Q11. Construct the triangle shown in Fig. 1.30 and circumscribe a circle around it.

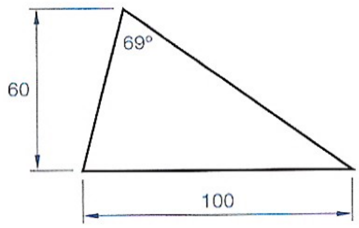


Fig. 1.30

Q12. Construct the triangle shown in Fig. 1.31 and inscribe a circle.

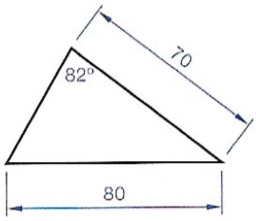


Fig. 1.31

Q13. Construct the triangle shown in Fig. 1.32 given the perimeter, altitude and one base angle.

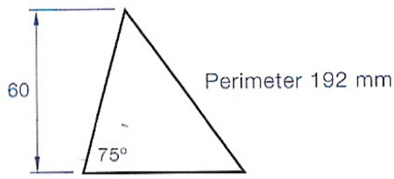


Fig. 1.32

Q14. Construct a triangle given the perimeter of 150 mm and the ratio of the sides as 5:4:6. Find the centroid of this triangle.

Questions Q15.–Q23.
Construct the following triangles:

Q15.

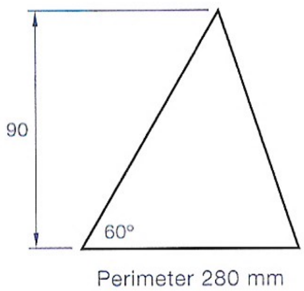


Fig. 1.33

Q16.

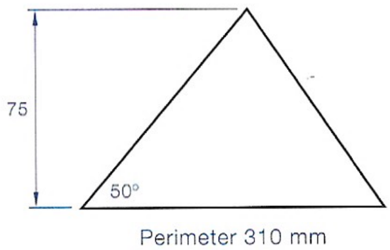


Fig. 1.34

Q17.

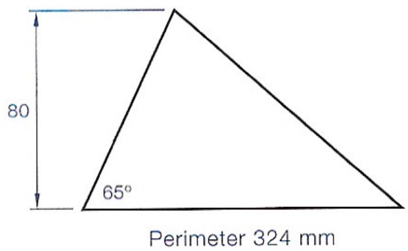


Fig. 1.35

Q18.

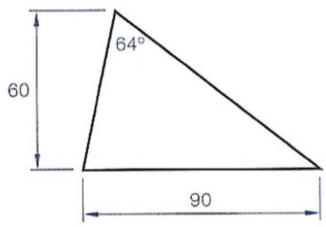


Fig. 1.36

Q19.

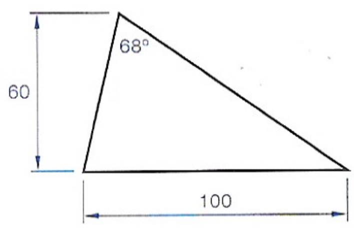


Fig. 1.37

Q20.

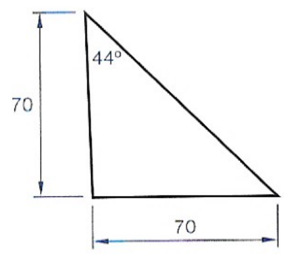


Fig. 1.38

1 Basic Constructions

SYLLABUS OUTLINE

Areas to be studied:

• Construction of plane figures. • Construction of loci.
• Circles in contact with points, lines and curves.

Learning outcomes

Students should be able to:

Higher and Ordinary levels

- Construct triangles, quadrilaterals and regular polygons of given side/altitude, inscribed and circumscribed about a circle.
- Apply the principles and properties of plane figures in a problem-solving setting.

Higher level only

- *Use the principle of loci as a problem-solving tool.*

Basic Constructions

To bisect a line AB.

To bisect a line is to divide the line in half.

(1) With A as centre and radius greater than half of AB, draw an arc.
(2) With B as centre, draw another arc having the same radius as Step 1.
(3) The arcs cross, top and bottom, giving the bisector.

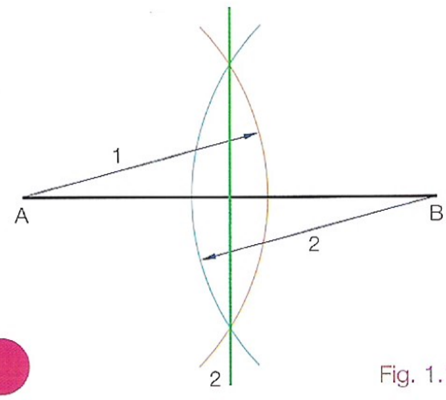


Fig. 1.1

To bisect an angle ABC.

