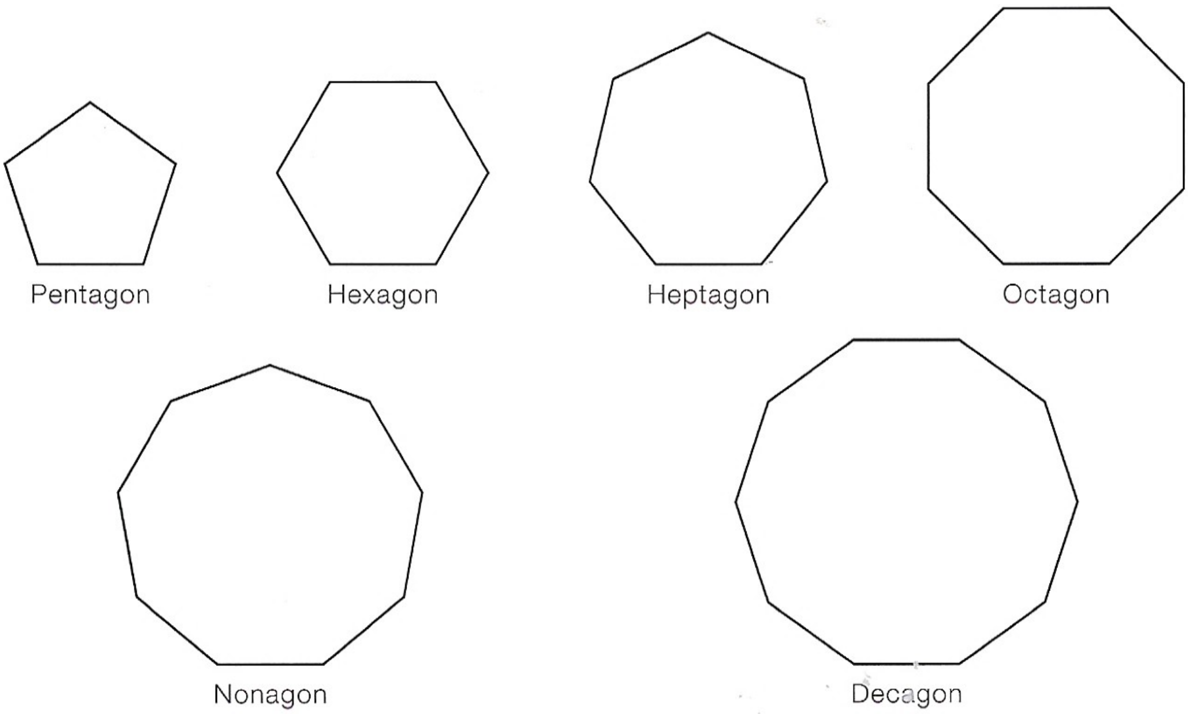

Fig. 1.42

Definition: A polygon is a plane figure having three or more sides. A regular polygon has all its sides of equal length and all its angles of equal measure.

Construction of a polygon given the base
Method 1: Protractor

The exterior angle for any regular polygon can easily be found by dividing the number of sides required into 360°.

Exterior angle = 360°/Number of sides

For example the exterior angle for a pentagon is 360°/5 = 72°

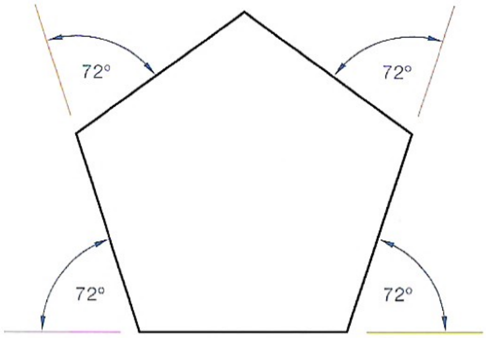

Fig. 1.43

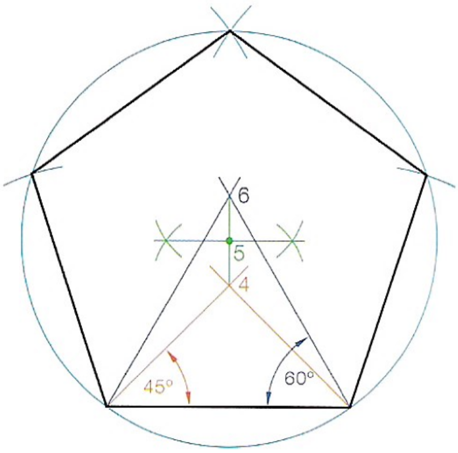

Fig. 1.44

Method 2: Triangle

(1) Draw the base.

(2) On the base, construct a 45° angle from each end as shown in Fig. 1.44. Number the apex as 4.