(1) With the vertex B as centre, draw an arc of any radius to hit the arms of the angle AB and BC.
(2) With D and E as centres, swing arcs of equal radius, so they cross as shown, at point F.
(3) F joined back to B bisects the angle.

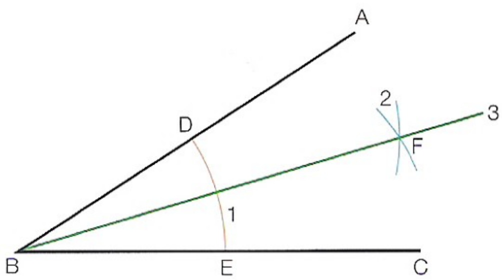


Fig. 1.2

To construct a perpendicular from A to a given line BC.

(1) With A as centre, scribe an arc having a radius long enough to cut the line AB in two places.
(2) With these two points as centre, draw two arcs of equal radius to cross at D.
(3) Join D back to A to form the perpendicular.

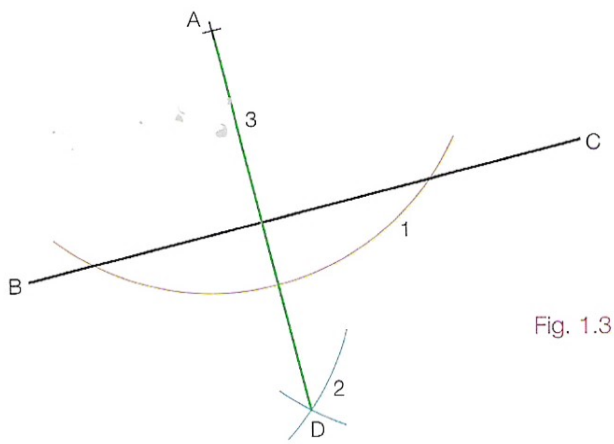


Fig. 1.3

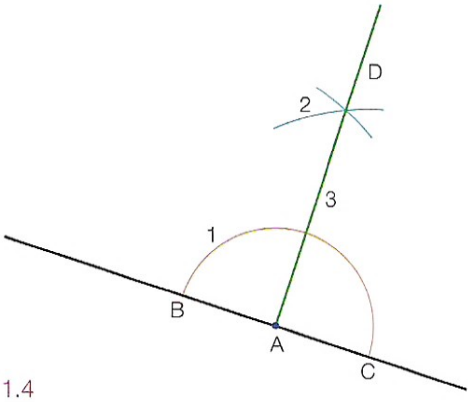


Fig. 1.4

To draw a perpendicular to a line from a point on the line.

(1) Draw a semicircle with A as centre, giving points B and C.
(2) Lengthen the compass and B and C as centres, scribe arcs to cross at point D.
(3) D joined back to A forms the perpendicular.

To divide a line into any number of equal parts.

(1) From one end of the line draw a line at any angle
(2) Using a compass, step down equal spaces on this line. In the diagram we wish to divide the line into four so therefore we step four equal spaces.
(3) Join the last division, point 4, to other end of line AB.
(4) Draw lines parallel to line B4 from points 3, 2 and 1. The line is now divided into four equal divisions.