(3) Produce another triangle on the same base having base angles of 60°. The apex of this triangle is numbered as 6.

(4) Join 4 to 6 and bisect giving point 5.

(5) With 5 as centre, scribe a circle to pass through both ends of the base.

(6) The base stepped round this circle will give the sides of the pentagon.

This method can be used for polygons with more sides but is not very accurate.

The spacing between the centres is the same as that between 4 and 5. As the number of sides increases the accuracy decreases.

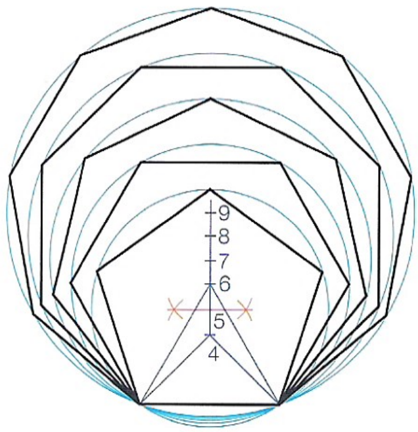

Fig. 1.45

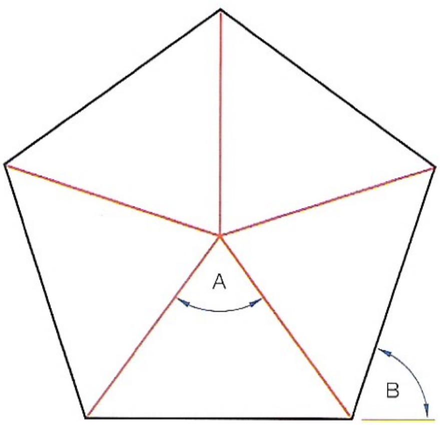

Fig. 1.46

Method 3: Angle at the centre of a polygon

For all regular polygons the angles formed by joining the vertices to the centre will all be equal and will also equal the exterior angle of the polygon.

In Fig. 1.46, Angle A = Angle B

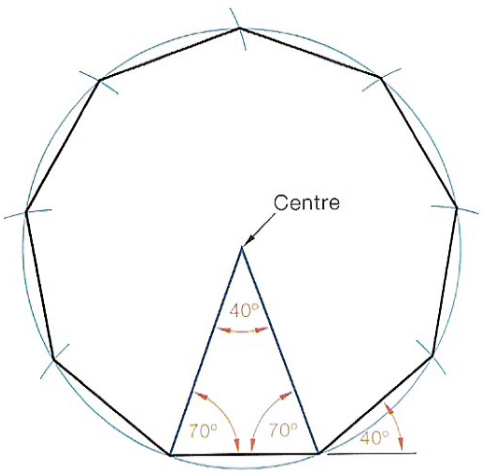

Fig. 1.47

Construct a regular nonagon given the base.

(1) Draw the base.

(2) Since the exterior angle for a nonagon is 40° ($360^\circ/9 = 40^\circ$) the interior angle will also be 40°. If the top angle of an isosceles triangle is 40°, then the two base angles will be 70°. (Angles in a triangle add up to 180°.) Draw the two base angles finding the nonagon centre at the triangle apex.

(3) Scribe the circle and step the base around it to find the remaining vertices.

To inscribe a regular polygon in a given circle.

(1) Draw the circle and the diameter.

(2) Divide the diameter into the same number of equal parts as there are sides needed on the polygon (7 parts for the heptagon in the diagram).

(3) With the diameter as radius, draw two arcs using the ends of the diameter as centre. The arcs cross at B.

(4) Draw a line from B through 2 on the diameter and extend to hit C on the circle.

(5) AC is one side of the polygon. Step the distance AC around the circumference finding the remaining sides (approximate).

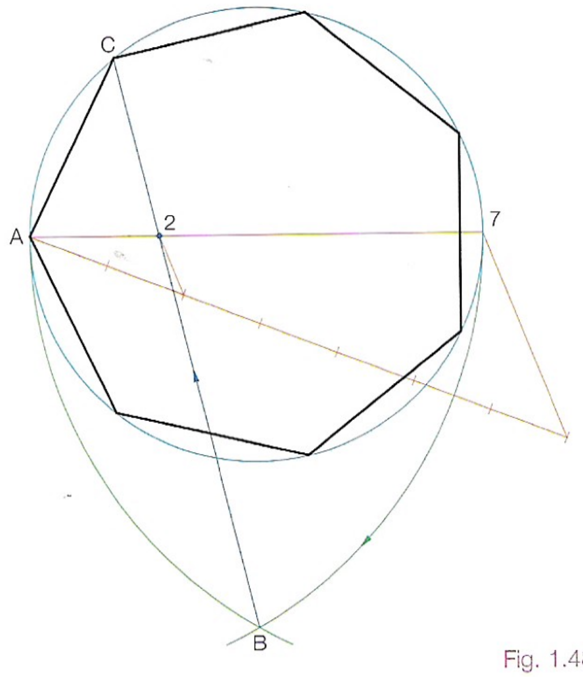

Fig. 1.48

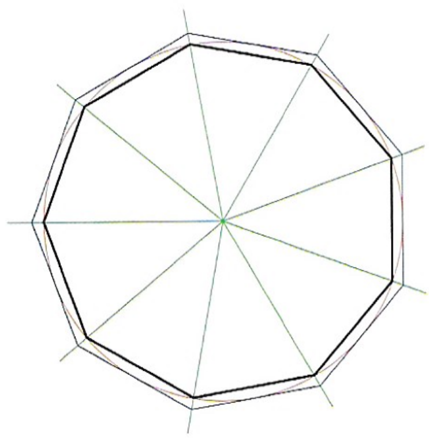

Fig. 1.49

To describe a regular polygon about a given circle.

(1) Draw the given circle.

(2) Inscribe the polygon as described above.

(3) Radiate lines from the centre of the circle through each vertex.

(4) Draw the sides of the required polygon parallel to the sides of the inner polygon.

Circles in Contact with Points, Lines and Curves

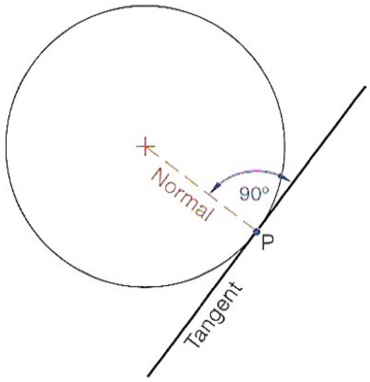

Fig. 1.50

To draw a tangent to a circle from a point P on the curve.

Join P back to the centre of the circle giving the normal. The tangent can be constructed perpendicular to the normal at P.

To draw a tangent to a circle from a point P outside the circle.

The solution is based on the fact that the normal is always perpendicular to the tangent and that the angle in a semicircle is always a right angle.

(1) Join P to C and bisect.
(2) Draw a semicircle on line CP to cut the circle at A.
(3) Point A is the point of contact and PA is the tangent.

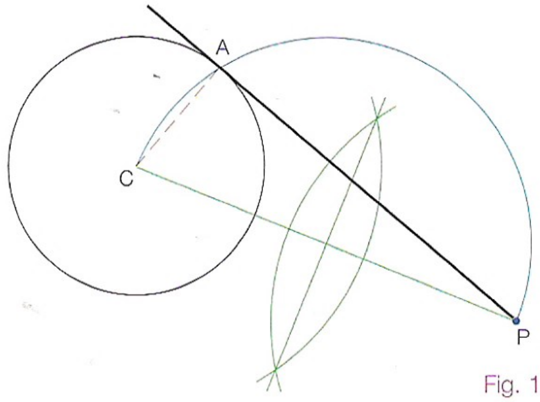

Fig. 1.51

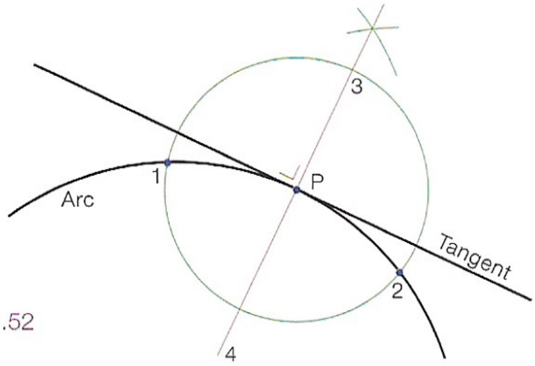

Fig. 1.52

Tangent to an arc when we do not have the centre.

(1) With P as centre, draw a circle finding points 1 and 2.
(2) Use points 1 and 2 to bisect, giving line 3,4.
(3) The tangent is drawn perpendicular to line 3,4.

To draw a tangent to a circle, parallel to a given line.

(1) With centre C, draw an arc across the given line at A and B.
(2) Bisect AB giving D.
(3) Join D back to C giving the normal.
(4) The point of contact is found and the tangent is drawn parallel to the original line.