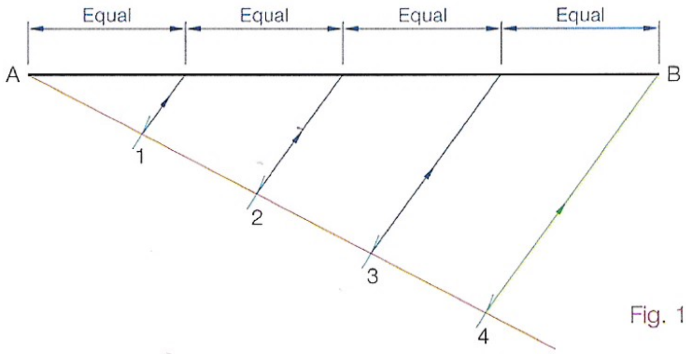


Fig. 1.5

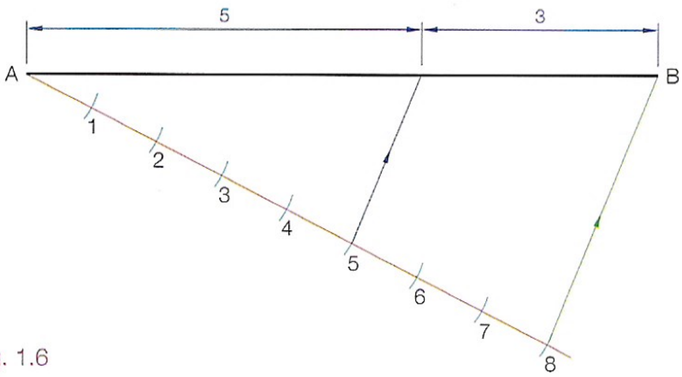


Fig. 1.6

To divide a line into a given ratio (e.g. 5:3).

(1) Add the numbers in the ratio 5 + 3 = 8.
(2) Set up as above for division of a line stepping eight equal steps.
(3) Join 8 to B.
(4) Draw from 5 parallel to B8 hence dividing AB into the required ratio.

To bisect an angle when we do not have the apex.

Method 1 (Fig. 1.7a)

(1) Draw a line parallel to AB and a set distance away.
(2) Draw a line parallel to CD and the same distance away.
(3) These lines intersect on the bisector at E.
(4) Repeat with a larger or smaller distance to find a second point on the bisector F.

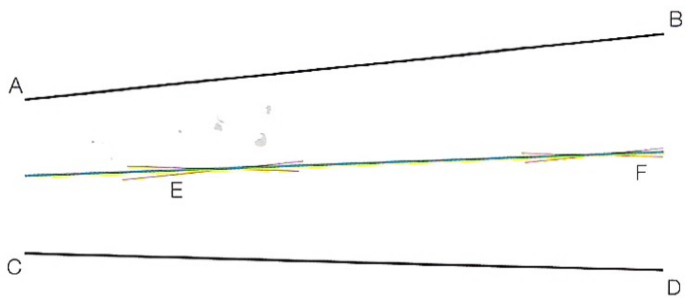


Fig. 1.7a

To divide a line into any number of equal parts.

Method 2 (Fig. 1.7b)

(1) Draw any line EF.

(2) Bisect all the angles to give G and H.

(3) G and H joined give the bisector.

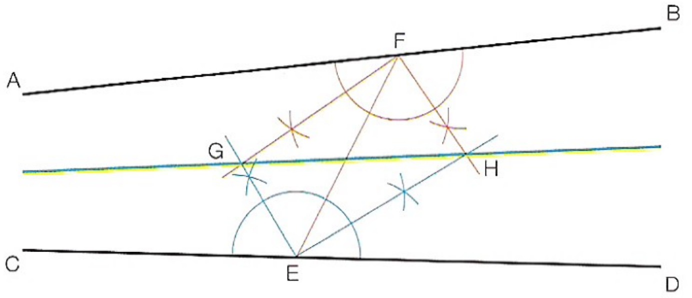


Fig. 1.7b

Triangles

Equilateral Triangle (Fig. 1.8)
All sides and angles equal

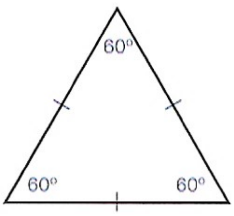


Fig. 1.8

Isosceles Triangle (Fig. 1.9)
Two sides and two angles equal

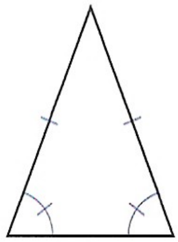


Fig. 1.9

Scalene Triangle (Fig. 1.10)
All sides and angles unequal

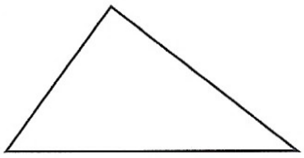


Fig. 1.10

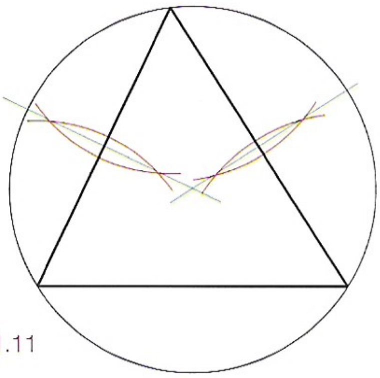


Fig. 1.11

To circumscribe a circle about a triangle (Fig. 1.11)

Bisect any two sides and extend the bisectors until they cross.
The point where they cross is the centre of the circle.