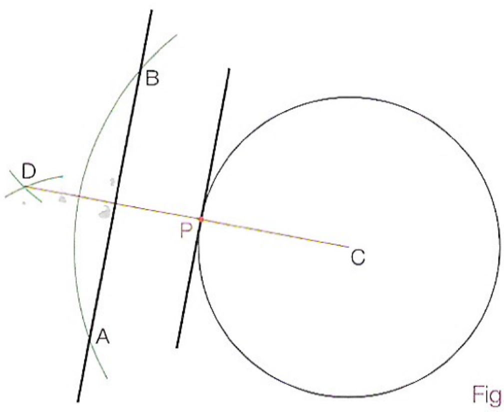

Fig. 1.53

External tangent to two unequal circles.

(1) Join the centres and bisect.
(2) Draw a semicircle.
(3) Step the radius of the small circle r inside the circumference of the large circle, giving point A.
(4) Draw the circle through A as shown.
(5) This circle crosses the semicircle at B which is a point on the normal.
(6) Draw the normal. The normal for the smaller circle will be parallel to this.
(7) Draw the tangent, Fig. 1.55.

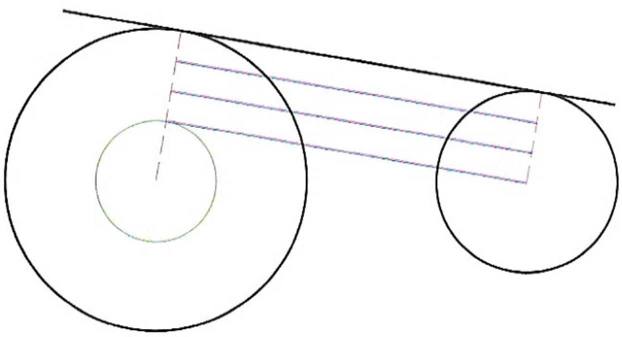

Fig. 1.55

T out r in
External tangent step the radius of the small circle in.

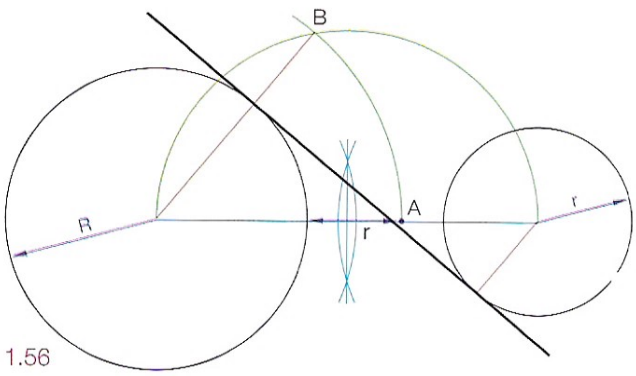

Fig. 1.56

This construction is based on the fact that when the tangent is internal, by increasing the radius of the larger circle the smaller circle must decrease by a similar amount if the tangents are to remain parallel. Increase the large circle by the radius r of the small circle will result in the small circle being reduced to a point.

T in r out
Internal tangent step the radius of the small circle out.

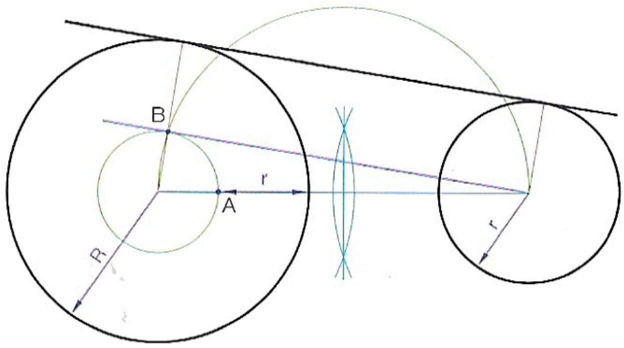

Fig. 1.54

This construction is based on the fact that if the two circles increase or decrease in radius by the same amount, the external tangents to them will remain parallel. The small circle is reduced to a point by subtracting its radius. The larger circle must be reduced in size by a similar amount. The problem is now reduced to that of drawing a tangent to a circle from a point outside it.

Internal tangent to two unequal circles.

(1) Join the centres and bisect.
(2) Draw a semicircle.
(3) Step the radius of the small circle r outside the circumference of the large circle giving point A.
(4) Draw the arc from A as shown.
(5) This arc crosses the semicircle at B which is a point on the normal.
(6) Draw the normals and the tangent.

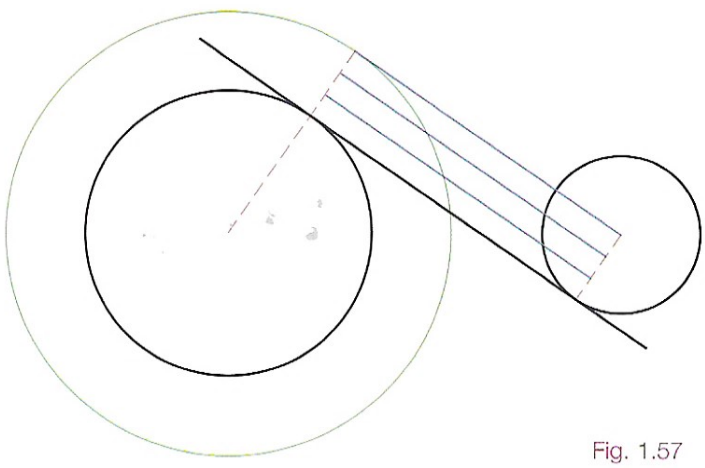

Fig. 1.57

Activities

Q1. Given the base of a pentagon as 40 mm. Construct the pentagon using two methods.

Q2. Construct a heptagon having a base of 35 mm using the triangle method.

Q3. Draw a circle of 90 mm diameter. Inscribe a pentagon in this circle.

Q4. Describe a nonagon about a circle of radius 40 mm.

Q5. Inscribe a pentagon in a circle of 90 mm diameter.

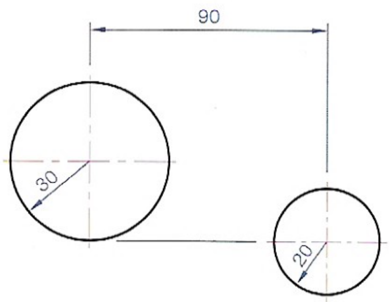

Fig. 1.60

Q8. Draw an external tangent to the two given circles, Fig. 1.60.

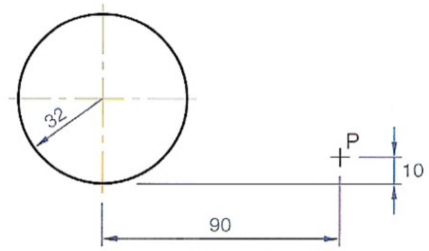

Fig. 1.58

Q6. Draw both tangents to the circle from point P showing clearly the points of contact, Fig. 1.58.

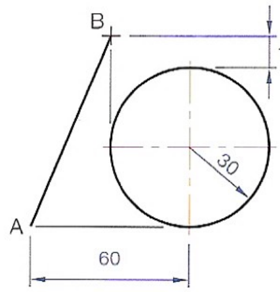

Fig. 1.59

Q7. Construct a tangent to a given circle, parallel to a given line AB, Fig. 1.59.

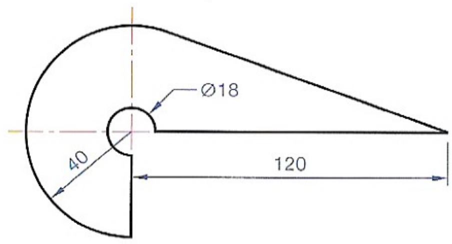

Fig. 1.61

Q9. Draw the centre square shown in the diagram Fig. 1.61.

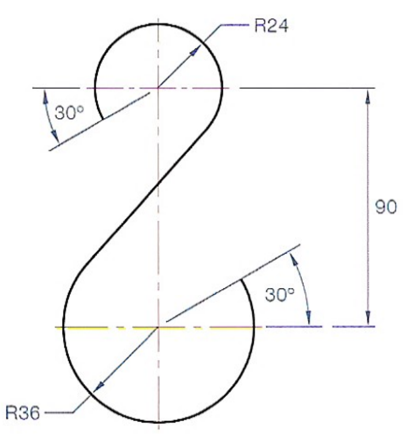

Fig. 1.62

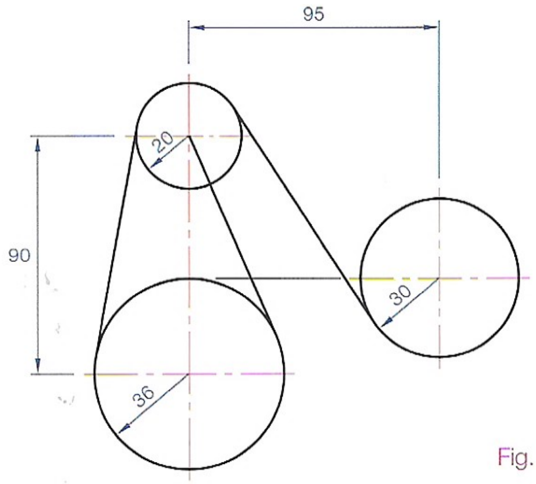

Fig. 1.63

Q10. Draw the metal hook shown in Fig. 1.62.

Q11. Draw the pulley system shown in Fig. 1.63.

Tangent Curves

To construct the tangent arc shown, to two given circles.

(1) From the centre of the large circle draw an arc equal to R + 50 = 30 + 50 = 80.