Medians and Centroid (Fig. 1.13)

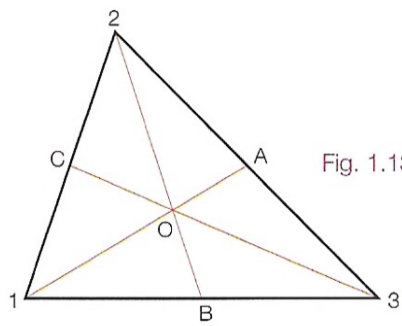


Fig. 1.13

If the midpoints of the sides of a triangle are joined back to the opposite vertex, then the lines produced are called the medians. The medians all cross at the one point called the centroid. The centroid, point O, divides each of the medians, 1A, 2B and 3C in the ratio 1:2.

To inscribe a circle in a triangle (Fig. 1.12)

Bisect any two angles.
Extend the bisectors to cross.
Where they cross is the centre of the circle.

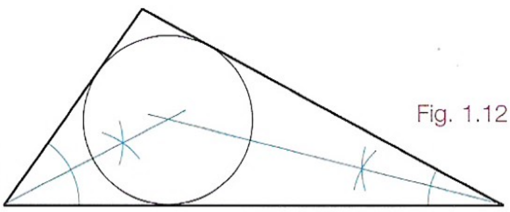


Fig. 1.12

The Circle

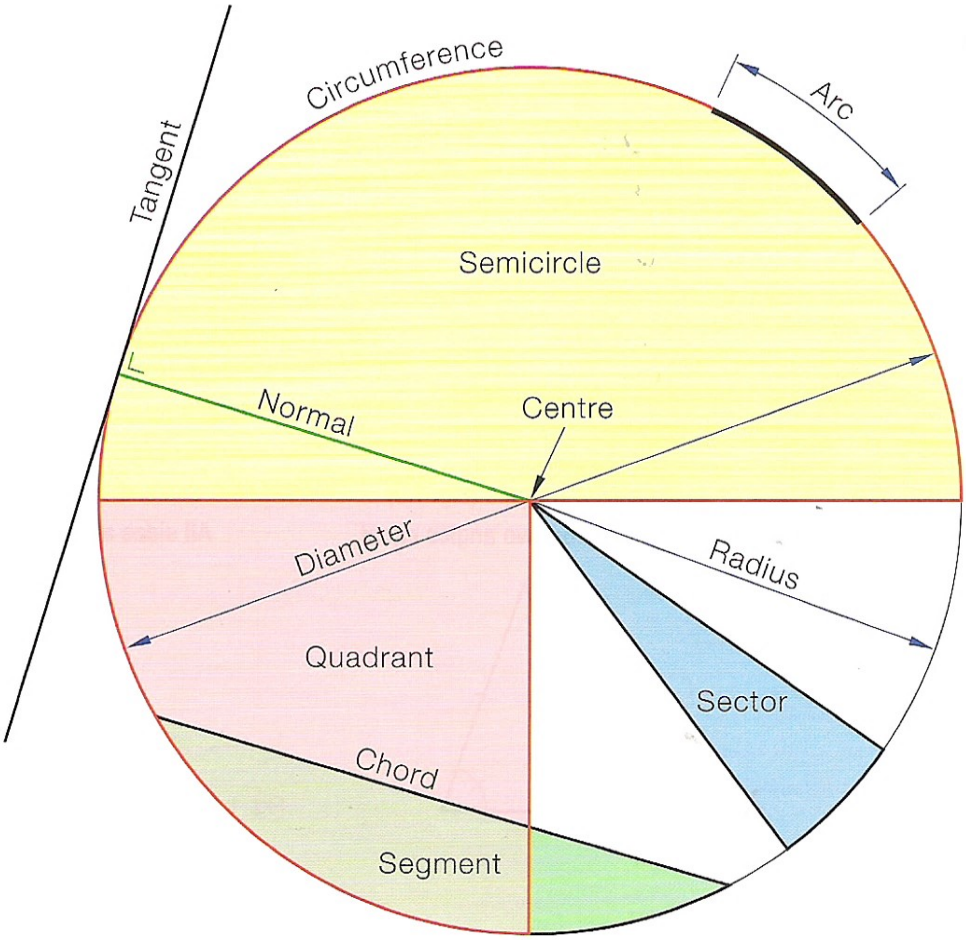


Fig. 1.14

Radius – Distance from the centre to the circumference.