(2) From the centre of the small circle draw an arc equal to r + 50 = 16 + 50 = 66.

(3) The centre is found where these two arcs cross. An arc of radius 50 mm will form a tangent to the two circles.

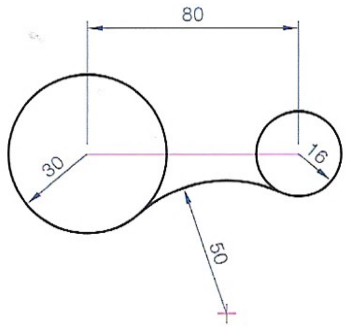

Fig. 1.64

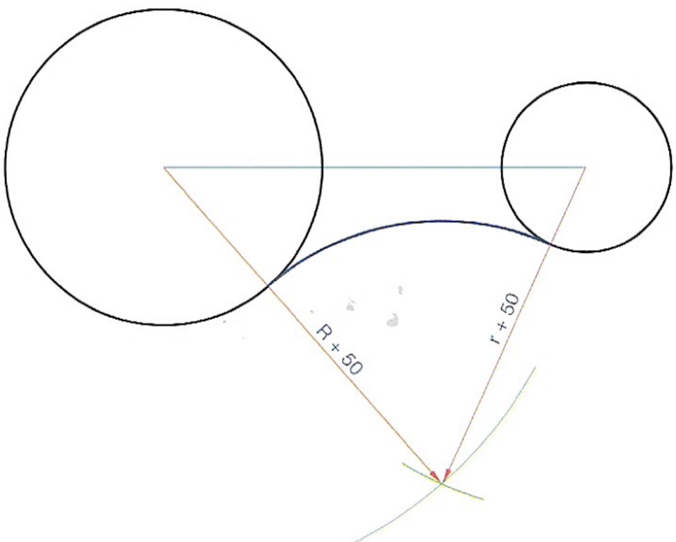

Fig. 1.65

To construct a tangent arc to enclose two given circles.

(1) From the centre of the large circle draw an arc equal to 100 – R = 100 – 30 = 70.

(2) From the centre of the small circle draw an arc equal to 100 – r = 100 – 16 = 84.

(3) The centre is found where these two arcs cross. An arc of radius 100 mm will form a tangent curve to the two circles.

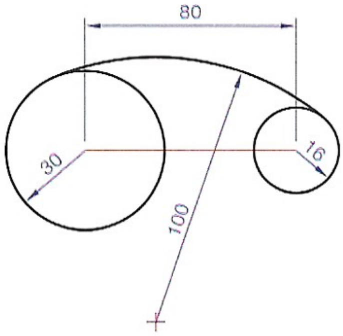

Fig. 1.66

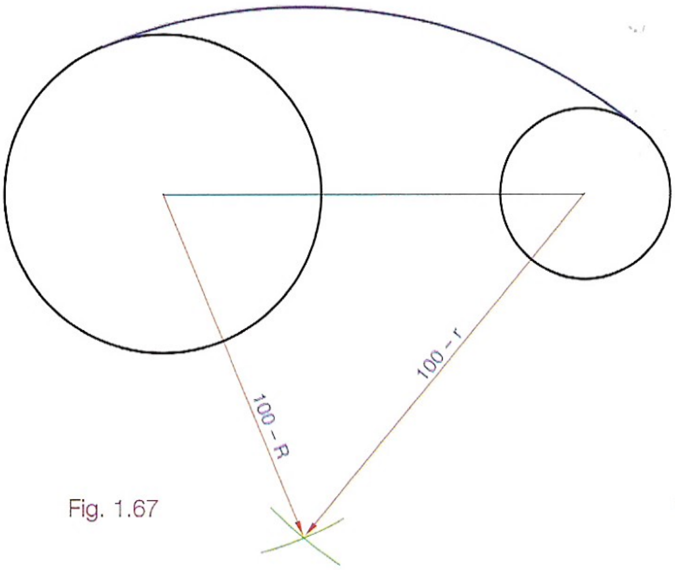

Fig. 1.67

To construct an internal tangential arc to two given circles.

(1) From the centre of the large circle draw an arc equal to 80 + R = 80 + 30 = 110.

(2) From the centre of the small circle draw an arc equal to 80 – r = 80 – 16 = 64.

(3) The centre is found where the two arcs cross. An arc drawn with radius 80 mm will form an internal tangent to the two circles.

There are four possible combinations of arcs to give four possible internal tangents.