Diameter – A straight line that passes through the centre touching the circumference at both ends. Symbol: ø

Chord – A straight line joining two points on the circumference.

Segment – The area enclosed between a chord and the circle circumference.

Sector – An area enclosed by two radii and the circumference.

Quadrant – A quadrant is a quarter of a circle.

Semicircle – Exactly half of a circle. The area between a diameter and a circumference.

Arc – A piece of the circumference.

Tangent – A straight line that touches the circle at one point, the point of contact.

Normal – A straight line perpendicular to a tangent drawn from the point of contact. The normal passes through the circle centre.

The line that bisects a chord will always pass through the centre point of the circle.

The line that bisects a chord will always pass through the centre point of the circle.
Therefore if you have two chords the centre of the circle can be found as shown in Fig. 1.15.

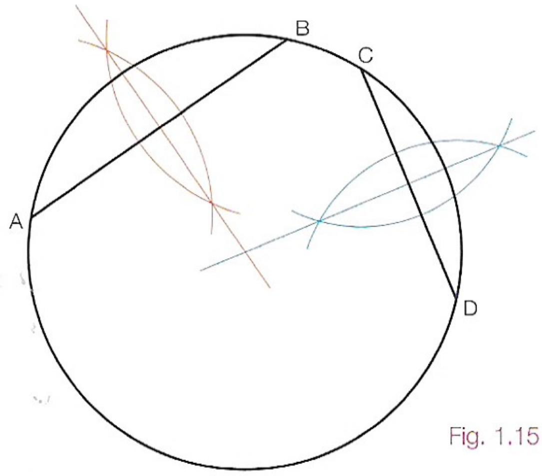


Fig. 1.15

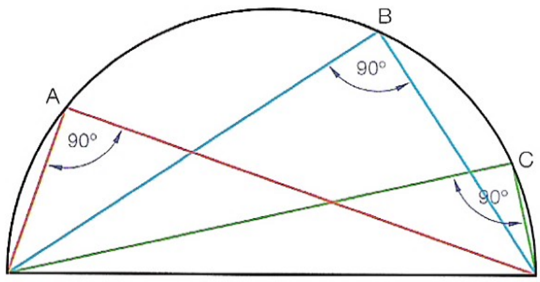


Fig. 1.16

The angle in a semicircle is always a right angle.

(1) Draw a semicircle.
(2) Choose any point A on the circumference.
(3) Join A back to the ends of the diameter.
(4) The enclosed angle is 90°.
Any point chosen on the circumference will give the same answer.

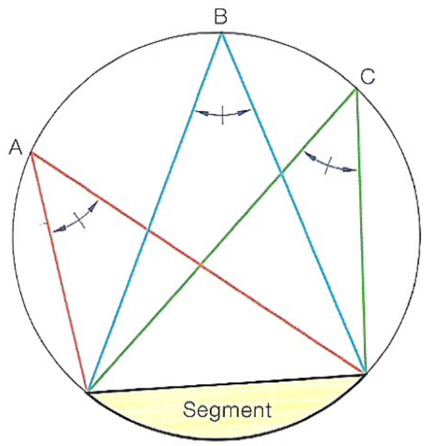


Fig. 1.17

Angles on the same segment of a circle are equal.

The angles at A, B and C will all be identical.

Note: If the segment is smaller than a semicircle, then the angle will be acute. When the segment equals a semicircle, the angle is a right angle as above. When the segment is greater than a semicircle, the angle is obtuse.

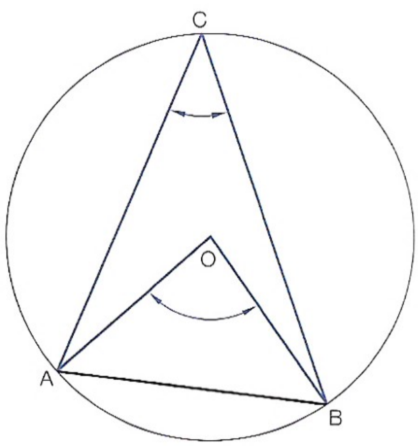


Fig. 1.18

The angle at the centre of a circle is twice that of the angle at the circumference standing on the same chord.

In the diagram we have the chord AB giving us the angle at the centre AOB and the angle at the circumference ACB.