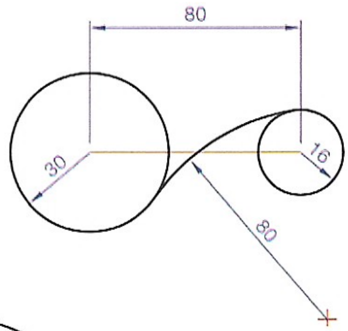

Fig. 1.68

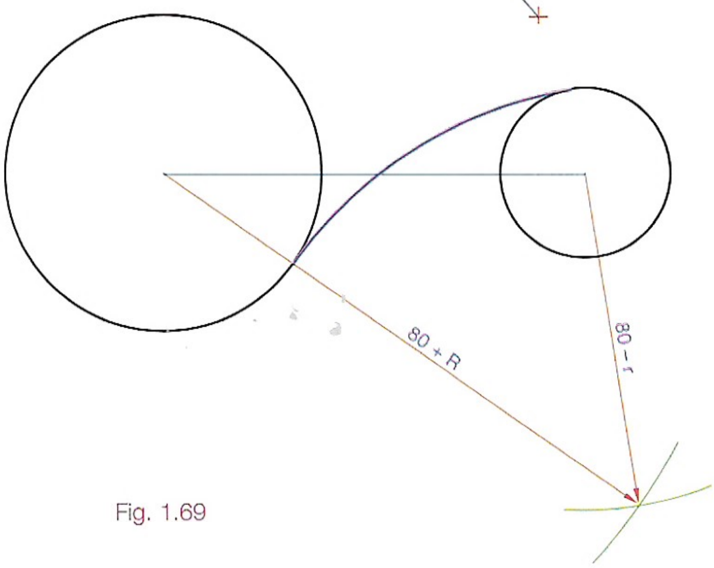

Fig. 1.69

Circles Touching Points and Lines

To draw a circle of given radius to touch two arms of an angle.

(1) Draw a line parallel to AC the radius of the required circle away.
(2) Draw a similar line parallel to AB.
(3) The lines cross, giving the circle centre.

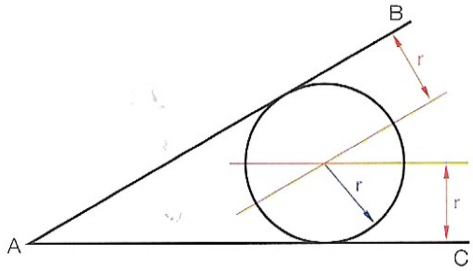

Fig. 1.70

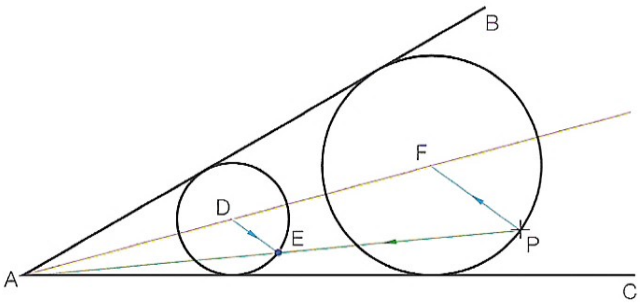

Fig. 1.71

To draw a circle to touch the two arms of an angle and a given point P.

(1) Bisect the angle BAC. The centre of the required circle must rest on this bisector.
(2) Draw the circle having its centre D on the bisector.
(3) Join point P back to the corner A.
(4) The line PA strikes circle D at E. Join D to E.
(5) From P draw a line parallel to line DE finding point F, the centre of the required circle.

To draw a circle to touch a given line AB at a given point D and to pass through a point P.

(1) AB will be a tangent to the circle and D will be the point of contact.
A perpendicular from D will therefore form the normal and will pass through the centre.
(2) Join DP.
(3) Bisect DP. The line DP will form a chord to the circle and therefore the bisector of this chord will also pass through the circle centre.
(4) Point C is found where the bisector and the perpendicular meet.

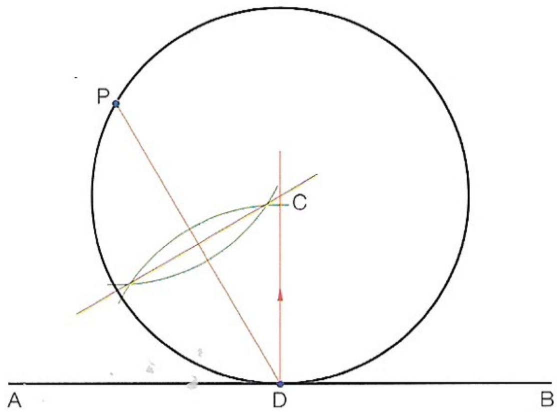

Fig. 1.72

To draw a circle to pass through two given points A and B and to touch a given line GH.