AOB = 2 × ACB

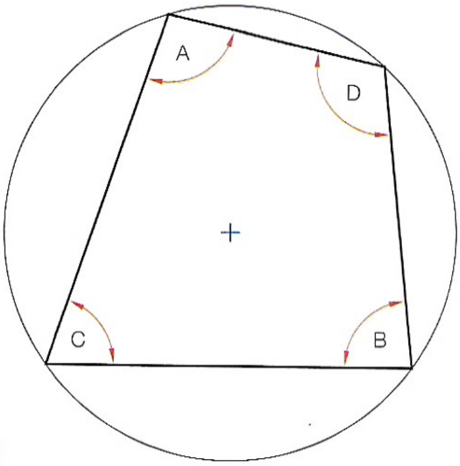


Fig. 1.19

Opposite angles of a cyclic quadrilateral add up to 180º.

A cyclic quadrilateral is one having all four corners on a circle.

Angle A + Angle B = 180°

Angle C + Angle D = 180°

For all quadrilaterals A + B + C + D = 360°

The angle between a tangent and a chord is equal to the angle in the opposite segment.

Angle A = Angle D

Proof: Angle C is a right angle because the angle in a semicircle is always 90°. Therefore Angle B + Angle D must also equal 90° because the sum of the angles in a triangle equal 180°.

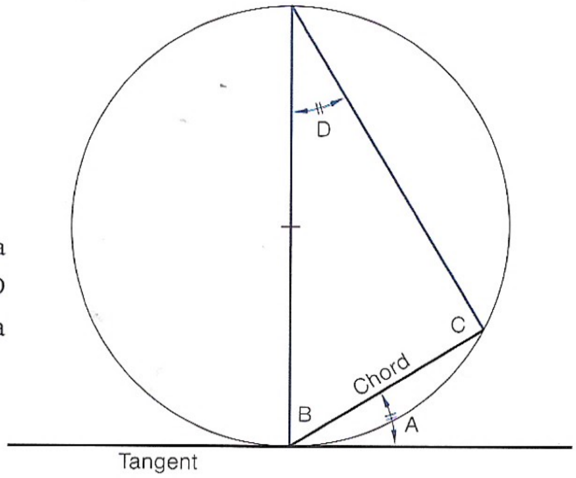
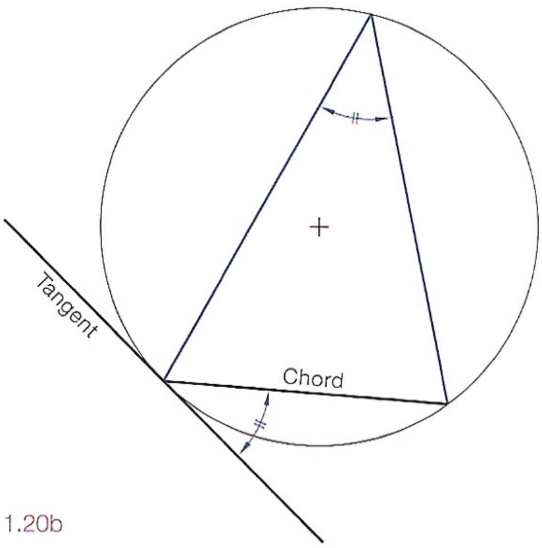


Fig. 1.20a



Angle B + Angle A also equal 90°.

Angle B + Angle D = Angle B + Angle A

Angle D = Angle A

Fig. 1.20b

Worked Problems

To construct a triangle given the base, the altitude and the top angle.

All triangles can be circumscribed by a circle. Bisect the base, treating it as a chord. The angle at the circumference is 80°, therefore the angle at the centre will be 160°.

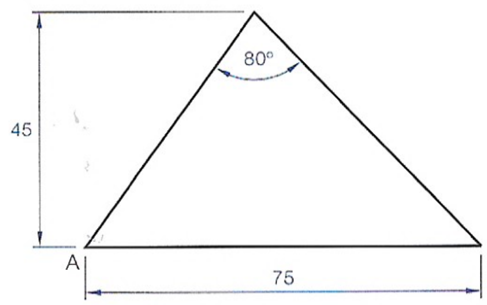


Fig. 1.21

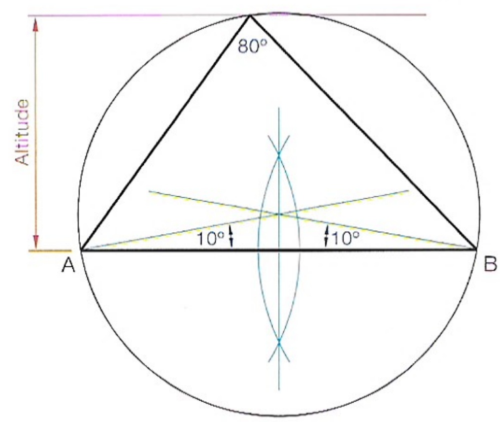


Fig. 1.22

If we create a 10° angle at A and another at B it will create 160° at the centre. Draw the circle. Draw the altitude line parallel to the base, giving two possible solutions for the third vertex of the triangles, Fig. 1.22.

To construct a triangle given the base, one side and the vertical (top) angle.

(1) Treat the base as a chord and bisect it to help find the centre of the circumscribing circle.
(2) Create the vertical angle at A thus forming a tangent.

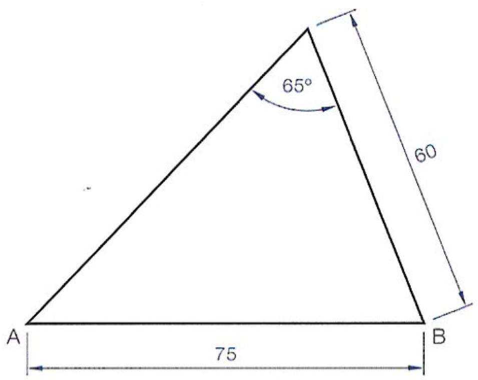


Fig. 1.23a

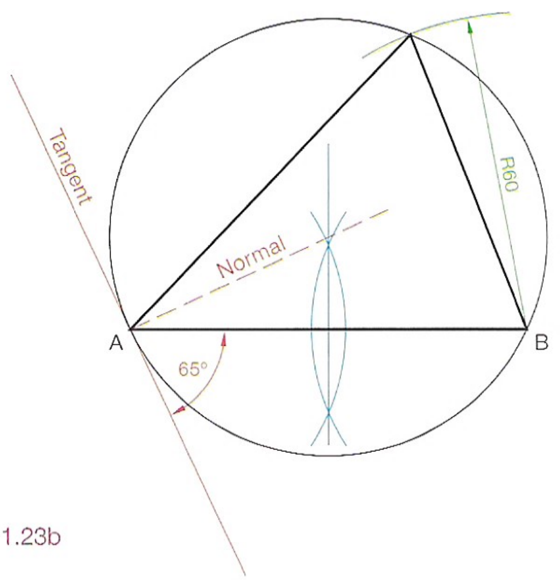


Fig. 1.23b

(3) Produce a normal to this tangent. The normal will pass through the centre of the circumscribing circle.
(4) Using where the normal and the bisector cross as centre, draw a circle to pass through A and B.
(5) The third corner is now located by swinging an arc of radius 60 mm from B to cross the circle, Fig. 1.23b.

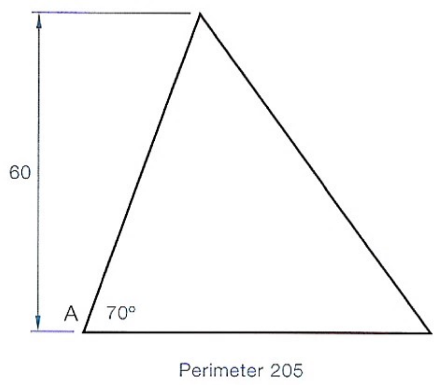


Fig. 1.24a

To construct a triangle given the perimeter, altitude and one base angle.

(1) Draw two parallel lines the altitude apart, i.e. 60 mm.
(2) Draw in the base angle.
This gives one side of the triangle, AB.
(3) Measure the length of AB and subtract it from the perimeter.
205 – 65 = 140
This is the length of the two remaining sides.

(4) Make AD equal to 140 mm.
(5) Join D to B and bisect.
(6) Extend the bisector to give point C.
ABC is the required triangle.

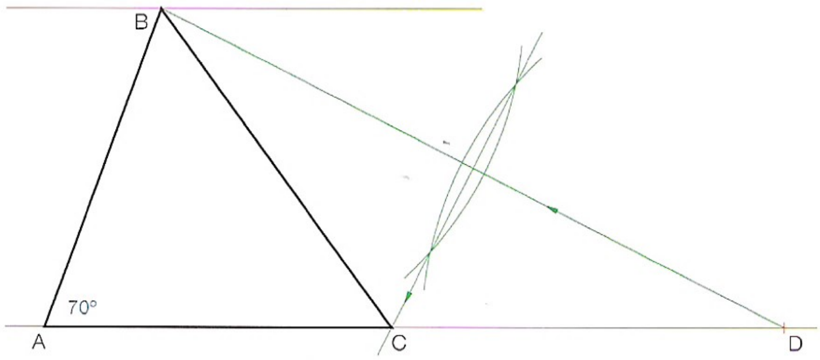


Fig. 1.24b

To construct a triangle given the perimeter and the ratio of the sides.
e.g. perimeter 220 mm, ratio of sides 5:3:4