(1) Join A to B and bisect (chord).
(2) Extend AB to the line GH, finding point C.
(3) Extend BC further to D so that the length of CA equals CD.
(4) Bisect the line DB and draw a semicircle.
(5) Draw CE perpendicular to DB.
(6) Make CF equal in length to CE. Point F is the point of contact.
(7) A perpendicular from F to the line GH will find the centre.
(8) Draw the circle.

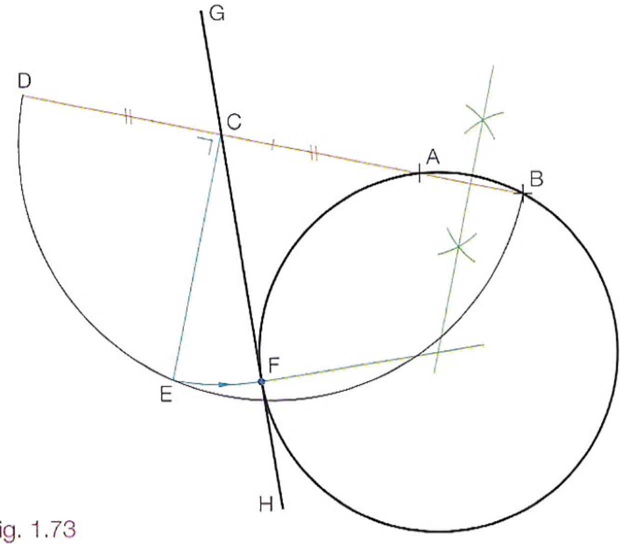

Fig. 1.73

To draw a circle to touch a given point p and a given circle.

(1) Draw a vertical line through the centre of the given circle. Line BAC.
(2) Draw a horizontal line through point P to C.
(3) Erect a perpendicular to line CP at point P. The centre of the required circle will be on this line.
(4) Join B to P, cutting the given circle at D. D will be the point of contact.
(5) A line from A through D to E will find the circle centre.

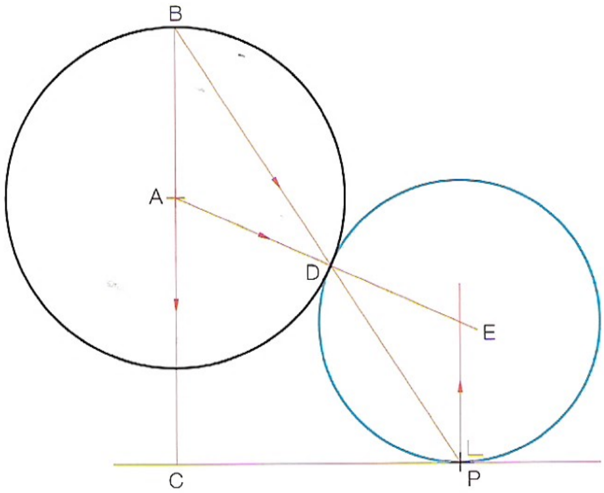

Fig. 1.74

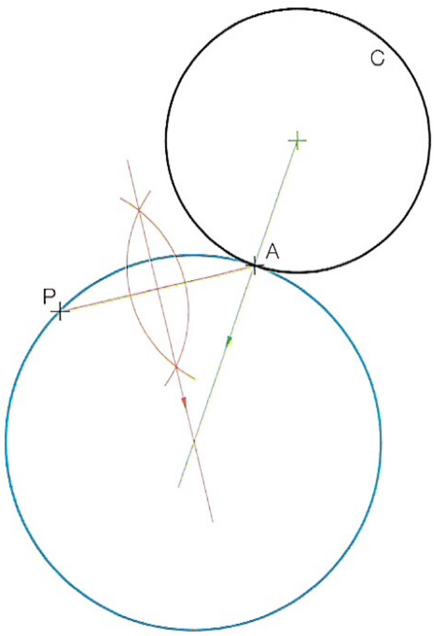

Fig. 1.75a

To draw a circle to touch a given circle C at a point A on the circumference and to pass through a given point

(1) Join A to P and bisect. AP will be a chord and therefore the bisector of this chord will pass through the centre.
(2) Join A and the centre of the original circle and extent to hit the bisector. If two circles are in contact then their centres and the point of contact will be in line.
(3) Draw the circle.

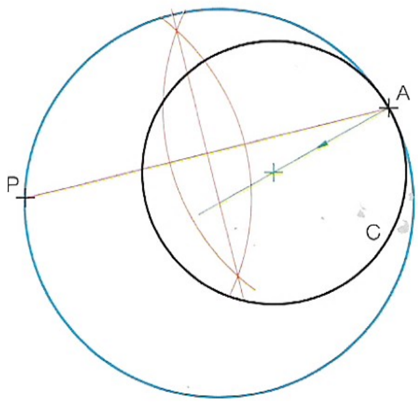

Fig. 1.75b