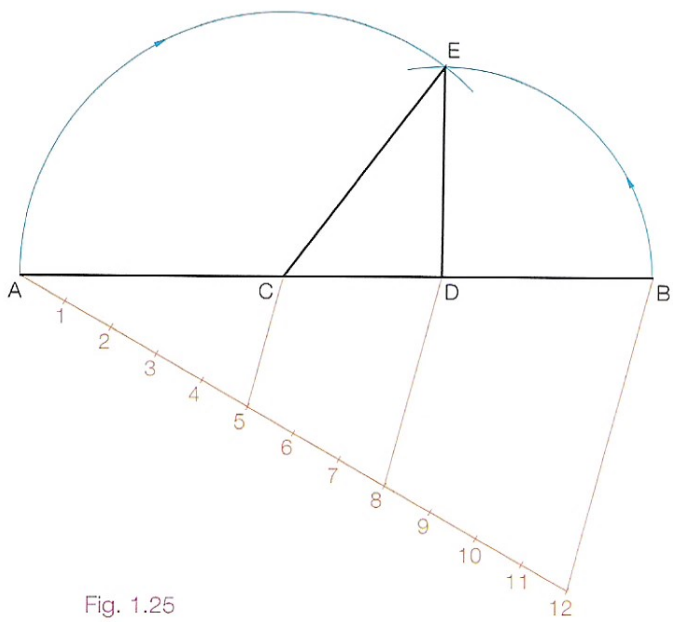


Fig. 1.25

(1) Draw a line AB equal to the length of the perimeter.
(2) Divide the line into the given ratio. AC, CD and DB are the lengths of the sides of the triangle.
(3) With C as centre and AC as radius, scribe an arc.
(4) With D as centre and DB as radius, scribe an arc.
(5) The arcs cross at E. CDE is the required triangle.

Activities

Q1. Draw a line AB, 120 mm long. Divide this line into 9 equal parts.

Q2. Draw a line AB, 120 mm long. Divide this line into 7 equal parts.

Q3. Draw a line AB, 120 mm long. Divide this line into the ratio 2:5:4.

Q4. Draw a line AB, 120 mm long. Divide this line into the ratio 3:5:5.

Q5. Given the line AB with a point P above it. Draw a perpendicular to AB from P using a compass only, Fig. 1.26.

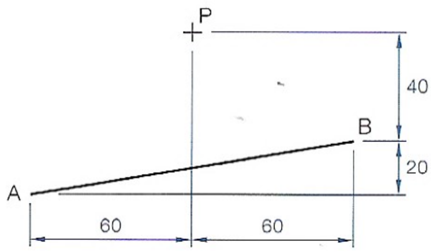


Fig. 1.26

Q6. Given a line AB and a point P on the line. Construct a perpendicular to AB from P using a compass, Fig. 1.27.

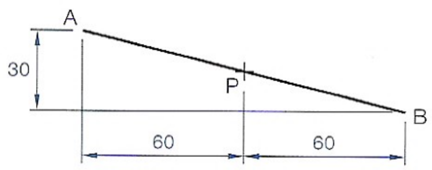


Fig. 1.27

Q7. Bisect the angle formed by the lines AB and CD, Fig. 1.28.

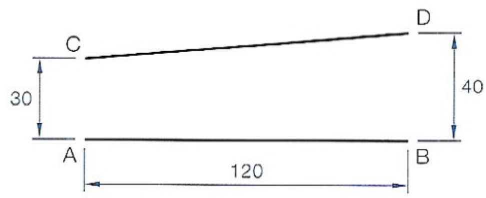


Fig. 1.28

Q8. Construct a triangle of sides 58 mm, 65 mm and 53 mm. Circumscribe a circle about this circle.

Q9. Construct a triangle of sides 95 mm, 45 mm and 75 mm. Inscribe a circle in this triangle.

Q10. Construct the triangle shown in Fig. 1.29. Find the centroid of this triangle.

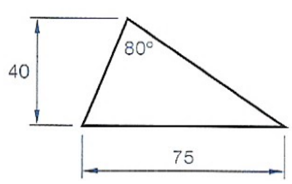


Fig. 1.29